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# Congestion and Game in Capacity:

## A Duopoly Analysis in the Presence of Network Externalities

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**ABSTRACT.** – We consider the case of a duopoly selling perfect substitutes except for their level of capacity. Firms are playing a two-stage game in which they take capacities as given when they play in prices and anticipate the price outcome when they play in capacities. We analyze the case where consumers are homogeneous and where they are differentiated in their willingness to pay to avoid congestion.

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## Encombrement et jeu en capacité : une analyse de duopole en présence d'externalités de réseaux

**RÉSUMÉ.** – Nous considérons le cas d'un duopole offrant sur le marché des biens parfaitement substituables, à l'exception de leur capacité. Les firmes jouent un jeu en deux étapes dans lequel elles considèrent données les capacités lorsqu'elles jouent en prix et anticipent le résultat en prix lorsqu'elles jouent en capacité. Nous analysons le cas où les consommateurs sont homogènes et celui où ils sont différenciés en termes du prix de réservation qu'ils sont prêts à payer pour éviter les encombrements.

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# 1 Introduction

A number of goods are characterised by the fact that the utility a consumer derives from buying them increases or decreases with the number of consumers who are also buying them. Such goods are commonly referred to as "**networks**" to which consumers decide whether or not to get linked. When its utility **increases** with the number of consumers, a good is said to be subject to (positive) **network externalities**. Some goods can be characterised by both effects. This happens in the case of a telephone system for example. As long as the number of customers remains low, the utility increases when a marginal user joins the network. However, if the number of customers is too large, the system gets congested and a marginal user decreases the utility. Clearly, negative externalities directly raise the question of the optimal investment in terms of network capacity.

Negative externalities occur in transportation and telecommunication systems. In such cases, each marginal user induces a cost to all other users. Typically, the marginal cost and the individual social cost do not coincide. This suggests that competitive equilibrium and social optimum need not necessarily coincide. Examples can be found in airline, bus and rail companies.

Another aspect of negative externalities is that when demand exceeds capacity, queues tend to develop, as it happens in shopping centers, movie theaters, etc. This gives scope to the owner of the commodity to differentiate prices and reallocate consumers in different queues using the price mechanism. Such a pricing scheme is used in the Tirupati temple in South India, where the access to the temple is restricted to a few doors with different prices. As a consequence, queues of different length develop, rich pilgrims choosing the expensive (but short) queues while those who care less for time go to the long (but cheap) queues. It can be shown that such a differentiated pricing scheme may improve welfare and profits. Thus, it can be used by a monopolist as well as by a planner (see CHANDER and LERUTH [1989]).

Negative network externalities (or congestion) have been studied in club theory and in the general equilibrium framework (see MARCHAND [1968] and LÉVY-LAMBERT [1968]) as well as in industrial organisation, usualy when the good is provided by a monopolist or a planner (see MILLS [1981], FREEMAN and HAVEMAN [1977]). However, the case of competing firms has received much less attention if one excepts the work of WILSON [1986] and REITMAN [1987] for the case of electricity. Congestion effects are a very common phenomenon affecting many goods such as transportation networks for example, or any good which can only be obtained by standing in a queue. Let us suppose that consumers are different in the way they are ready to pay to avoid congestion (just as they are ready to pay in order to join a large network in the presence of an externality). In that case, there is obviously scope for providing the same good with different degrees of congestion (and at different prices) to consumers who would form queues of different sizes. Several such examples can be provided.

A differentiated price system has been implemented in the buses of New-Delhi. Buses are all identical except for the price charged inside, either 1 R*e* or 2 R*s*, which is written on the front. When a bus comes, potential consumers consider the level of congestion in the bus (it may be huge) and the price. The system works fairly well with the 1 R*e* buses clearly (much) more crowded than the 2 R*s* ones. In all these cases, the quality of the service in a particular category mainly depends on the level of congestion achieved in that category. Another good example is provided by WILSON [1986] and REITMAN [1987] who study a pricing system for electricity where, according to the price he pays (or to the category he decides to belong to), a consumer is more or less likely to be disconnected in case of excess demand. The idea is that, capacity being limited, those who pay most (first category) are never disconnected. Those who are in the second category only get disconnected if the first ones absorb the whole capacity, etc.

We study here the impact of congestion on optimal strategies of two competing firms selling products which have the same (average) intrinsic qualities but not necessarily the same capacity. Although the intrinsic quality is the same for both products, it may happen that, at equilibrium, the difference of price and/or capacity leads to products which are no more perfect substitutes. This happens when the levels of congestion are different for both goods, whether it is because of a capacity which is lower for one good than for the other, or because demand is different.

The two main points which are stressed in this paper are the equilibrium in prices and the capacity levels selected by both firms. We consider a game in which prices and capacities are outcomes of a two stage sub-game perfect equilibrium: firms choose optimal prices taking capacities as given and select their own capacity bearing in mind how their decision will affect the outcome in prices. Note that in the presence of negative externalities, a price game can always be played as demands are well defined (see CHANDER and LERUTH [1989]) while in the case of positive externalities, it may lead to demand indetermination (see DE PALMA and LERUTH [1989]).

The simplest model of price competition was introduced by Bertrand and relies on the assumption that products are *perfect substitutes* and consumers *homogeneous in their willingness to pay for quality*. In section 2, we extend that model to the case of negative network externalities and show that Bertrand competition is relaxed. In section 3, we assume that consumers are vertically differentiated. We show that congestion benefits the firms and harms consumers' welfare. We also show that firms tend to offer differentiated capacities although offering the same capacity would not completely jeopardize profits. Conclusions can be found in section 4.

# 2 Homogeneous Consumers

## 2.1. Price Game with Exogeneous Capacities

Two firms denoted by 1 and 2 sell an homogeneous good at prices $p_1$ and $p_2$ respectively. We assume throughout the paper that firms are producing at constant marginal cost, set equal to zero without loss of generality. Each firm chooses its price so as to maximize its profit. Firms are playing a non cooperative Nash game in prices. We also assume that each consumer buys one unit of the good which gives him the highest utility. However, if none of the available goods gives him a positive utility, he can choose the zero option, which is not to buy anything at all.

In the classical case (absence of any externality or differentiation), the argument developped by Bertrand is that each firm can gain the whole market by undercutting the price charged by the other one. We know that the outcome of such a game is a zero-price, zero-profit solution.

We now extend the above model to take capacity and level of congestion of the goods into account. For that purpose, we assume that a consumer who buys from firm $i$ ($i=1, 2$) derives a utility given by

$$U_i = V - p_i - \theta . \frac{Q_i}{C_i} \tag{1}$$

where $C_i$ is the capacity of good $i$ and $Q_i$ the number of consumers who buy from firm $i$. Thus, $[Q_i/C_i]$ is the level of congestion of good $i$.[1] Consumers are characterised by the same parameter $\theta$ which represents their willingness to pay in order to avoid congestion.[2] For example, if congestion can be modelled by a deterministic queue, $C_i$ is equal to $2s_i$ where $s_i$ represents the capacity of the bottleneck; this expression measures the average waiting time for a consumer who joins the queue (see NEWELL [1983]). V is a constant which represents the "intrinsic" quality of the good or, in other words, the utility one customer would derive from consuming the good, would it be available for free and would he be alone to consume it.

We make the further assumption that the potential market size (*i.e.* the set of would-be users) is normalized to one. Thus, the following situations

1. Thus, $Q_i/C_i$ should be interpreted as a density: the number of consumers per unit of capacity.
2. Note that (1) is a reformulation of the surplus function used by MUSSA and ROSEN [1978] and ITOH [1983]. Note also that $\theta$ could have been set equal to 1 without loss of generality. However, we maintain this notation because it naturally leads to the generalisation made in the next section.

can arise:

(I) V is large enough compared to $\theta(Q_i/C_i)$ and $p_i$, so that each consumer buys one unit (from one firm or the other). In that case, $U_1$ and $U_2$ are strictly positive. This situation is referred to as situation (I) and we have $Q_2 = 1 - Q_1$.

(II) All consumers derive a surplus equal to zero, but all are buying one unit anyway. This situation is referred to as situation (II) and is characterized by $Q_1 + Q_2 = 1$ and $U_1 = U_2 = 0$.

(III) Values of parameters and variables are such that some consumers are not willing to buy and prefer the zero option. In such a case, all consumers, even those who are buying, derive a zero surplus. This situation is referred to as situation (III). It is characterized by $Q_1 + Q_2 < 1$.

These cases can be summarized in the following way:

$$U = U_1 = U_2 \geqslant 0$$

with

$$Q_1 + Q_2 \leqslant 1$$

and

$$Q_1 + Q_2 < 1 \text{ implies } U = 0.$$

Note that it may also happen that V is lower than $p_i$, $i = 1, 2$. This would imply that, even in the absence of any congestion, no consumer is willing to buy any of the goods. In such a case, profits are trivially equal to zero and the situation is without any practical interest.

We can now write profits, which are given by

$$\pi_i = p_i \,.\, Q_i - K(C_i), \qquad i = 1, 2. \tag{2}$$

The cost fuction K(.) does not depend on quantities sold. It is assumed to be an increasing function of capacity. In this section, capacities ($C_i$) are supposed to be fixed because they are outcomes of the first stage game. We consider here the second stage where firms play in prices in order to maximize profits. In various situations however, capacities cannot be changed and have been determined once for all. Thus, this game has a practical interest of its own.

### (i) Situation (I)

At equilibrium, the three situations considered above can arise. We first consider situation (I) where all consumers derive the same strictly positive surplus. In that case, using (1), we have

$$p_1 + \theta \,.\, \frac{Q_1}{C_1} = p_2 + \theta \,.\, \frac{Q_2}{C_2}. \tag{3}$$

For the sake of convenience, we introduce parameters $R_1$ and $R_2$ which are respectively equal to $1/C_1$ and $1/C_2$. Thus, equation (3) can be rewritten as

(3′) $$p_1+\theta.R_1.Q_1=p_2+\theta.R_2.Q_2.$$

Using $Q_2=1-Q_1$, we obtain

(4) $$Q_1=\frac{\theta.R_2+p_2-p_1}{\theta.(R_1+R_2)}=\frac{C_1.[\theta+C_2.(p_2-p_1)]}{\theta.(C_1+C_2)}$$

which is the demand associated with firm 1, firm 2's being symmetric.

At this stage, we would like to emphasize the fact that equation (3) has been used in the context of transportation to model the route choice of car users; in that case, it expresses the fact that, at equilibrium, the utility drivers derive from using any of the available routes (from a given origin to a given destination) are the same and larger than the utility corresponding to routes which are not used. This equation is known in transportation science under the name of Wardrop Principle.

As demand is well defined through equation (4), we can solve the price game and look for a Nash equilibrium. By definition, the pair of prices $(p_1^*, p_2^*)$ is a Nash equilibrium if and only if those prices satisfy the following equations:

$$\pi_i(p_i^*, p_j^*) \geqslant \pi_i(p_i, p_j^*), \quad \text{for any } p_i \geqslant 0; \quad i, j=1, 2, \quad j \neq i.$$

By substituting equation (4) into the profit functions given by (2), we have

$$\pi_i=\frac{p_i.\theta.R_j+p_i.(p_j-p_i)}{\theta.(R_1+R_2)}-K\left(\frac{1}{R_i}\right); \qquad i, j=1, 2, \quad j \neq i.$$

Maximizing $\pi_i$ with respect to $p_i$, we obtain the following first order conditions [3]

$$\frac{\partial \pi_i}{\partial p_i}=0 \quad \Leftrightarrow \quad 2p_i=p_j+\theta.R_j; \qquad i, j=1, 2, \quad j \neq i.$$

Therefore, equilibrium prices are given by

(5) $$p_i^*=\frac{\theta.(2R_j+R_i)}{3}; \qquad i, j=1, 2, \quad j \neq i.$$

Solution (5) is indeed a Nash equilibrium as each profit function is quasi-concave in its own price.

At equilibrium, demand associated with each firm is given by [see equations (4) and (5)]

$$Q_i^*=\frac{2R_j+R_i}{3.(R_i+R_j)}; \qquad i, j=1, 2, \quad j \neq i.$$

3. It is easy to check that the second order conditions are satisfied.

Note also that

$$p_i^* = \theta \,.\, Q_i^* \,.\, (R_i + R_j); \qquad i, j = 1, 2, \quad j \neq i.$$

This means that equilibrium prices $p_i^*$ are proportional to market size of firm $i$ and also to parameter $q$. Equilibrium profit of firm $i$ ($i = 1, 2$) is therefore given by

$$\pi_i(R_i, p_i^*, R_j, p_j^*) = \frac{\theta \,.\, (2R_j + R_i)^2}{9 \,.\, (R_i + R_j)} - K\left(\frac{1}{R_i}\right)$$

$$= \frac{\theta \,.\, (2C_i + C_j)^2}{9 \,.\, (C_i + C_j) C_i C_j} - K(C_i), \qquad j \neq i.$$

It is important to see that solution (5) generalises the Bertrand equilibrium characterized by $R_1 = R_2 = 0$. The presence of congestion does not spoil existence of a static Nash equilibrium. We even see that congestion tends to decrease competition among firms in the sense that undercutting prices is not always a profitable strategy, even from the point of view of one firm. Indeed, too low a price implies a high level of congestion and the good becomes less attractive. Thus, the force which was driving prices down to costs in the pure Bertrand game tends to disappear because of congestion. In some sense, firms benefit from congestion. They know that if their goods get congested, they tend to compete less. For example, equation (5) shows that equilibrium prices decrease when capacity increases (or when $R_i$ decreases). However, this does not imply that congestion is necessarily profitable to firms. Indeed, as congestion grows, consumers get worse off first because they pay a higher price and second because they face congestion. As a consequence, they may prefer the zero option if the level of congestion is really too high. This is because price solution (5) is only valid when consumers derive a strictly positive utility from their consumption. This will happen if and only if

$$V - p_i - \theta \,.\, R_i \,.\, Q_i > 0; \qquad i = 1, 2,$$

or

(6) $$V > \frac{\theta \,.\, (2R_1 + R_2) \,.\, (R_1 + 2R_2)}{3 \,.\, (R_1 + R_2)} = V_u.$$

When condition (6) is not fulfilled, consumers derive no surplus from their consumption. Let us consider the following situation. Suppose V is given and capacity levels $C_i$ are such that goods get congested with a small number of consumers. It may be the case that, even if it behaves as a pure monopolist, each firm can only attract a few consumers. Indeed, its good would otherwise be so congested that price would fall, jeopardising profit. If congestion effect is high enough, it may happen that by maximising profits independantly, firms do not cover the whole market. Anyway, all consumers, whether they buy or not get a surplus equal to zero and we are in situation (III). We shall now see under which condition on V, $R_1$ and $R_2$, situation (III) holds.

### (ii) Situation (III)

Firms behave as pure monopolists and as such try to extract the largest possible surplus from consumers. This leads to

$$V - \frac{\theta . Q_i}{C_i} - p_i = 0,$$

or

$$Q_i = \frac{C_i . (V - p_i)}{\theta}$$

and maximising $\pi_i$ with respect to $p_i$ leads to

(7)
$$p_i^* = \frac{V}{2}$$

and

(8)
$$Q_i^* = \frac{C_i . V}{2 . \theta}; \qquad \pi_i^* = \frac{C_i . V^2}{4\theta} - K(C_i).$$

Again, it is easy to show that solution (7) is a Nash equilibrium, but it can only be the solution of our problem if equilibrium quantities are such that

(9)
$$Q_1 + Q_2 < 1.$$

Condition (9) is equivalent

$$V < \frac{2\theta}{C_1 + C_2} = \frac{2\theta . R_1 . R_2}{R_1 + R_2} = V_1.$$

Clearly, this last condition states that the "intrinsic" quality V of the good has to be low enough so that consumers get easily discouraged from buying it. If it is not so, firms have an incentive to expand their market so that condition (9) is violated because they compete so as to attract more consumers. It is important to see that here, firms try to increase their capacity so as to attract more consumers. Put another way, congestion does not benefit the firms. This clearly corresponds to the intuition that a more congested good allows firms to extract less surplus from consumers. In the previous case, congestion decreased competition and profits were going up.

Note that one always has $V_1 < V_u$, which suggests that there are cases, namely V in $[V_1, V_u]$, where market is covered and where consumers get no surplus. This is typically happening in situation (II).

### (iii) Situation (II)

Thus, in this case, our candidate Nash equilibrium would be characterized by a pair of prices $(p_1^*, p_2^*)$ satisfying

$$U_1 = U_2 = 0, \qquad Q_1 + Q_2 = 1.$$

This is equivalent to

$$(10) \qquad p_1^* = p_2^* = V - \frac{\theta}{C_1 + C_2} = V - \frac{\theta . R_1 . R_2}{R_1 + R_2}$$

and we have

$$(11) \qquad Q_i^* = \frac{C_i}{C_1 + C_2}$$

which leads to equilibrium profits

$$(12) \qquad \pi_i^* = \frac{C_i . [V . (C_1 + C_2) - \theta]}{(C_1 + C_2)^2} - K(C_i).$$

It is very easy to see that solution (10) is a Nash equilibrium for values of V in $[V_1, V_u]$. We can also represent as in figure 1 the reaction function of firm 1 if $p_2$ is fixed by equation (10). We see that for those values of V, we have a corner solution.

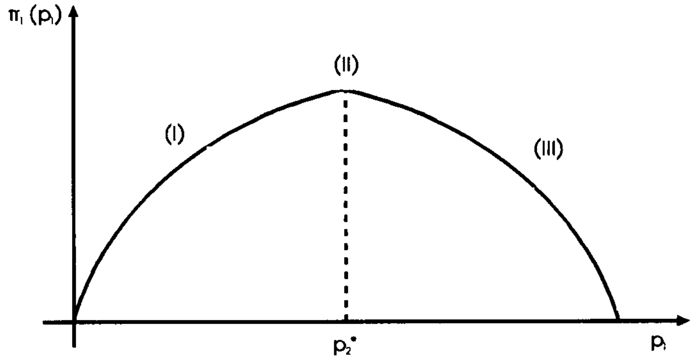


FIGURE 1

Finally, we present in figure 2 equilibrium profits (where costs are neglected) and prices obtained under situation (I), $V > V_u$, situation (II), V in $[V_1, V_u]$ and situation (III), $V < V_1$. As expected, we can see that once V is larger than $V_u$, neither profits nor prices get affected by V (unlike for lower values of V). This is so because, once V is large enough to ensure that all consumers buy one unit of the good, competition between firms increases and jeopardizes the potential advantage of consumers' higher willingness to pay.

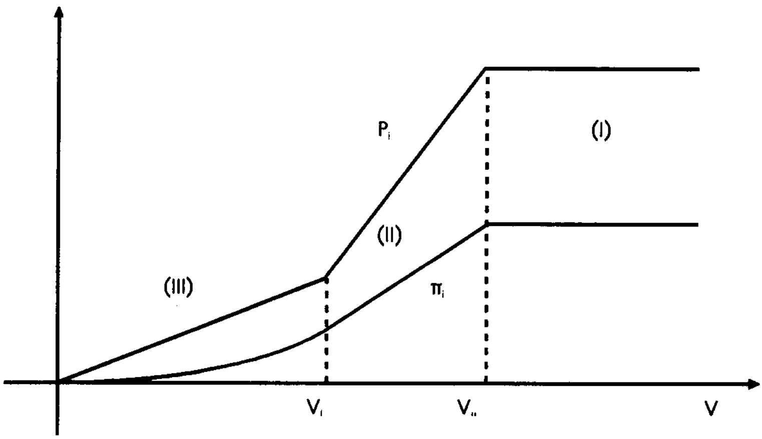


FIGURE 2

## 2.2. **Price Game with Endogeneous Capacities**

We now consider the situation where capacities are no more fixed but are outcomes of a two-stage game. In order to solve this problem, we also need to specify the cost function K(.) and we assume that $K(C_i) = k . C_i$. This simply means that the marginal cost of capacity is constant. Formally, the situation can be represented as follows:

(i) at the second stage, capacities are fixed and firms play a game in prices, which leads to a Nash equilibrium characterised by $p_1^*(C_1, C_2)$ and $p_2^*(C_1, C_2)$.

(ii) at the first stage, firms play the game in capacities and anticipate the outcome of the price game. Thus, they maximise

$$\pi_i(C_i, C_j, p_i^*(C_i, C_j), p_j^*(C_i, C_j)).$$

By definition,

$$(C_i^*, C_j^*, p_i^*(C_i^*, C_j^*), p_j^*(C_i^*, C_j^*))$$

is a Nash equilibrium if and only if

$$\pi_i(C_i^*, C_j^*, p_i^*(C_i^*, C_j^*), p_j^*(C_i^*, C_j^*)) \geqslant \pi_i(C_i, C_j^*, p_i^*(C_i, C_j^*), p_j^*(C_i, C_j^*))$$

for any $C_i \geqslant 0$, $i, j = 1, 2, j \neq i$.

We study the first stage game in the case of situations (I), (II) and (III) and make the analysis in terms of $R_i (= 1/C_i)$ and $R_j (= 1/C_j)$ instead of $C_i$ and $C_j$ for reasons of convenience.

If we first with situation (I), we know that the price game outcome is given by (5). Profits can be written as

$$\pi_i = \frac{\theta . (2R_j + R_i)^2}{9.(R_i + R_j)} - \frac{k}{R_i}.$$

For the first stage game in capacities, we need to compute $\partial\pi_i/\partial R_i$ :

$$\frac{\partial\pi_i}{\partial R_i} = \frac{\theta . R_i . (2R_j + R_i)}{3.(R_i + R_j)^2} + \frac{k}{R_i^2}.$$

We see that this expression will always be strictly positive. Note that the sign of $\partial\pi_i/\partial R_i$ does not depend on the particular form of the cost function: each duopolist tries to reduce its capacity. This is because increased congestion implies decreased competition.

However, reducing capacity also implies reducing consumers' surplus, so that there is a point at which consumers may prefer not to buy any of the available goods. Situation (III) corresponds to that case and profits are given by (8)

$$\pi_i = \frac{V^2}{4\theta . R_i} - \frac{k}{R_i}$$

and the capacity game leads

$$\frac{\partial\pi_i}{\partial R_i} = -\frac{V^2 - 4\theta . k}{4\theta . (R_i)^2}.$$

If $(V^2 - 4\theta . k)$ is negative, each firm (behaving as a monopolist) tends to decrease its capacity to the minimum. The reason is that consumers are not able to pay for any capacity and this case is without practical interest. On the other hand, if the same expression is positive, firms increase capacity. In the case of a general capacity cost function with increasing marginal cost, firms would increase capacity to the point where marginal revenue is equal to marginal cost. In the case of a linear cost function, they tend to increase capacity to the maximum. However, by doing so, they provide larger surplus to consumers and there is a point where they will start competing again. That is characterized by $Q_1 + Q_2 = 1$.

The two previous results clearly show that the two stage game equilibrium will be such that all consumers buy. The reason is that if some consumers do not buy, firms behave as monopolists and we have seen that under that condition, they increase capacities and provide the good to all consumers. But at equilibrium, consumers also get no surplus. This is so because if they do (so that we are in situation I), firms tend to decrease capacity. Thus, at equilibrium, the market has to be characterized by situation (II). Using (12), we have

$$\pi_i = \frac{R_j.[V.(R_i+R_j)-\theta.R_i.R_j]}{(R_i+R_j)^2} - \frac{k}{R_i}, \qquad i=1, 2.$$

The first order condition in $R_i$ gives

$$\frac{-R_j.[V.(R_i+R_j)+\theta.R_j.(R_j-R_i)]}{(R_i+R_j)^3} + \frac{k}{(R_i)^2}, \qquad i=1, 2.$$

As both conditions for $i=1$ and 2 are symmetric, the solution (capacities and profits) is given by

$$C_1 = C_2 = \frac{V}{4k}; \qquad \pi_1 = \pi_2 = \frac{V^2 - 4\theta k}{4V}$$

and

$$p_1 = p_2 = \frac{V^2 - 2\theta k}{V}.$$

First, note that profits are always positive as we have assumed in the previous section that $V^2 > 4\theta k$.[4] Second, note that this solution holds if and only if V belongs to $[V_1, V_u]$. Replacing the value of $C_1$ and $C_2$ in $V_1$ and $V_u$, it is a matter of simple algebra to see that this will only be true if $V^2 > 4\theta k$ and $V^2 < 6\theta k$ (for $V > V_1$ and $V < V_u$ respectively). The first condition being verified, we just have to check the second one.

If $V^2 > 6\theta k$, the candidate solution leads prices such that all consumers derive a positive surplus so that we are actually in situation (I). However, we know that firms then prefer to reduce capacity in order to avoid competition. They will do so as long as $V_2 > V_u$ is violated. In $V^2 = V_u$, we have

$$R_1 = R_2 = \frac{2V}{3\theta}, \qquad C_1 = C_2 = \frac{3\theta}{2V}.$$

It is easy to prove that this solution is indeed a Nash equilibrium. It leads to

$$p_1 = p_2 = \frac{2V}{3}$$

4. This condition is also sufficient to insure the second order condition.

and

$$\pi_1 = \pi_2 = \frac{2V^2 - 9k\theta}{6V}.$$

It is interesting to see that although profits increase with V, as one would expect, capacities, on the other hand, decrease when V is large. This is because when V is large, firms tend to avoid competition by reducing capacity.

# 3 Vertically Differentiated Consumers

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We have seen in the previous section that when consumers are homogeneous, the equilibrium in capacity (and in prices) is always symmetric although profits are not driven down to zero. We now assume that consumers are differentiated in their willingness to pay to avoid congestion. This simply means that some consumers are ready to pay a high price in order to enjoy a less congested good, while others prefer to pay less because they do not care so much for congestion. Such an approach is useful because it may lead to an equilibrium in which firms try to provide differentiated networks.

We assume that each consumer is characterized by a parameter θ which represents his willingness to pay to avoid congestion. We also assume that the distribution of the θ's is uniform with density 1 over the [0, 1] interval. The surplus $S(\theta, i)$ derived by a consumer θ who joins network $i$ (or buys good $i$) is a simple generalisation of (1),

$$S(\theta, i) = V - \frac{\theta . Q_i}{C_i} - p_i = V - \theta . Q_i . R_i - p_i; \qquad i = 1, 2,$$

The constant V is here (unlike in the previous section) supposed to be large enough to give a positive surplus to all consumers. This assumption is restrictive. It was made to keep tractability of the model and corresponds to the absence of demand elasticity. In the context of a price-game, it proves usefull although it cannot be made without loss of generality. We have already mentionned that it affects the number of firms on the market and we shall clearly stress its implications on these results.

PROPERTY 1: If $p_i > p_j$, one must have $R_i . Q_i < R_j . Q_j$

This property simply means that the most expensive network should also be the less congested one. Indeed, suppose it is not true, consumers who would be linked to a more expensive and congested network, would prefer to join another one. This would remain true until so many consumers have left the expensive network that it becomes the less congested one. This property does not prove that the process necessarilly stabilizes at a certain level of congestion (this is proved in CHANDER and LERUTH [1989]). It simply gives a necessary condition for the coexistence of such networks. It also implies that, without any loss of generality, we can suppose that firm 1 is selling the less congested and most expensive network while firm 2 is selling the other one. For reasons of convenience, we use the following notation: firm 1 will be referred to as firm $s$ (for strong) and firm 2 as firm $w$ (for weak). Using standard properties of the model, we can now define demand.

We know that there exists a consumer who is characterized by $\theta^*$ and is just indifferent between both options. Thus, $\theta^*$ can be obtained by equating both surpluses. We have

$$\theta^* . R_s . (1-\theta^*) + p_s = \theta^* . R_w . \theta^* + p_w. \tag{13}$$

Thus, (13) can be solved and leads to

$$\theta^* = \frac{R_s + \sqrt{R_s^2 - 4.(R_s + R_w).(p_w - p_s)}}{2.(R_s + R_w)}. \tag{14}$$

The other solution to (13) is to be rejected because $\theta^*$ must be increasing in $p_s$. Indeed, if $p_s$ is increasing, some consumers will prefer good $w$ because considering the respective levels of congestion, they derive more utility from the extra money they save then they loose from increased congestion. This argument simply states that demand is downwards sloping. As long as $p_s$ and $p_w$ are such that, for $R_w$ and $R_s$ given, the value of $\theta^*$ given in (14) is defined and belongs to [0, 1], those prices will indeed generate the demand associated with each good: all those consumers whose $\theta$ belongs to [$\theta^*$, 1] will prefer network $s$ while all the others will prefer network $w$.

$Q_s$ and $Q_w$ being the quantities sold by firms $s$ and $w$ respectively, profits are given by

$$\pi_s = p_s . Q_s - K(C_s) = p_s . (1-\theta^*) - K(C_s),$$
$$\pi_w = p_w . Q_w - K(C_w) = p_w . \theta^* - K(C_w).$$

We can now consider the price game.

## 3.1. Price Game with Exogeneous Capacities

As in the previous section, firms take capacities as given and maximise profits with respect to prices. Although demands can explicitely be defined as functions of prices, the direct maximisation of profit functions turns out to be difficult. Thus, it is preferable to use implicit equations. We have

$$\frac{\partial \pi_s}{\partial p_s} = (1-\theta^*) - p_s . \frac{\partial \theta^*}{\partial p_s} = 0 \tag{15}$$

$$\frac{\partial \pi_w}{\partial p_w} = \theta^* + p_w . \frac{\partial \theta^*}{\partial p_w} = 0. \tag{16}$$

Using equation (14), we can derive the value of $(\partial\theta^*/\partial p_s)$ and $(\partial\theta^*/\partial p_w)$. For the sake of tractability, we again use the convention that $R_s$ (resp. $R_w$) is equal to $1/C_s$ (resp. $1/C_w$). This leads to

$$(17)\qquad \frac{\partial\theta^*}{\partial p_s} = -\frac{\partial\theta^*}{\partial p_w} = \frac{1}{2\,\theta^*.(R_s+R_w)-R_s}.$$

Substituting (17) into (15) and (16) leads to equilibrium prices

$$(18)\qquad p_s^* = (1-\theta^*).\Omega,$$

$$(19)\qquad p_w^* = \theta^*.\Omega$$

where $\Omega = 2\,\theta^*.(R_s+R_w)-R_s$. Note that (18) and (19) are implicit equations as $\theta^*$ is defined through (14) and is itself a function of the prices.

We can now use equations (18) and (19) in (13) to derive $\theta^*$ as a function of $R_s$ and $R_w$. This leads to

$$5\,\theta^{*2}.(R_s+R_w)-\theta^*.(5\,R_s+2\,R_w)+R_s=0,$$

or

$$(20)\qquad \theta^* = \frac{5\,R_s+2\,R_w+\sqrt{5\,R_s^2+4\,R_w^2}}{10.(R_s+R_w)}.$$

The other solution has to be rejected as it would be strictly smaller than (14). If we replace this expression of $\theta^*$ in (18) and (19), we obtain explicit formulations of the candidate equilibrium prices in terms of $R_s$ and $R_w$. They are given by

$$(21)\qquad p_s = \frac{[5\,R_s+8\,R_w-\sqrt{5\,R_s^2+4\,R_w^2}].[2\,R_w+\sqrt{5\,R_s^2+4\,R_w^2}]}{50.(R_s+R_w)},$$

$$(22)\qquad p_w = \frac{[5\,R_s+2\,R_w+\sqrt{5\,R_s^2+4\,R_w^2}].[2\,R_w+\sqrt{5\,R_s^2+4\,R_w^2}]}{50.(R_s+R_w)}.$$

Since we assume that good $s$ is more expensive, its level of congestion is the lowest (see *property* 1). We can easily find conditions on $R_s$ and $R_w$ under which our candidate equilibrium solution satisfies that constraint. Indeed, one must have

$$(23)\qquad R_s.(1-\theta^*) < R_w.\theta^*.$$

Using (20), it can be shown after some manipulations that inequality (23) is equivalent to

$$R_s < R_w.$$

This last inequality shows that firm $s$ is more expensive, but also provides larger capacity. This is not surprising as one expects that large capacity leads to less congestion. However, we still do not know whether firm $s$ uses that large capacity to provide high quality to very few consumers (and extracts a maximum surplus from them) or to attract more consumers than its competitor (and gain in quantity what it looses on individual surplus). We can give a straightforward answer to that question. Indeed, using (14) and (19), we see that $p_s > p_w$ implies $1-\theta^* > \theta^*$, or $\theta^* < 0.5$. Thus, firm $s$ does not only provide larger capacity and a less congested good, but it also has a larger market share.

Also note that if $R_s = R_w$, both prices are equal and $\theta^* = 0.5$. This does not mean that when goods are perfect substitutes, consumers make their purchase decision in such a way that those who are characterized by large $\theta^*$'s buy a certain one while the rest buys the other one. This corresponds to a limit case. If goods are offered at the same price and have the same capacity, consumers share randomly and equally between both options.

## 3.2. Price Game with Endogeneous Capacities

As in the case where consumers are homogeneous in their willingness to pay, we now consider a game where capacity levels are decided in a first stage and prices in the second stage. As usual, firms keep in mind the implications fo their first stage decisions on the outcome of the second stage.

In the price game, we have derived the value of equilibrium prices and market shares for given capacities (or $R_i$). Profits at the first stage can thus be written as

$$\pi_s = p_s . (1-\theta^*) - K(C_s) = (1-\theta^*)^2 . \Omega - K(C_s),$$
$$\pi_w = p_w . \theta^* - K(C_w) = (\theta^*)^2 . \Omega - K(C_w),$$

where prices are given by (21) and (22), and where

$$\theta^* = \frac{5R_s + 2R_w + \sqrt{5R_s^2 + 4R_w^2}}{10.(R_s + R_w)},$$
$$\Omega = \frac{2R_w + \sqrt{5R_s^2 + 4R_w^2}}{5}.$$

We shall first consider the maximisation of $\pi_w$ with respect to $R_w$ (and not $C_w$ again for reasons of convenience). Neglecting capacity costs for the time being, we see that $\partial\pi_w/\partial R_w$ has the same sign as

$$(24) \quad R_w . \sqrt{5R_s^2 + 4R_w^2} . (7R_s + 4R_w) - 10R_s^3 + 4R_w^3 + 5R_s^2 R_w + 14R_s R_w^2.$$

If we recall the fact that capacity offered by firm $w$ is always lower than the one offered by firm $s$, we must have $C_s \geqslant C_w$, which implies $R_s \leqslant R_w$. As a consequence, (24) is strictly positive. This shows that in the absence of capacity costs, firm $w$'s best strategy is to decrease its capacity to the minimum. That result is naturally reinforced if there are costs of capacity. But it also clearly depends upon the assumption that all consumers buy one and only one unit of one of the goods, which means that global demand is perfectly inelatic. Indeed, would it sell too low a quality, firm $w$ would face a shrinking market because some of its consumers (those with a low valuation of quality) would prefer not to buy anything at all. However, the effect of cross-product elasticity is taken care of through $\theta^*$. What the positive sign of (24) really shows is that the weak firm will indeed tend to provide a good of low capacity. Thus, we have

$$C_w = 0, \quad \text{or} \quad R_w = +\infty.$$

Note finally that if quality is not linked to congestion (as in GABZEWICZ and THISSE [1980], or in SHAKED and SUTTON [1982]), absence of elasticity also leads to the result that the weak firm provides zero quality. It is a matter of simple algebra to prove it and the intuition is that firms, not constrained by shrinking demand, tend to differentiate products as much as possible.

We now consider the case of firm $s$ and again neglect costs of capacity for reasons of tractability. We see that $\partial\pi_s/\partial R_s$ has the same sign as

$$(25) \qquad \begin{cases} -(5R_s^2 + 15R_sR_w + 4R_w^2).\sqrt{5R_s^2+4R_w^2} \\ +25R_s^3 + 35R_s^2R_w + 20R_sR_w^2 - 8R_w^3. \end{cases}$$

As $R_w$ tends to converge to $+\infty$, it is clear that expression (20) is always negative, except in $R_s = R_w$, which corresponds to a minimum. This shows that firm $s$ tends to decrease $R_s$ as much as possible and means that it tries to provide maximum capacity. Again, this result can be obtained in the case where quality does not depend upon congestion (as shown in SHAKED and SUTTON [1982], who also neglect the costs of quality). The absence of capacity costs clearly plays a key role here, as firm $s$ may not find it profitable to increase its capacity to the maximum if the cost of doing so is not compensated by an increased flow of money from consumers.

Nevertheless, we see that firms $s$ and $w$ tend to differentiate their products through capacity. The result is very similar to the one obtained in the absence of congestion where firms tend to differentiate quality. However, one important difference has to be mentionned. Suppose costs of capacity are very high: firms may find it profitable to provide low quality and they will loose some of their incentive to differentiate capacities. In the absence of congestion, such a low difference of quality completely jeopardizes profits because of increased competition. Indeed, goods being almost homogeneous, each firm has an icentive to undercut prices so as to regain a part of the market share owned by competitors. If goods are perfectly homogeneous, we have the standard Bertrand argument under which both profits go down to zero. We again see that once goods get congested, undercutting prices is a move which may prove harmful: it may only attract a few consumers because the good gets too congested. In that case, the few extra consumers may not compensate for the loss made through the low price on the remaining ones.

Again, the intuition is that congestion reduces competition among firms. From a welfare point of view, it harms the consumers in two ways: one is the increased congestion (and the decreased utility) and the other is the lower level of competition (and thus the lower capacities and higher prices).

# 4 Conclusions

In this paper, we show that firms gain monopoly power by selling congested goods. This is not necessarily intuitive, as one would expect that consumers being worse off when the good is congested, it is more difficult to extract surplus from them. However, we have seen that competition among firms is decreasing with the presence of congestion. We have also shown that when consumers derive a high utility from the good, competing firms tend to decrease capacities in order to avoid competition. One implication is that congested goods are offered at a high price and low quality (or high congestion) to consumers. Similar conclusions have been obtained in the study of deregulation of telecommunications networks (see Amiel and Rochet [1987], for example). Note that, although it depends on the cost function of capacity, we have seen that a monopolist would tend to offer a less congested good, while duopolists would try to avoid competition by decreasing capacity. This suggests that the presence of several firms offering highly substitutable congested facilities may actually decrease total welfare.

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