ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 18 — 1990

Economic Theory and Structural Time Series Models for Aggregate Consumption

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ABSTRACT. – In this paper, we formulate a structural time series model for aggregate quarterly nondurable consumption by using the life cycle consumption hypothesis under uncertainty to obtain a model for the trend cycle component. The seasonal components are assumed to sum to a white noise. The model is analyzed using nondurable consumption expenditures per capita in constant prices for the Netherlands for the period 1967-1984. Special attention is paid to the implications of a structural change in labor income for the trend-cycle component of consumption. The model is found to be fairly well in agreement with the time series properties of consumption. An additional test based on the relationships between the income and consumption processes yields less favourable results and throws doubt on the appropriateness of the life cycle model.

Théorie économique et modèle structurels univariés de la consommation agrégée

RÉSUMÉ. – Partant de l'hypothèse du cycle de vie d'un consommateur, nous obtenons un modèle univarié pour la composante non saisonnière de la consommation agrégée. Les composantes saisonnières sont supposées s'agréger en un bruit blanc. Le processus ainsi obtenu pour la consommation non désaisonnalisée se présente sous la forme d'un modèle structurel à composantes non observables. Le modèle est estimé à partir de données trimestrielles sur la consommation par habitant de biens non durables aux Pays-Bas dans la période 1967-1984. Nous nous intéressons en particulier aux implications théoriques et empiriques d'un changement structurel dans le processus de la variable exogène revenu du travail pour le modèle du cycle de vie.

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1 Introduction

Autoregressive integrated moving average (ARIMA) processes put forward by Box and Jenkins [1971] provide a wide class of models for univariate time series forecasting and seasonal adjustment. In the Box-Jenkins approach, the main tools for specifying a suitable model are the autocorrelation (ACF) and partial autocorrelation function. More recently, structural time series models have been introduced and used in forecasting (see e.g. Harvey and Todd [1983], Gersch and Kitagawa [1983]) and in the decomposition of economic series into trend, cyclical and seasonal components (e.g. Engle [1978], Nerlove *et al.* [1979], Nelson and Plosser [1982], Harvey [1985] and Maravall [1985]). Structural time series models are formulated in terms of simple ARIMA schemes for the trend, cyclical, seasonal and irregular components of the series. The processes for the components are specified in such a way that the implied model for the series is in accordance with the sample information on autocorrelations and forecast function.

Although structural time series models (STMs) are derived from prior information, the implications from economic theory are usually not explicitly incorporated into the specification of STMs. In his important contribution to the life cycle consumption hypothesis, Hall [1978] has shown how a model for intertemporal decision making under uncertainty can be brought to bear on the serial correlation properties of a single economic time series. In this paper, we use the life cycle consumption hypothesis under uncertainty to obtain a model for the trend-cycle component of aggregate consumption.

The seasonal components are assumed to sum to a white noise. Given that the trend-cycle and seasonal components are assumed to be stochastic, no irregular component is included in the model. The model is estimated and analyzed using seasonally unadjusted quarterly data on nondurable consumption expenditures per capita in constant prices for the Netherlands for the period 1967-1984.

The aim of the paper is to incorporate the implications from economic theory jointly with information on the seasonals into a STM and to examine these implications by means of univariate time series procedures.

In section 2, we derive the implications of the life cycle hypothesis under uncertainty for the time series properties of consumption. We relate the univariate stochastic model for consumption to that of the underlying exogenous process for income. This allows us to use the results of the empirical analysis of the income series to check the appropriateness of the model for consumption. The results are obtained for the utility function with constant absolute risk aversion combined with normally distributed consumption.

Section 3 contains the empirical results which show that a STM incorporating the implications of the stochastic life cycle model is fairly well in agreement with the time series properties of the data. However, an examination of the relationships between the consumption and income processes is less confirmative in this respect.

Finally, section 4 is devoted to concluding remarks.

STMs can be estimated by state space methods. Instead, we apply the method of asymptotic least squares (see e. g. GOURIÉROUX *et al.* [1985]) to get efficient parameter estimates, standard errors and test statistics for the restrictions implied by the life cycle model. This computationally convenient method is briefly outlined in an appendix.

2 The Stochastic Implications of the Life Cycle Hypothesis

The life cycle consumption hypothesis was put forward by MODIGLIANI and BRUMBERG [1955] some thirty years ago. Among the multitude of articles on life cycle consumer behavior, it is worth to mention HALL [1978]. He formulates the life cycle hypothesis as an intertemporal decision problem under uncertainty and he shows that the first order conditions for the intertemporal optimization have straightforward implications for the serial correlation properties of time series data on consumption. In particular, his model implies that the marginal utility of consumption is generated by a first order autoregressive process. Recently, the life cycle hypothesis has been studied by several authors, see e. g. BILSON [1980], FLAVIN [1981], HANSEN and SINGLETON [1982, 1983], MIRON [1986], MUELLBAUER [1983], WICKENS and MOLANA [1983] and DEATON [1987] for a survey. CHARPIN [1987] has shown that for the utility function with constant absolute risk aversion, when labor income is Gaussian, consumption is also normally distributed (see WINDER [1988] for an alternative derivation of this result).

To make some of the considerations explicit, we assume that at each time period t, the expectation of an additive intertemporal von Neumann-Morgenstern utility function U is maximized,

$$\text{(1)} \qquad E_t \sum_{i=0}^{T-t} \beta^i U(c_{t+i}),$$

with $U' > 0$, $U'' < 0$, where c_{t+i} denotes consumption, U' and U'' are the first and second derivatives of U with respect to c. T denotes the life-time and β is a time preference parameter, $0 < \beta < 1$. The period to period budget constraint is given by

$$\text{(2)} \qquad c_{t+i} = -a_{t+i} + (1+r)\,a_{t+i-1} + y_{t+i},$$

where a_{t+i} and y_{t+i} denote real assets and real labor income respectively and r is the real interest rate, which is assumed to be constant.

The first order conditions for a maximum of (1) correspond to Hall's result that the marginal utility U' follows an AR (1)-process

$$(3) \qquad \mathrm{E}_t \mathrm{U}'(c_{t+i}) = \beta(1+r)\,\mathrm{E}_t \mathrm{U}'(c_{t+i+1}), \qquad i=0, \ldots, \mathrm{T}-t-1,$$

and $a_\mathrm{T}=0$. Assuming that $a_\mathrm{T}=0$ is sensible if a_T denotes real assets after deduction of the bequest.

Notice that we do not assume stationarity of the income process. We only require the existence of the conditional moments appearing in the life cycle model. The choice of the utility function leads to specific requirements for the income process. For instance, for a quadratic utility function, solving the Euler equations only requires the existence of the conditional moments $\mathrm{E}_t y_{t+i}$, that is we have certainty equivalence.

To arrive at an operational model, it is necessary to choose a specific functional form for the utility function. In this paper we take the utility function with constant absolute risk aversion parameter $\gamma > 0$, $\mathrm{U}(c) = -\gamma^{-1}\exp(-\gamma c)$. Without additional assumptions it is in general impossible to find an explicit solution for c_t. Therefore, we assume that $(c_{t+1}, c_{t+2}, \ldots, c_\mathrm{T})$ conditional on the information available at time t is normally distributed. The Euler equations in (3)

$$\mathrm{E}_t \exp(-\gamma c_{t+i}) = [\beta(1+r)]^{-i}\exp(-\gamma c_t), \qquad i=1, \ldots, \mathrm{T}-t$$

can be expressed as

$$(4) \qquad \mathrm{E}_t c_{t+i} = c_t + i\delta + \frac{1}{2}\gamma\sigma_t^2(c_{t+i}),$$

where we define $\delta = \gamma^{-1}\ln[\beta(1+r)]$.

With the consumption innovation $\varepsilon_{t+1} = c_{t+1} - \mathrm{E}_t(c_{t+1})$, (4) yields for $i=1$

$$(5) \qquad \Delta c_{t+1} = \delta + \frac{1}{2}\gamma\sigma_t^2(c_{t+1}) + \varepsilon_{t+1}.$$

Hence, consumption follows a random walk with drift.

In Winder and Palm [1989] it is shown that an analysis of the Euler equations is only appropriate when no unanticipated change in the income process occurs. In order to trace the effects of structural changes in the income process it is necessary to derive the closed form solution of the maximization problem faced by the consumer. In appendix A we express c_t as a function of characteristics of the income process and parameters of the maximization problem only. When we assume that the change in labor income is generated by an ARMA-process, the implications with respect to structural changes are as follows:

(1) A change in the constant term leads to a step change in the consumption model (5).

(2) A change in the variance of the income innovation implies besides a step change, a persistent change in both the variance and the drift of the random walk process (5).

(3) A change in one of the ARMA-parameters will have a persistent effect on both the drift of the random walk process and the variance of the consumption innovation. In that case, we also have a step change in the consumption model (5).

The derivation opens up the possibility to express the consumption innovation ε_{t+1} in terms of characteristics of the income process. More specifically, in appendix A it is shown that the consumption innovation is a linear transformation of the income innovation. This implication and hence the relationship between the variances of both innovations offers, besides the random walk specification, an additional opportunity to test the model. In the next section we will carry out a test of the above life cycle model.

3 Empirical Evidence

Our concerns will now be to analyze the implications of the life cycle theory for the time series properties of seasonally unadjusted quarterly real per capita consumption of nondurable in the Netherlands for the period 1967.I-1984.IV.

We relate the univariate model for consumption explicitly to the model generating labor income. This allows us to remedy the LUCAS [1976] critique by taking into account the impact of a structural change in the income process for the stochastic properties of consumption. Finally, we jointly model the seasonal and other components of consumption.

A short description of the series is given in appendix B. Per capita expenditures on nondurables have been deflated by the price index for nondurable consumption. The base year is 1980. We like to point out that the budget share of nondurable consumption has been fairly stable in the Netherlands during the sample period, so that it seems justified in first instance to limit the analysis to nondurables. An alternative would be to include the services of durables into the series. This would require additional assumptions on the relationship between the size of these services and the stock of durables. The consumption and income series used in the empirical analysis are given in figure 1. The income series is seasonally adjusted. Unfortunately, we do not have seasonally unadjusted data on labor income at our disposal. Since we want to carry out an integrated analysis of consumption and income, we used the same income series as in WINDER and PALM [1989]. Although the choice of an adjusted income series may be questioned, the use of it is not prohibitive since the rational consumer is capable to anticipate on the seasonal fluctuations in income when making his consumption decision. The income series covers the

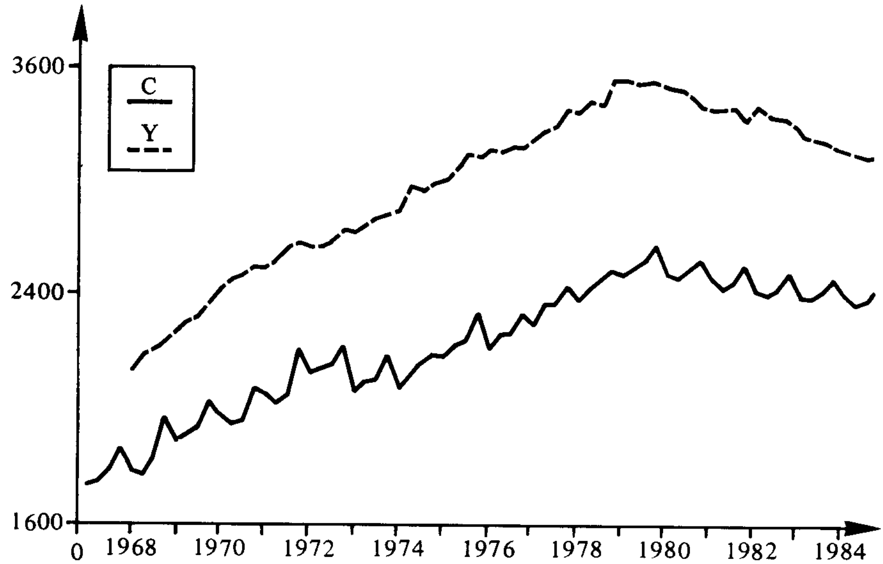

FIGURE 1

Nondurable consumption and labor income (including transfers) per capita in the Netherlands, expressed in Dutch guilders in prices of 1980, quarterly seasonally unadjusted consumption figures, 1967.I-1984.IV.

period 1968.I-1984.IV. From the figure, it is obvious that income and consumption start to decrease in 1979 and 1980 respectively. The fall of income leads that of consumption. In terms of the life cycle model, a negative drift parameter of the univariate model for consumption (5) can only occur when the time preference parameter β is smaller than $(1+r)^{-1}$. It is sensible to assume that $\beta(1+r)$ has been roughly constant until 1979 after which it decreased due to a drop of β as a result of a policy change aiming at a drastic reduction of public deficits or a drop of the real interest rate (which indeed happened in the Netherlands at the end of the seventies).

Alternative explanations are that the risk aversion γ or $\sigma_t^2(c_{t+1})$ decreased (provided $\beta < (1+r)^{-1}$) or that the consumer interprets a structural break in the income process as a large negative income innovation. In the subsequent analysis, we assume that the time preference parameter β has changed in 1980. Using the closed form solution given in appendix A, it can easily be shown that the decrease in β implies, besides the permanent drop in the drift of the random walk process, a positive step change in the consumption level (see WINDER and PALM [1989] for a formal derivation of this result in a similar context).

As the consumption data are seasonally unadjusted, we model the seasonality jointly with the dynamics of the life cycle theory. More specifically, we assume that consumption c_t consists of two independent unobserved components $c_t = \tilde{c}_t + s_t$. Its nonseasonal component $\tilde{c}_t$ is explained by the life cycle model. The seasonal component s_t is assumed to be such that the sum over four subsequent quarters is white noise.

Formally, we have

$$\text{(6)} \qquad \psi(L)\,s_t = \mu_t, \qquad \psi(L) = 1 + L + L^2 + L^3, \qquad \mu_t \sim \text{IIN}(0, \sigma_\mu^2).$$

When the "trend-cycle" component $\tilde{c}_t$ is generated by the life cycle model (4), we get for the annual change of c_t ($\Delta_4 = 1 - L^4$)

$$\text{(7)} \qquad \Delta_4 c_t = \psi(L)\,\tilde{\delta} + \psi(L)\,\varepsilon_t + \Delta\mu_t,$$

with $\psi(L)\,\tilde{\delta} = \psi(L)\,\delta + \frac{1}{2}\gamma \sum_{i=0}^{3} \sigma_{t-1-i}^2 (\tilde{c}_{t-i})$.

According to (7), the annual change of c_t is generated by a restricted third order MA-process with mean $\psi(L)\,\tilde{\delta}$. When we assume that the mean of $\Delta\tilde{c}_t$ has changed in 1980, we introduce a dummy variable for the period 1980-1984.

Before estimating the model (7) for consumption, we will analyze the income series. Figure 1 suggests that the slope of the income series has changed in 1970.IV and in 1978.IV. Winder and Palm [1989] specify the following model for income

$$\text{(8)} \qquad \underset{}{\Delta y_t} = \underset{(7.81)}{40.46\, d_{1t}} + \underset{(8.56)}{25.19\, d_{2t}} - \underset{(3.81)}{13.01\, d_{3t}} + \nu_t - \underset{(3.72)}{.428\ \nu_{t-1}}$$

with $\sigma_\nu^2 = 809.6$, where d_{it} is a dummy variable having the value 1 in subperiod i, and 0 otherwise and t-ratio's are reported between parentheses. The three sample subperiods are respectively 1968.II-1970.IV, 1971.I-1978.IV and 1979.I-1984.IV. The slope coefficients of (8) are significantly different from each other. A Lagrange Multiplier test of the null hypothesis that ν_t in (8) has a constant variance against the alternative hypothesis that the disturbance has an ARCH-structure of order $p = 1$ or 4 respectively, yields insignificant values. From this and visual inspection of the income series we infer that the income process is homoscedastic. Under the assumption that the changes in the constant term of (8) were unanticipated, the occurrence of breaks implies only step changes in the consumption level. These can be taken account of by including dummy variables in the random walk model. Together with the implications of the change in the time preference parameter (assumed to have arisen in 1979.IV), we have

$$\text{(9)} \qquad \Delta\tilde{c}_t = \beta_1 d_{1t} + \beta_2 d_{2t} + \beta_3 d_{3t} + \beta_4 d_{4t} + \beta_5 d_{5t} + \varepsilon_t$$

with $d_{1t} = 1$ in the period 1967.II-1979.IV, $d_{2t} = 1$ in 1980.I-1984.IV, $d_{3t} = 1$ in 1971.I, $d_{4t} = 1$ in 1979.I and $d_{5t} = 1$ in 1979.IV (zero otherwise). The coefficients β_1 and β_5 are positive, β_2, β_3 and β_4 are negative. For a derivation of this result in a similar context, we refer to Winder and Palm

(1989). For observed consumption, we therefore get after substitution

$$(10) \quad \Delta_4 c_t = \beta_1 \tilde{d}_{1\,t} + \beta_2 \tilde{d}_{2\,t} + \beta_3 \tilde{d}_{3\,t} + \beta_4 \tilde{d}_{4\,t} + \beta_5 \tilde{d}_{5\,t} + \psi(L)\,\varepsilon_t + \Delta\mu_t$$

with $\tilde{d}_{1\,t} = 4$ in 1968.I-1979.IV

3 in 1980.I

2 in 1980.II

1 in 1980.III

$\tilde{d}_{2\,t} = 1$ in 1980.I

2 in 1980.II

3 in 1980.III

4 in 1980.IV-1984.IV

$\tilde{d}_{3\,t} = 1$ in 1971.I-1971.IV

$\tilde{d}_{4\,t} = 1$ in 1979.I-1979.IV

$\tilde{d}_{5\,t} = 1$ in 1979.IV-1980.III.

The disturbance of model (10) has an error-component structure which can be expressed as a (restricted) third order MA-process – say $\psi(L)\,\varepsilon_t + \Delta\mu_t = (1 - \varphi_1 L - \varphi_2 L^2 - \varphi_3 L^3)\,\xi_t$ with $\xi_t \sim IN(0, \sigma_\xi^2)$.

Notice that even when μ_t is autocorrelated up to order two, i.e. μ_t is generated by an MA(2)-process, the disturbance of $\Delta_4 c_t$ in (10) can still be represented as an MA(3)-process, though its error component structure is then different from that assumed above.

It is interesting to note that when we assume that the utility U depends on $c_t^* = \psi(L)\,c_t$, the rational habit formation approach proposed by MUELLBAUER [1988] implies that $\Delta_4 c_t$ is white noise. The mean of $\Delta_4 c_t$ also shifts as a result of a structural change in the income process. For the details, we refer to WINDER [1988].

Estimates of the unrestricted model have been obtained by Maximum Likelihood (ML) method. Fully efficient estimates of the restricted model (10) have subsequently been obtained by the method of asymptotic least squares (ALS) based on the ML estimates. The method of ALS is briefly outlined in appendix C. For more details, we refer to GOURIÉROUX *et al.* [1985] and KODDE *et al.* [1989]. Results for model (10) are reported in table 1. Asymptotic *t*-ratio's are given in parentheses. The Wald statistic which tests the 2 restrictions implied by the error-component structure of (10) is reported. The Box-Pierce (BP) and the Ljung-Box (LB) statistics based on *p* residual serial correlations are given for several values of *p*.

Several comments are in order about the results in table 1. In terms of residual serial correlation, the models behave fairly well. The restrictions implied by the error components are not rejected at conventional significance levels. The variance of the trend-cycle component is highly significant. For the seasonals, the variance is not significant suggesting that a deterministic specification for the seasonals might be in accordance with the sample information. We like to notice however that the model with seasonal dummy variables only performs badly in terms of diagnostic

TABLE 1

Univariate Time Series Model for Consumption, 1967.I-1984.IV.

	$\Delta_4 c_t$			
	ML (unrestricted)		ALS	
Coefficients:				
β_1	16.4	(4.88)	16.2	(4.80)
β_2	−8.6	(1.60)	−8.6	(1.58)
β_3	61.1	(1.97)	62.5	(2.06)
β_4	40.7	(1.34)	43.9	(1.46)
β_5	−16.4	(.54)	−18.2	(.46)
φ_1	−.76	(6.81)	–	–
φ_2	−.48	(3.51)	–	–
φ_3	−.52	(4.59)	–	–
σ^2_ξ	1,210.5	–	–	–
σ^2_ε	–	–	676.9	(4.59)
σ^2_μ	–	–	16.1	(1.21)
Wald test	4.15			
BP/LB test:				
$p=4$	.38	.40	–	–
$p=8$	3.50	3.65	–	–
$p=12$	4.84	5.06	–	–
$p=16$	15.27	15.95	–	–

tests and parameter estimates. The estimates of β_1 and β_2 have the expected sign. This is not the case for β_3, β_4 and β_5, although with the exception of β_1 and β_3 none of the parameters has a t-value larger than 2. The t-value for the hypothesis $H_0: \beta_1 = \beta_2$ is 3.878. These findings suggest that the change in the slope of consumption as a result of the change in the preferences has to be accounted for in the model. The impact of structural changes in the income process for consumption seems to be less important empirically.

Using the results of appendix A, it can be shown that

$$\varepsilon_t = (1 - \theta + \theta \eta_{T-t-1}^{-1})\, \nu_t,$$

with $\eta_{T-t-1} = \sum_{i=0}^{T-t-1} (1+r)^{-i}$, θ is the MA-parameter of (8) and ν_t and ε_t are the income and (trend-component) consumption innovation respectively. Since $\hat{\theta} = .428$ and $0 < \eta_{T-t-1}^{-1} < 1$, we have as an implication of the theoretical model that $\sigma^2(\varepsilon_t) < \sigma^2(\nu_t)$. A comparison of the estimates shows that this restriction is satisfied by the point estimates. Using the point estimates of $\sigma^2(\varepsilon_t)$, $\sigma^2(\nu_t)$ and θ, we find for η_{T-t-1} the value 1.250. Since the quarterly real interest rate r should be rather small, we can approximate $T-t$, that is the remaining life time of the representative consumer in period t, after manipulating the geometric sum η_{T-t-1} by $1.250(1+r)^{-1}$. As in WINDER and PALM [1989] we find for reasonable values of r very small values for $T-t$. This finding casts some doubts on the model, in particular on a strict microeconomic interpretation of the estimates from aggregate data.

Although the life cycle model together with the assumed process of the seasonal components provides a satisfactory description of the serial correlation properties of the data on consumption, not all results from the analysis of the relationships between the income and consumption processes implied by the theoretical model are realistic. In this respect the size of the variance of the consumption innovation relative to that of the income innovation leads to the conclusion that consumption is not smooth enough.

4 Concluding Remarks

In this paper, we used the life cycle consumption hypothesis under uncertainty to obtain a fully specified stochastic model for the trend-cycle component of aggregate nondurable quarterly consumption in the Netherlands. We also expressed the innovation of the trend-cycle component of consumption in terms of the income innovation and we showed what – in the light of the LUCAS [1976] critique – the implications of structural changes in the income process are for consumption.

In the empirical part, seasonality is jointly modeled with the life cycle hypothesis. The highly parsimonious model implied by the life cycle consumption theory and the stochastic specification for the seasonals in consumption lead to a simple STM which was found to be fairly well in agreement with the serial correlation properties of the data. We also examined the interrelationships between the income and consumption processes implied by the theoretical model. The implications of structural changes in the income process were not found to be significant. A comparison of the variances of the consumption and income innovations showed that although the empirical results satisfy the implication that consumption should be smoother than income, the size of the variances led to the conclusion that consumption is not smooth enough. These findings throw doubts on the appropriateness of the life cycle model to explain our data set.

Our analysis shows how information from dynamic optimization models under uncertainty can be incorporated into univariate STM's with the aim to test the theory using univariate time series methods. Theory can also be used in STM's to improve forecasts or to improve the extraction of signals from time series. The method of ALS was found to be very useful in estimating and testing STM's with nonlinear restrictions on the parameters. The analysis can be extended in several ways. Under the assumption of stochastic interest rates, the life cycle hypothesis generates restrictions between the stochastic processes for consumption and interest rates. In line with the work of HANSEN and SINGLETON [1982, 1983], these restrictions can be analyzed using a bivariate model. The extremely small value of the remaining life time found in the empirical analysis suggests that a model of intertemporal optimization with a planning time span shorter than the life time should be more appropriate. WINDER and PALM [1989] develop such a model. A model with moving planning horizon, that implies a consumption function with an error correction mechanism, yields satisfactory empirical results for seasonally adjusted data.

The Relationship between Income and Consumption in the Stochastic Life Cycle Model

1. *The marginal process of consumption*

In section 2 it was shown that the life cycle model with the utility function implying constant absolute risk aversion and the additional assumption of normality of consumption, conditionally on the past, yields

$$\text{(11)} \qquad E_t(c_{t+i}) = c_t + i\delta + \frac{1}{2}\gamma\sigma_t^2(c_{t+i}), \qquad i = 1, \ldots, T-t.$$

We note that for the exponential utility function Charpin [1987] derives a closed form solution for consumption and shows that the normality of c_t is implied by that of labor income.

Similarly for period $t+1$, the Euler equations are

$$\text{(12)} \qquad E_{t+1} c_{t+i} = c_{t+1} + (i-1)\delta + \frac{1}{2}\gamma\sigma_{t+1}^2(c_{t+i}), \quad i = 2, \ldots, T-t.$$

As a result of conditional normality of $(c_{t+1}, \ldots, c_T)$, $\sigma_{t+1}^2(c_{t+i})$ is independent of c_{t+1}. Therefore (12) leads to

$$\text{(13)} \qquad E_t c_{t+i} = E_t c_{t+1} + (i-1)\delta + \frac{1}{2}\gamma\sigma_{t+1}^2(c_{t+i}).$$

From (11) we get an expression for $E_t c_{t+1}$, which after substitution into (13) yields

$$\text{(14)} \qquad E_t c_{t+i} = c_t + i\delta + \frac{1}{2}\gamma[\sigma_{t+1}^2(c_{t+i}) + \sigma_t^2(c_{t+1})], \quad i = 2, \ldots, T-t.$$

By comparing (14) with (11), we have

$$\text{(15)} \qquad \sigma_t^2(c_{t+i}) = \sigma_{t+1}^2(c_{t+i}) + \sigma_t^2(c_{t+1})$$

for $i = 2, \ldots, T-t$. Along the same lines, a more general result can be derived

$$\text{(16)} \qquad \sigma_t^2(c_{t+i}) = \sigma_{t+j}^2(c_{t+i}) + \sigma_t^2(c_{t+j}),$$

$i = j+1, \ldots, T-t$ with $j = 0, \ldots, T-t-1$. It follows that the covariance matrix of $(c_{t+1}, \ldots, c_T)$ conditional on information at time t is subject to the restrictions

$$\text{(17)} \qquad \sigma_t^2(c_{t+i}, c_{t+j}) = \sigma_t^2(c_{t+i}), \qquad i > j.$$

From (17) it is obvious that the conditional process of c_t (or Δc_t) cannot be stationary. Aggregation of consumption across consumers with a different expected lifetime could possibly induce stationarity of aggregate consumption.

Now we shall express the characteristics of the process of c_t in terms of those of the distribution of labor income.

If we substitute (11) into the expected value of the lifetime budget constraint

$$(18) \qquad \sum_{i=0}^{T-t} (1+r)^{-i} E_t c_{t+i} = (1+r) a_{t-1} + \sum_{i=0}^{T-t} (1+r)^{-i} E_t y_{t+i},$$

we obtain the consumption function relating consumption to the expected value of life time wealth

$$(19) \quad c_t \eta_{T-t} + \delta \sum_{i=1}^{T-t} i(1+r)^{-i} + \frac{1}{2} \gamma \sum_{i=1}^{T-t} (1+r)^{-i} \sigma_t^2 (c_{t+i})$$
$$= (1+r) a_{t-1} + y_t + \sum_{i=1}^{T-t} (1+r)^{-i} E_t y_{t+i},$$

where $\eta_K = \sum_{i=0}^{K} (1+r)^{-i}$. FRIEDMAN'S [1957] permanent income hypothesis arises as a special case of (19) when the "constant" term on the left-hand side (l.h.s.) is zero. Along the lines of deriving (19) we get for period $t+1$

$$(20) \quad c_{t+1} \eta_{T-t-1} + \delta \sum_{i=1}^{T-t-1} i(1+r)^{-i}$$
$$+ \frac{1}{2} \gamma \sum_{i=2}^{T-t} (1+r)^{1-i} \sigma_{t+1}^2 (c_{t+i})$$
$$= (1+r) a_t + y_{t+1} + \sum_{i=2}^{T-t} (1+r)^{1-i} E_{t+1} y_{t+i}.$$

Since the conditional variances $\sigma_{t+1}^2 (c_{t+i})$ do not depend on c_{t+1}, the variance of consumption expressed in terms of the moments of the income process becomes

$$(21) \qquad \sigma_t^2 (c_{t+1}) = \eta_{T-t-1}^{-2} \sigma_t^2 \left[y_{t+1} + \sum_{i=2}^{T-t} (1+r)^{1-i} E_{t+1} y_{t+i} \right].$$

To express the consumption innovation ε_{t+1} defined as $\varepsilon_{t+1} = c_{t+1} - E_t c_{t+1}$ in terms of the characteristics of the income process, substitute $(1+r) a_{t-1} + y_t = a_t + c_t$ into (19), premultiply the result by $(1+r)$

and substract it from (20) to yield

$$(22)\quad c_{t+1}-c_t=\delta+\varepsilon_{t+1}-\frac{1}{2}\gamma\eta_{T-t-1}^{-1}\left\{-\sigma_t^2(c_{t+1})+\sum_{i=2}^{T-t}(1+r)^{1-i}[\sigma_{t+1}^2(c_{t+i})-\sigma_t^2(c_{t+i})]\right\},$$

where

$$(23)\quad \varepsilon_{t+1}=\eta_{T-t-1}^{-1}\left[y_{t+1}-E_t y_{t+1}+\sum_{i=2}^{T-t}(1+r)^{1-i}(E_{t+1}y_{t+i}-E_t y_{t+i})\right].$$

Substituting (15) into (22) for $\sigma_{t+1}^2(c_{t+i})$, we get

$$(24)\quad c_{t+1}-c_t=\delta+\frac{1}{2}\gamma\sigma_t^2(c_{t+1})+\varepsilon_{t+1}.$$

To discuss the implications of the above for empirical work, we assume that the change of labor income is generated by a stationary process with moving average representation

$$(25)\quad \begin{aligned} &y_{t+1}=y_t+\gamma^*+\sum_{i=0}^{\infty}\psi_i v_{t+1-i}, \qquad \psi_0=1,\\ &\sum_{i=0}^{\infty}\psi_i^2<\infty, \qquad v_t\sim \mathrm{IIN}(0,\sigma_v^2), \end{aligned}$$

where γ^* is the drift parameter. Then the innovation and the variance of consumption in (23) and (21) become

$$(26)\quad \varepsilon_{t+1}=\alpha_{T-t-1}v_{t+1} \quad \text{and} \quad \sigma_t^2(c_{t+1})=\alpha_{T-t-1}^2\sigma_v^2 \text{ respectively,}$$

where $\alpha_K=\eta_K^{-1}\sum_{i=0}^{K}(\psi_0+\psi_1+\ldots\psi_i)(1+r)^{-i}$. The consumption innovation is a linear transformation of the income innovation. Contrary to a remark by MUELLBAUER [1983] that the variance of the change in the consumption should be smaller than the variance of the innovation of the income process, from (26), it results that the variance of Δc_{t+1} can be smaller as well as larger than the variance of Δy_{t+1}. Moreover, as already pointed out above, the process for Δc_{t+1} is not stationary. The drift parameter in (24) depends on $\sigma_t^2(c_{t+1})$ which depends on the remaining life time $T-t$.

Using (16), $\sigma_t^2(c_{t+i})$ can be expressed in terms of parameters of the income process only. It is now possible to determine the level of consumption c_t from (19), provided the first and second moments of the income process, actual income y_t and a_{t-1} are given.

2. *The consequences of a structural shift in the income process*

We assume, for reasons of simplicity that income follows a random walk, that is $\psi_i=0$, $i>0$ in (25). The results can easily be generalized to the case of an arbitrary ARIMA-process. Then,

$$\text{(27)} \qquad \begin{cases} \mathrm{E}_{t+j}y_{t+i} - \mathrm{E}_{t+j-1}y_{t+i} = \nu_{t+j}, & i > j, \\ \mathrm{E}_t y_{t+i} = i\gamma^* + y_t, & \end{cases}$$

and from (26) it follows that $\varepsilon_{t+1} = \nu_{t+1}$ and

$$\text{(28)} \qquad \sigma_t^2(c_{t+i}) = i\,\sigma_\nu^2.$$

If in period $t+1$, γ^* and/or σ_ν^2 unexpectedly change to become $\bar{\gamma}^*$ and $\bar{\sigma}_\nu^2$ respectively, we have that $\mathrm{E}_t y_{t+i} = i\gamma^* + y_t$ and $\sigma_t^2(c_{t+i}) = i\sigma_\nu^2$, $i > 0$ and $\mathrm{E}_{t+1}y_{t+i} = (i-1)\bar{\gamma}^* + y_{t+1}$ and $\sigma_{t+1}^2(c_{t+i}) = (i-1)\bar{\sigma}_\nu^2$, $i > 1$.

Substituting these expressions into (24) gives

$$\text{(29)} \quad c_{t+1} - c_t = \delta + \frac{1}{2}\gamma\sigma_\nu^2 + (\bar{\gamma}^* - \gamma^*) + \eta_{T-t-1}^{-1}\left[\frac{1}{2}\gamma(\sigma_\nu^2 - \bar{\sigma}_\nu^2) + \bar{\gamma}^* - \gamma^*\right]\left(\sum_{i=1}^{T-t-1} i(1+r)^{-i}\right) + \nu_{t+1}.$$

We see that a structural change in the income process affects the parameters in the consumption process. The difficulty in applied work will be to determine the moment and the nature of the structural break in the model for consumption. An analysis of the income data will probably yield useful information concerning the features of the univariate process for c_t. The implications of changes in γ^* and σ_ν^2 for the dynamics of c_t are different.

When in period $t+1$, γ^* and/or σ_ν^2 change into $\bar{\gamma}^*$ and $\bar{\sigma}_\nu^2$ respectively, $\Delta c_{t'+1}$ is generated by (24) with

$$\sigma_{t'}^2(c_{t'+1}) = \sigma^2(\nu_{t'+1}) = \sigma_\nu^2 \qquad \text{for} \quad t' < t.$$

For $t' = t$, $\Delta c_{t'+1}$ is generated by (29) with $\sigma^2(\nu_{t'+1}) = \bar{\sigma}_\nu^2$, and for $t' > t$, we have

$$\text{(30)} \qquad c_{t'+1} - c_{t'} = \delta + \frac{1}{2}\gamma\bar{\sigma}_\nu^2 + \nu_{t'+1}, \qquad \sigma^2(\nu_{t'+1}) = \bar{\sigma}_\nu^2.$$

We see that a persistent change in γ^* only affects the constant term in the model for c_{t+1}. The consequences of a persistent change in σ_ν^2 are permanent and twofold, first, a step change of the constant term in the process for consumption, which is completed after two periods, and second a change of the innovation variance. In other words, increased uncertainty about labor income is translated into a larger drift of the consumption process and larger fluctuations around that level. From the discussion of this example, it becomes obvious that it is important for the empirical analysis of consumption to correctly assess the nature of structural changes in the income process.

Sources of the Data

The quarterly series on nondurable consumption per capita in prices of 1980 has been computed as the sum of consumption expenditures per capita on food, beverages, services and other nondurables. Monthly indices on these series are published in Centraal Bureau voor de Statistiek, *Maandstatistiek Binnenlandse Handel en Dienstverlening*, Staatsuitgeverij, 's-Gravenhage. Annual figures on expenditures which are published in Centraal Bureau voor de Statistiek, *Nationale Rekeningen*, Staatsuitgeverij, 's-Gravenhage, have been used to transform the indices into monthly expenditures per capita expressed in prices of 1980. The monthly figures have then been aggregated into quarterly data.

The observations for the first and fourth quarter of 1975 are replaced by the average of the corresponding quarters in 1974 and 1976. In Centraal Planbureau, *Centraal Economisch Plan* 1976, Staatsuitgeverij, 's-Gravenhage, the irregular behavior in 1975 is explained as an advance of sales in the first quarter from the second and third quarters. The high level of consumption in the fourth quarter is due to an increase of sales as a result of a change in the excise tax at the beginning of 1976.

Quarterly data on labor and transfer income for 1968.I-1984.IV have been kindly provided by the Centraal Planbureau. To obtain per capita figures in prices of 1980, the nominal series has been divided by the price index of total consumption and by the size of the population.

An Example of ALS Estimation

To outline the method of asymptotic least squares (ALS), we consider the univariate model (10) with error components

$$\Delta_4 c_t = \sum_{i=1}^{5} \beta_i \tilde{d}_{it} + \psi(L)\varepsilon_t + \Delta\mu_t. \tag{31}$$

When we ignore the error-component structure, $\Delta_4 c_t$ can be expressed as a third order MA process

$$\Delta_4 c_t = \sum_{i=1}^{5} \beta_i \tilde{d}_{it} + (1 - \varphi_1 L - \varphi_2 L^2 - \varphi_3 L^3)\xi_t, \tag{32}$$

where ξ_t is white noise with variance σ_ξ^2. The model (32) can be estimated by ML yielding an estimate $\hat{\gamma}$ of $\gamma = (\beta', \varphi_1, \varphi_2, \varphi_3, \sigma_\xi^2)'$, with $\beta = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5)'$. The parameters γ are related to the parameters of interest $\alpha = (\beta', \sigma_\varepsilon^2, \sigma_\mu^2)'$, by the following (use the second moments)

$$\begin{bmatrix} \beta \\ (1+\varphi_1^2+\varphi_2^2+\varphi_3^2)\sigma_\xi^2 \\ (-\varphi_1+\varphi_2\varphi_1+\varphi_3\varphi_2)\sigma_\xi^2 \\ (-\varphi_2+\varphi_3\varphi_1)\sigma_\xi^2 \\ -\varphi_3\sigma_\xi^2 \end{bmatrix} = \left[\begin{array}{c|cc} I_5 & & 0 \\ \hline & 4 & 2 \\ 0 & 3 & -1 \\ & 2 & 0 \\ & 1 & 0 \end{array}\right] \times \begin{bmatrix} \beta \\ \hline \\ \\ \sigma_\varepsilon^2 \\ \sigma_\mu^2 \end{bmatrix} \tag{33}$$

or alternatively in matrix notation

$$g(\gamma) = A\alpha, \tag{34}$$

where g is a vector of functions in γ and A is the matrix in (33) with known coefficients.

Given a consistent estimate of γ, $\hat{\gamma}$, ALS minimizes the distance of $g(\hat{\gamma}) - A\alpha$ in the metric of a nonsingular matrix S^{-1}, i.e.

$$\min_{\alpha} [g(\hat{\gamma}) - A\alpha]' S [g(\hat{\gamma}) - A\alpha], \tag{35}$$

which yields the ALS estimate

$$\hat{\alpha} = (A'SA)^{-1} A' S g(\hat{\gamma}). \tag{36}$$

The optimal choice of S is $S^* = \left[\frac{\partial g}{\partial \gamma'} \Omega \frac{\partial g'}{\partial \gamma}\right]^{-1}$, where Ω is the asymptotic covariance matrix of $\hat{\gamma}$. When S^* is used, and $\hat{\gamma}$ has a large sample normal distribution, the large sample distribution of $\hat{\alpha}$ is

(37) $$\sqrt{T}(\hat{\alpha}-\alpha) \underset{a}{\sim} N(0, [A' S^* A]^{-1}).$$

When $\hat{\gamma}$ is used with the corresponding optimal weighting matrix S^*, the ALS method yields an estimator of α which is asymptotically equivalent with the ML estimator. In the present example, the efficient ALS-estimator can be obtained as a generalized least squares estimator of the model

(38) $$g(\hat{\gamma}) = A\alpha + u, \text{ with weighting matrix } S^*.$$

In applied work, a consistent estimate has to be substituted for Ω. Notice finally that the minimum value of the objective function (35) multiplied by the number of observations yields a Wald statistic for testing the two restrictions implied by the error components.

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