ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 15/16 — 1989

Uncertainty, Capacity and Flexibility: the Monopoly Case

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ABSTRACT. – It is generally expected that profit maximisation leads a firm to choose a more flexible plant the more uncertain its demand function is and/or the more variable is the sequence of quantities to produce. In this paper we make explicit the precise conditions under which this intuitive argument is valid. We show that a sufficient condition is that the increase in uncertainty must involve only those states of demand for which the firm is initially active, that is for which it is able to cover quasi-fixed and variable costs. These are common assumptions in most flexibility choice models. But we also show that a reverse relation between the variability in demand and plant flexibility may exist under reasonable cost and demand conditions, even under the most common definition of increased uncertainty in demand and the most acceptable notion of plant flexibility. We give an example of such an inverse relation.

Incertitude, capacité et flexibilité : le cas du monopole

RÉSUMÉ. – On admet généralement que la maximisation de ses profits conduit une firme à choisir des installations d'autant plus flexibles que la demande face à laquelle elle est confrontée est d'autant plus incertaine et/ou que la suite de ses niveaux de production est d'autant plus volatile. Dans cet article, nous précisons les conditions sous lesquelles on devrait observer une telle relation. On montre qu'une condition suffisante de validité est que l'accroissement d'incertitude ne concerne que les états de la demande pour lesquels la firme produisait initialement, c'est-à-dire pour lesquels elle était en mesure de couvrir ses coûts de fonctionnement. Ce sont les hypothèses posées dans la plupart des modèles qui traitent de la flexibilité. Mais on montre aussi qu'un accroissement d'incertitude peut amener une firme à choisir des installations moins flexibles sous des conditions de demande et de coûts raisonnables. On donne un exemple non pathologique d'une telle relation.

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1 Introduction

It is G. Stigler [1939] who first pointed out that a firm must in general decide between different types of equipment, each giving rise to a different average cost function. He suggested in particular that the firm must choose between an equipment allowing it to get an average cost curve relatively flat over a broad range of production levels and an equipment giving it a more V-shaped cost curve such that the latter average cost is lower inside a narrow interval of outputs but larger outside, with a fast increasing difference outside the interval.

Neglecting the problems raised by multiproduction, two polar models capture the essential aspects of situations where the firm is faced with such tradeoffs. In the first one, the firm must produce only once; when choosing its equipment the firm has only imperfect information about its demand function, but this uncertainty is later partially or totally resolved before deciding its effective production level. In the second one, the firm has to produce fluctuating quantities over time with no uncertainty about the quantities which must be produced. From the point of view of the logic of choices among different kinds of plants the two models are equivalent first if there is no specifically dynamic costs, *i.e.* costs resulting from the mere fact of changing the output levels, and second if holding inventories is not allowed (or prohibitively costly).

In this context, it is generally expected or stated (a folk theorem) that profit maximization leads the firm to choose a more flexible plant the more uncertain its demand function is or the more variable is the sequence of quantities to produce. We show in this paper that this intuitive statement is indeed valid under conditions which we make explicit. Those conditions, which are sufficient conditions, state that the increase in uncertainty must involve only those states of demand for which the firm is in fact active, that is for which it is able to cover its incremental cost (setup cost and variable cost). We also show that a reverse relation between the variability in demand and plant flexibility may exist under reasonable cost and demand conditions, even when one uses the most common definition of increased uncertainty in demand and the most reasonable acceptation of the notion of plant flexibility. We illustrate such a case where the increase in demand uncertainty does involve states of (low) demand for which the firm prefers to shut down, possibly temporarily.

The intuitive reasoning behind this result are twofold. Suppose that the firm's optimal size and flexibility imply that it would not produce if demand falls below some (endogenous) benchmark. A reduction in flexibility reduces the minimum value of average cost. Then two cases must be distinguished. In the first case such a reduction will reduce also the abovementioned benchmark; hence the firm would now produce for some realizations of demand for which it was not producing before the reduction in flexibility. If the increase in uncertainty makes those realizations of demand more probable than before, it *may* be profitable – in terms of expected profits – for the firm to reduce its flexibility in order to produce in those realized states even if such an action reduces its profits in some other realizations of demand. In the second case the reduction in flexibility increases the lower bound of demand under which the firm will not be producing. If the increase in uncertainty reduces the probability of the states of demand in the neighborhood of the initial benchmark, then the less flexible technique may be more profitable. This is indeed the case in the example of section 5 below. However, if the increase in uncertainty involves only realizations of demand above the above-mentioned benchmark, then such an increase always implies an increase in flexibility. There is no reason to believe that the aforementioned cases are pathological; it is in fact quite generic and therefore, the inverse relationship may be empirically quite relevant.

The model with which we will work is exposed in section 2. In section 3 we review the main definitions of flexibility found in the literature. In section 4 we demonstrate our results on the relationship between uncertainty and flexibility. An example of a greater uncertainty implying the choice of a less flexible plant is given in section 5. In the conclusion we interpret our results in the spirit of the JONES and OSTROY [1984] work on the relation between flexibility and uncertainty and in the spirit of the FREIXAS and LAFFONT [1984] work on the irreversibility effect.

2 The Model

In this paper we adopt the first of the two frameworks referred to in the introduction. It will be assumed that a risk neutral monopoly firm must make two decisions: at t_1, it must choose some plant, with a particular installation cost which will be denoted by K(.). When picking an equipment, the inverse demand functions is known only in probability.

The inverse demand function is assumed linear: $p = \max\{\alpha - \beta q, 0\}$ where α is a random variable with support $[\underline{\alpha}, \bar{\alpha}]$, density function $f(.)$ and cumulative function F(.) which can be seen either as prior beliefs or as an objective probability. Each equipment is inducing some variable cost function TVC(q; .). Before t_2 the true value of α is disclosed and at t_2 the firm chooses its production level q.

For any chosen plant, only the alternative costs associated to the different production levels are relevant in period 2. In order to keep matters simple let us assume that the scrap and/or residual value of the plant is always zero, whatever be the production level, so that the firm is faced with variable costs which are zero at zero production level. Define the capacity of the plant as the production level which minimizes the average variable costs. Let us denote by x this production level and by a the minimum average variable cost. In order that the average cost curve be relatively flat in the neighborhood of the capacity, the marginal cost curve must be locally not too steep. The most simple case is the linear one for which the slope is an index of plant flexibility: the smaller the slope of the marginal cost curve is the more flexible the plant is (see in that spirit STIGLER [1939], MARSCHAK and NELSON [1962], ORR [1967], VIVES [1986*a*], [1986*b*]). So let:

$$\mathrm{MVC} = r + \rho(q - x), \qquad r > 0, \quad \rho > 0$$

be the marginal cost function. The marginal cost must be equal to the average variable cost at $q = x$, hence:

$$r = a$$

and the total variable cost takes the following form:

$$\mathrm{TVC} = \int_0^q [a + \rho(z - x)]\, dz = (a - \rho x) q + \rho \frac{q^2}{2} + k$$

where k is the integration constant. In order to determine this constant, we know that the average variable cost is minimized at $q = x$, [1] so that

$$\frac{d\mathrm{AVC}}{dq} = \frac{\rho}{2} - \frac{k}{q^2}\bigg|_{q=x} = 0$$

hence :

$$k = \frac{1}{2} \rho x^2.$$

If the marginal cost is linear the total variable cost function has necessarily the following form where $\gamma = \frac{1}{2}\rho$

$$\text{(1)} \qquad \mathrm{TVC} = \begin{cases} 0 & \text{if } q = 0 \\ \gamma x^2 + (a - 2\gamma x) q + \gamma q^2 & \text{if } q > 0 \end{cases}$$

with $a - 2\gamma x \geqslant 0$.

In the expression (1), the triplet (a, γ, x) is chosen at t_1. To each given triplet of plant characteristics is associated a cost $\mathrm{K}(a, \gamma, x)$ and the set of admissible triplets is a subset D of R^3. The K(.) function and the boundary of D are not purely technical data but are determined also by prices of factors chosen in t_1, and prices of factors chosen in t_2. For the model to be meaningful it must be assumed, if K(.) is differentiable, that $\partial \mathrm{K}/\partial a \leqslant 0$ (a decrease in the minimum average cost for any given capacity and flexibility imply larger equipment outlays), $\partial \mathrm{K}/\partial x \geqslant 0$ (a larger capacity for a given minimum average cost and flexibility requires a more costly equipment), and

1. The capacity level could be defined as usual in terms of total average costs. For any given (α, γ, x) the production level minimizing the average total cost is $y = [\mathrm{K}(a, \gamma, x)/\gamma + x^2]^{1/2}$.

$\partial K/\partial\gamma \leqslant 0$ (a greater flexibility for a given capacity and minimum average cost cannot be obtained without larger expenses).

Suppose that the boundary of the set of admissible triplets can be described locally as the solution of $L(a, \gamma, x)=0$, with $L(a', \gamma', x') \geqslant 0$ if $(a', \gamma', x') \in D_1$, and $L(.)$ differentiable [note that $L(.)$ may depend on the boundary point considered]. Because it is always possible to pick a less efficient technique we must have $\partial L/\partial a > 0$ and $\partial L/\partial\gamma > 0$. Hence on the boundary $\partial a/\partial\gamma = -(\partial L/\partial\gamma)/(\partial L/\partial a) < 0$. Now it seems that the sign of $\partial L/\partial x$ can be either positive, nil or negative so that $\partial a/\partial x$ and $\partial\gamma/\partial x$ have no definite signs.

3 Review of General Definitions of Flexibility

The only studies (to our knowledge) in which general definitions of flexibility are given, and appropriate for the above model, are those of Marschak and Nelson [1962] and Jones and Ostroy [1984]. Marschak and Nelson study a decision problem with the following structure. The decision-maker chooses among several actions at t_1 and t_2, $t_2 > t_1$, after receiving, before each choice, some signal about the state of the world, the information conveyed by the message received before t_2 being at least as precise as the information received before t_1. His payoff depends on the selected actions and the state of the world. Formally let D_i be the set of actions open at t_i, d_i an element in D_i, Z_i the set of signals which can be received before t_i, z_i an element in Z_i, W the set of states of the world, w an element in W, and $\pi: D_1 \times D_2 \times W \rightarrow R$ the payoff function. The decision maker knows the joint distribution of (z_1, z_2, w). In the model presented in section 2 the decision maker gets a message of zero informational content before t_1, and a signal revealing the true state of the world before t_2. D_1 is the set of admissible triplets (a, γ, x) and D_2 the set of production levels q, *i.e.* $D_2 = R_+$.

Marschak and Nelson propose three alternative and different definitions of the greater flexibility of an initial position. The first definition assumes that the set of actions which can be chosen in t_2 depends on the action taken in t_1. Let us denote by $\mathscr{D}_2(d_1)$ the set of admissible actions in t_2 if d_1 is the position selected in t_1. Then d_1 is more flexible than d_1' if the set of admissible actions in t_2 starting from d_1 is larger than the set starting from d_1'. Formally:

Definition 1: The initial position d_1 is more flexible than the position d_1' if:

$$\mathscr{D}_2(d_1') \subseteq \mathscr{D}_2(d_1) \tag{2}$$

In terms of the irreversibility effect literature (HENRY [1974], FREIXAS and LAFFONT [1984]) d_1 is more flexible than d'_1 if d_1 is less irreversible than d'_1. Definition 1 is the only one, among the four definitions we will review, which is equivalent to the definition of irreversibility.

Two problems may be raised with such a definition in our context. First it does not take account of the cost of the additional actions allowed by the choice of d_1 instead of d'_1. According to the definition 1 a position d_1 would be more flexible than a position d'_1 even under such extreme conditions that:

(3)
$$\forall: w \in W, \quad d_2 \in \mathscr{D}_2(d_1), \quad d'_2 \in \mathscr{D}_2(d'_1): \\ \pi(d_1, d_2, w) < \pi(d'_1, d'_2, w)$$

which does not seem consistent with what is suggested by the words "more flexible". Second, it is always possible in the model of section 2, to produce any quantity whatever the triplet (a, γ, x) chosen at t_1. Hence, according to the definition 1, all plants would be equally flexible. [2]

The second definition of Marschak and Nelson takes into consideration the cost differences and assumes that the payoff function can be decomposed into a revenue depending on the state of the world and the action picked at t_2 and a cost depending on the choices made at t_1 and t_2:

(4)
$$\pi(d_1, d_2, w) = R(d_2, w) - C(d_1, d_2).$$

This is the case in our model where the revenue is a function of the production level $q = d_2$ and the state of the world $w = \alpha$, and the total cost (installation plus variable costs) is a function of the triplet $(a, \gamma, x) = d_1$ and the production level $q = d_2$.

An initial position d_1 is now qualified as more flexible than a position d'_1 if the two following conditions are satisfied. First for any given hypothetical cost differential θ there always exists some action d_2 for which the effective cost differential between (d_1, d_2) and (d'_1, d_2) will be larger than θ. Second for any second period choice d_2, the extra cost (possibly negative) induced by the choice of d_1 instead of d'_1 is bounded above.

DEFINITION 2: Suppose $\pi(d_1, d_2, w) = R(d_2, w) - C(d_1, d_2)$, then the initial position d_1 is more flexible than d'_1 if:

1.

(5)
$$\forall \theta > 0: \quad \exists d_2 \in D_2: \quad C(d'_1, d_2) - C(d_1, d_2) \geqslant \theta$$

2.

(6)
$$\exists \theta \geqslant 0: \quad \forall d_2 \in D_2: \quad C(d_1, d_2) - C(d'_1, d_2) \leqslant \theta.$$

2. For a model of plant choice, definition 1 would be operational for the following kind of tradeoffs: at t_1 a maximal production capacity is chosen and at t_2 an effective production level. A plant with a larger capacity would be more flexible, and this whatever would be the respective installation and variable costs of the two plants.

Note that if the set of admissible actions in t_2 depends on the initial position, it is always possible to consider that actions out of the set of admissible ones [outside $\mathscr{D}_2(d_1)$] are infinitely expensive. With this convention, in the context of plant capacity choice mentioned above, the two definitions 1 and 2 would induce the same flexibility ranking.

Applied to our model, definition 2 allows for a total lexicographic ordering of all triplets (a, γ, x) in D_1 according to the value of the coefficient γ. A commitment (a, γ, x) is more flexible than (a', γ', x') first if $\gamma < \gamma'$, whatever be (a, x) and (a', x') or second, when $\gamma = \gamma'$ if $(a - 2\gamma x) < (a' - 2\gamma x')$. For given (a, γ, x) and (a', γ', x') in D_1 the installation cost difference is given by $K(a, \gamma, x) - K(a', \gamma', x')$; let us denote it by Δ. For either $[\gamma < \gamma']$, or $[\gamma = \gamma'$ with $(a - \gamma x) < (a' - \gamma x')]$, there exists some production level $\bar{q}$ which depends on (a, γ, x), (a', γ', x') and Δ such that for any $q > \bar{q}$ we have $TVC(q; a', \gamma', x') - TVC(q; a, \gamma, x) > \Delta$; hence (5) is satisfied. Now let $\hat{q}$ be the production level for which the difference in variable costs is maximized:

$$\hat{q} = \arg\max_{q} \{TVC(q; a, \gamma, x) - TVC(q; a', \gamma', x')\} \tag{7}$$

and denote by Γ the difference in total variable costs at $\hat{q}$. It is clear that (6) is verified for $\theta = \max\{0, \Delta + \Gamma\}$. [3]

For the cases where the payoff function is not decomposable according to (4), Marschak and Nelson propose a third definition. Let us first define $\mathscr{D}_2^*(z_2, d_1)$ as the set of actions in $\mathscr{D}_2$ for which the expected profit at t_2 given the signal z_2 and the initial position d_1 is maximized:

$$\mathscr{D}_2^*(z_2, d_1) \equiv \arg\max_{d_2 \in \mathscr{D}_2} E\{\pi(d_1, d_2, w) \mid z_2\}. \tag{8}$$

Let $\pi_2^*(z_2, d_1)$ be the maximized expected profit at t_2 given z_2 and d_1:

$$\pi_2^*(z_2, d_1) = E_w\{\pi(d_1, d_2^*(z_2, d_1), w) \mid z_2\} \tag{9}$$

where $d_2^*(z_2, d_1) \in \mathscr{D}_2^*(z_2, d_1)$.

An initial position d_1 is thought as more flexible than d_1' if, firstly, for any given hypothetical expected profit differential θ, there exists some signal z_2 for which the difference between $\pi_2^*(z_2, d_1)$ and $\pi_2^*(z_2, d_1')$ is larger than θ and secondly, the increment in expected profits allowed by the choice of d_1 instead of d_1' is bounded whatever the signal z_2 received before t_2.

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3. It should be noted that in definition 2 only costs are taken into consideration. Hence it is possible for the d_2 which appears in the conditions (5) and (6) to be totally irrelevant. For example in our model, an initial position (a, γ, x) may be more flexible than (a', γ', x') even if for any $q \in (0, \bar{\alpha}/\beta)$ (*i.e.* any production level for which the revenue would be positive in the most favorable state of the world $\alpha = \bar{\alpha}$), the total variable cost with the (a, γ, x) technique is larger that the total variable cost with the (a', γ', x') technique, whatever the installation costs of two kinds of equipments, *i.e.* even with an installation cost for (a, γ, x) larger than for (a', γ', x').

DEFINITION 3: The initial position d_1 is more flexible than d_1' if:

1.

$$\forall \theta > 0: \ \exists z_2 \in Z_2: \quad \pi_2^*(z_2, d_1) - \pi_2^*(z_2, d_1') > \theta \tag{10}$$

2.

$$\exists \theta > 0: \ \forall z_2 \in Z_2: \ \pi_2^*(z_2, d_1') - \pi_2^*(z_2, d_1) \leqslant \theta \tag{11}$$

This third definition does not always imply the same order among the initial positions as the definition 2 when the decomposition (4) is possible. In our model for example, no equipment can be qualified as more flexible in the sense of definition 3 than any other one if $\bar{\alpha} < \infty$. Indeed for any technique (a, γ, x), given $\alpha \leqslant \bar{\alpha} < \infty$, the maximum profit over variable costs, *i.e.* $\max\{\max[\alpha - \beta q, 0] q - \mathrm{TVC}(q; a, \gamma, x)\}$, is bounded. Hence (10) cannot be satisfied. On the contrary if $\bar{\alpha} = \infty$, then the ordering of techniques induced by the definition 3 is the same as the ranking implied by the definition 2: the equipment (a, γ, x) is more flexible than the equipment (a', γ', x') if either $[\gamma < \gamma']$, for any given (a, x) and (a', x'), or $[\gamma = \gamma'$ and $(a - \gamma x) < (a' - \gamma' x')]$. There always exists some production level q for which the difference $\mathrm{TVC}(q; a, \gamma, x) - \mathrm{TVC}(q; a', \gamma', x')$ is as large as desired and there always exists some value of α for which at this production level the revenue is larger than the variable costs under the most expensive technique (in terms of variable costs), *i.e.* the (a', γ', x') technique. Hence condition (10) is satisfied. Concerning now the condition (11), let us remark that the maximum difference in profits over variable costs between the techniques (a, γ, x) and (a', γ', x'), when considering all the productions levels and all the values of α, is equal to the maximum difference in the sole variable costs:

$$\begin{aligned} &\operatorname*{Max}_{q, \alpha} [(\max\{\alpha - \beta q, 0\} q - \mathrm{TVC}(q; a', \gamma', x')) \\ &\quad - (\max\{\alpha - \beta q, 0\} q - \mathrm{TVC}(q; a, \gamma, x))] \\ &= \operatorname*{Max}_{q} \{\mathrm{TVC}(q; a, \gamma, x) - \mathrm{TVC}(q; a', \gamma', x')\} \end{aligned} \tag{12}$$

For either $[\gamma < \gamma']$ or $[\gamma = \gamma'$ and $(a - \gamma x) < (a' - \gamma x')]$, this maximum takes a finite value Γ [the same as defined in (7)]. Let $\theta = \max\{0, \Delta + \Gamma\}$ [where $\Delta = \mathrm{K}(a, \gamma, x) - \mathrm{K}(a', \gamma', x')$ as above]; then (11) is satisfied for this θ.

The decision problem investigated by Jones and Ostroy is similar to the Marschak and Nelson one. A decision maker must choose among several possible actions at t_1 and t_2. His payoffs depend on his choices and the state of the world. At t_1 the choice is made on the basis of some prior beliefs and between t_1 and t_2 the decision maker receives a signal about the state of the world. We can use the notations introduced in the preceding paragraphs.

Jones and Ostroy assume that the payoff function is decomposable in three additive components:

$$\pi(d_1, d_2, w) = r(d_1, w) + u(d_2, w) - c(d_1, d_2, w) \tag{13}$$

where $r(d_1, w)$ is the payoff generated by the position chosen at t_1, $u(d_2, w)$ the payoff specific to the chosen action at t_2 and $c(d_1, d_2, w)$ a switching cost from d_1 to d_2. Let us define $D_2(d_1, w, \theta)$ as the set of actions at t_2 which, given the initial position d_1, imply in the state w, a switching cost at most equal to θ:

$$(14) \qquad D_2(d_1, w, \theta) = \{d_2 \mid c(d_1, d_2, w) \leqslant \theta\}.$$

The switching cost function $c(.)$ is required to satisfy two conditions: impossibility of negative switching costs and existence of a terminal position with zero switching cost. By impossibility of negative switching costs it must be understood that:

$$(15) \qquad \forall: d_1 \in D_1, \quad w \in W, \quad \theta < 0: \qquad D_2(d_1, w, \theta) = \varnothing$$

and by existence of a terminal position with zero switching cost, that there exists some function $h: D_1 \to D_2$ such that:

$$(16) \qquad \forall: d_1 \in D_1, \quad w \in W: \qquad h(d_1) \in D_2(d_1, w, 0).$$

An initial position d_1 is, according to Jones and Ostroy, more flexible than d_1' if, for any state of the world and any given upper bound on the switching costs, the set of actions available at t_2 given d_1 and the given upper bound on the switching costs includes, except for $h(d_1)$, the similar set obtained from d_1'.

DEFINITION 4: The initial position d_1 is more flexible than d_1' if:

$$(17) \qquad \forall: w \in W, \quad \theta \geqslant 0: \qquad D_2(d_1', w, \theta) \setminus \{h(d_1)\} \subset D_2(d_1, w, \theta)$$

The definition of Jones and Ostroy is given for any decision problem but they have clearly in mind financial portfolio choices and they try to capture the notion of more or less flexibility of such portfolios. If there exists some set of financial assets which are available at the start of two consecutive periods, each asset having a life of at least two periods, the decision maker is given the option of staying in the second period with this initial portfolio. The switching cost is then zero. Generally any change in the portfolio structure implies some transaction costs, hence to any given initial position d_1 will correspond only one terminal position $h(d_1)$ with zero switching cost: the terminal position consisting in keeping the initial portfolio. Jones and Ostroy are then lead to subtract $h(d_1)$ from $D_2(d_1', w, \theta)$ in (17). Without this subtraction no portfolio would be ever more flexible than any other. But note that in a general decision problem, the set of actions at t_2 with zero switching cost given some initial position d_1, may include more than one element; $h(d_1)$ would then be a set-valued function.

This conceptual apparatus can be applied to our problem by identifying the $r(d_1, w)$ function to the installation cost, $-K(a, \gamma, x)$, the $u(d_2, w)$ function to the revenue, $\max\{\alpha - \beta q, 0\} q$, and the switching cost $c(d_1, d_2, w)$ to the variable cost function $TVC(q; a, \gamma, x)$. It is the only identification system with which the two conditions of "impossibility of negative switching cost" and "existence of a terminal position with zero switching cost" can be obtained. The first condition corresponds trivially to the fact that variable costs are non negative and the second one to the fact that these costs are zero if it is decided not to produce at t_2. Then some technique (a, γ, x) will be more flexible than another one (a', γ', x') if for any given upper bound θ to the variable costs, the set of production levels with variable cost not larger than θ with the technique (a, γ, x) includes the similar set with the technique (a', γ', x'). Clearly this criterion does not define a complete order on the set of available techniques. If for example $[x \neq x', a=a'$, and $\gamma=\gamma']$ or $[x=x', a < a'$ and $\gamma > \gamma']$ the two equipments cannot be ordered. It is only if $x=x'$, $a \leqslant a'$ and $\gamma < \gamma'$ that (a, γ, x) can be said to be more flexible than (a', γ', x').

4 The Characterization of Optimal Choices

The total variable cost function derived in section 2 is

$$(1) \qquad \mathrm{TVC} = \begin{cases} 0 & \text{if} \quad q=0 \\ \gamma x^2 + (a-2\gamma x) q + \gamma q^2 & \text{if} \quad q>0. \end{cases}$$

We consider that the firm chooses the technological triplet (a, γ, x) in the first stage (before α is observed) and then q in the second stage (after α is observed).[4] We can solve first for the optimal value of q as a function of α and β for given values of a, γ and x, say $q^* = q(\alpha, \beta; a, \gamma, x)$. Straightforward derivation leads to the optimal output function

$$(18) \qquad q(\alpha, \beta; a, \gamma, x) = \frac{1}{2(\beta+\gamma)} [\alpha - (a - 2\gamma x)]$$

and the variable profit function

$$(19) \qquad \mathrm{V}\pi(\alpha, \beta; a, \gamma, x) = \frac{1}{4(\beta+\gamma)} [\alpha - (a-2\gamma x)]^2 - \gamma x^2.$$

The firm will produce only if it covers its variable costs, that is only if $\mathrm{V}\pi(.) \geqslant 0$. This leads to a lower bound on α, say $\alpha(\beta, a, \gamma, x)$, such that $q(\alpha, \beta; a, \gamma, x) = 0$ if $\alpha < \alpha(\beta, a, \gamma, x)$. This lower bound can be found as the smallest value of α for which $\mathrm{V}\pi(.)$ is non negative. Clearly, $\alpha(\beta, a, \gamma, x) > a - 2\gamma x$; moreover, $\mathrm{V}\pi(.)$ is a quadratic function of α and

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4. We consider only the monopolist's decisions and therefore we rule out strategic considerations (see VIVES [1986*a*], [1986*b*] and SPENCER and BRANDER [1987] and competitive behavior considerations (see TISDELL [1968], SHESHINSKI and DRÈZE [1975], MILLS [1986]) in the choice of γ.

is negative at $\alpha = a - 2\gamma x$. We must therefore find the larger root of $V\pi(.) = 0$. This root is $\alpha(\beta, a, \gamma, x) = a - 2x\gamma^{1/2}(\gamma^{1/2} - (\beta+\gamma)^{1/2})$. Since $\gamma^{1/2} - (\beta+\gamma)^{1/2} < 0$, we have $\alpha(\beta, a, \gamma, x) > a - 2\gamma x$ as expected. Hence we can rewrite the optimal output function in the second stage as

$$\text{(20)} \quad q(\alpha, \beta; a, \gamma, x) = \begin{cases} 0 & \text{if} \quad \alpha < \alpha(\beta, a, \gamma, x) \\ \dfrac{1}{2(\beta+\gamma)}[\alpha - (a - 2\gamma x)] & \text{if} \quad \alpha \geqslant \alpha(\beta, a, \gamma, x) \end{cases}$$

with

$$\partial\alpha(.)/\partial a = 1, \qquad \partial\alpha(.)/\partial x = -2\gamma^{1/2}(\gamma^{1/2} - (\beta+\gamma)^{1/2}) > 0$$

and

$$\partial\alpha(.)/\partial\gamma = -2x + x[(\beta+\gamma)^{1/2}\gamma^{-1/2} + \gamma^{1/2}(\beta+\gamma)^{-1/2}]$$

which is positive since the factor in square bracket is increasing in β and equal to 2 for $\beta = 0$. Hence the lower bound on α is increasing with a, x and γ: the larger the minimum average variable cost is or the larger the efficient production scale (capacity) is or the more inflexible the technology is, the more likely it is that the firm will not produce in the second stage. Moreover for $\alpha > \alpha(\beta, a, \gamma, x)$ we have

$$\partial q(.)/\partial a = -\frac{1}{2}(\beta+\gamma)^{-1} < 0, \qquad \partial q(.)/\partial x = \gamma(\beta+\gamma)^{-1} \in (0, 1),$$

$$\partial q(.)/\partial\gamma = \frac{1}{2}(-\alpha + a + 2\beta x)(\beta+\gamma)^{-2}$$

which is positive [negative] for $\alpha \langle [\rangle] a + 2\beta x$. It is interesting to note here that $q(\alpha, \beta; a, \gamma, x)$ is discontinuous at $\alpha = \alpha(\beta, a, \gamma, x)$ while $V\pi(\alpha, \beta, a, \gamma, x)$ is continuous but not differentiable at the same point. We can now characterize the optimal choices of a, γ, x in the first stage.

Since the triplet (a, γ, x) is chosen before α is revealed, it will be chosen so as to maximize the expected profit

$$\text{(21)} \quad E\pi(\alpha, \beta; a, \gamma, x) = \int_{\alpha(\beta, a, \gamma, x)}^{\bar{\alpha}} V\pi(\alpha, \beta; a, \gamma, x) f(\alpha)\, d\alpha - K(a, \gamma, x).$$

First order conditions are as follows, where we make use of the fact that $V\pi(\alpha(\beta, a, \gamma, x); a, \gamma, x) = 0$:

$$\text{(22)} \quad \partial E\pi/\partial a = -\frac{\partial K}{\partial a} + \int_{\alpha(\beta, a, \gamma, x)}^{\bar{\alpha}} \frac{\gamma}{2(\beta+\gamma)}[\alpha - (a - 2\gamma x)] f(\alpha)\, d\alpha = 0$$

$$\text{(23)} \quad \partial E\pi/\partial\gamma = -\frac{\partial K}{\partial\gamma} + \int_{\alpha(\beta, a, \gamma, x)}^{\bar{\alpha}} \frac{x}{\beta+\gamma}[\alpha - (a - 2\gamma x)] f(\alpha)\, d\alpha - \int_{\alpha(\beta, a, \gamma, x)}^{\bar{\alpha}} \frac{\gamma}{4(\beta+\gamma)^2}[\alpha - (a - 2\gamma x)]^2 f(\alpha)\, d\alpha - \int_{\alpha(\beta, a, \gamma, x)}^{\bar{\alpha}} x^2 f(\alpha)\, d\alpha = 0$$

$$(24) \qquad \partial \mathrm{E}\pi/\partial x = -\frac{\partial \mathrm{K}}{\partial x} + \frac{\gamma}{\beta+\gamma}\int_{\alpha(\beta,\, a,\, \gamma,\, x)}^{\bar{\alpha}} [\alpha-(a-2\gamma x)]\, f(\alpha)\, d\alpha - \int_{\alpha(\beta,\, a,\, \gamma,\, x)}^{\bar{\alpha}} 2\gamma x f(\alpha)\, d\alpha = 0$$

We saw in section 3 how flexibility can be defined. In general, two technologies (a, γ, x) and (a', γ', x') will not be comparable flexibility-wise unless they happen to satisfy some restrictive conditions. Let (a^*, γ^*, x^*) be the expected profit maximizing technology satisfying the first order conditions above. Introducing more uncertainty in the distribution function $\mathrm{F}(\alpha)$ will bring a change in the chosen technology but to compare the new technology with the previous one on the basis of flexibility requires a basis of comparison which must now be introduced. In the spirit of Stigler [1939], Marschak and Nelson [1962] and Jones and Ostroy [1984], we will use the following definition of flexibility.

Definition 5: A technology (a, γ, x) is more flexible than another technology (a', γ', x') if there exists a finite interval Q (convex) such that the average variable costs satisfy

$$(25) \qquad \begin{cases} \mathrm{AVC}(q;\, a, \gamma, x) \geqslant \mathrm{AVC}(q;\, a', \gamma', x') & \text{for } \; q \in \mathrm{Q} \\ \mathrm{AVC}(q;\, a, \gamma, x) < \mathrm{AVC}(q;\, a', \gamma', x') & \text{for } \; q \notin \mathrm{Q} \end{cases}$$

Moreover, for $x = x'$, an increase in γ', the coefficient of inflexibility, should lead to a new interval Q′ such that $\mathrm{Q}' \subset \mathrm{Q}$.

These properties allow for an ordinal ranking of technologies characterized by a constant x but for different values of γ. They can be obtained by adjusting the technological parameter a as a function of γ and x. Therefore, instead of considering a as a free variable, we will consider it as completely determined by the choice of γ and x through a function $a(\gamma, x)$. Alternatively, we could restrict the feasible set D_1, introduced in section 2, in such a way that the optimal choices (a, γ, x) in D_1 satisfy the function $a = a(\gamma, x)$. A sufficient condition for the intervals Q and Q′ to exist and satisfy the above requirements is that $a(\gamma, x)$ be decreasing and convex with respect to γ as we now show.

Let $\gamma' > \gamma$; we want the two average variable cost functions $\mathrm{AVC}(a(\gamma, x), \gamma, x)$ and $\mathrm{AVC}\,(a(\gamma', x), \gamma', x)$ to intersect twice in order to obtain the desired interval Q. Hence

$$a(\gamma', x) - 2\gamma' x + \frac{\gamma' x^2}{q} + \gamma' q = a(\gamma, x) - 2\gamma x + \frac{\gamma x^2}{q} + \gamma q$$

should have two positive roots. Such is the case if $a(\gamma, x)$ is decreasing in γ. Let the smaller root be q_- and the larger be q_+. For the interval Q to shrink as γ' increases, we must have $\partial q_-/\partial\gamma' > 0$ and $\partial q_+/\partial\gamma' < 0$. Such will be the case if $a(\gamma, x)$ is convex in γ. From now on, we will assume that $a(\gamma, x)$ is decreasing and convex in γ. By doing so, we restrict ourselves to a family of technologies which can be compared flexibility-wise for a given x. The AVC functions for a given x and different γ appear in Figure 1.

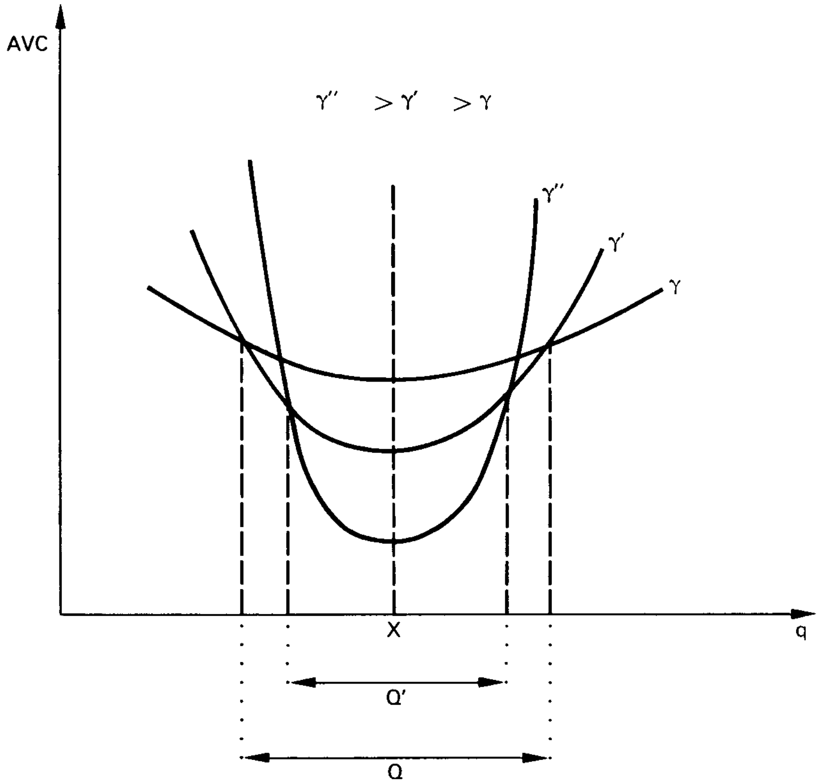

FIGURE 1

In order to compare two situations in terms of the uncertainty they represent, we will use the concept of mean preserving spread (MPS) of ROTHSCHILD and STIGLITZ [1970]. Considering a given distribution $F(\alpha)$ and the associated technology chosen by the monopolist, say $(a(\gamma, x), \gamma, x)$, we will introduce a MPS on $F(\alpha)$ and look at the implied change in the technology chosen in order to find out if γ has increased or not.

Considering $a(\gamma, x)$, we can rewrite the variable profit (19) and the first order conditions (23) and (24) by substituting $\alpha(\beta, \gamma, x)$ for $\alpha(\beta, a(\gamma, x), \gamma, x)$ and $a(\gamma, x)$ for a before taking derivatives. We obtain

$$(19') \qquad V\pi(\alpha; \gamma, x) = \frac{1}{4(\beta+\gamma)}[\alpha-(a(\gamma, x)-2\gamma x)]^2-\gamma x^2$$

with $a(\gamma, x) > 2\gamma x$ by assumption

$$\text{(23')}\quad \partial E\pi/\partial\gamma = -\frac{\partial K}{\partial\gamma} + \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \frac{1}{2(\beta+\gamma)}[\alpha-(a(\gamma,x)-2\gamma x)] \times\left[-\frac{\partial a(\gamma,x)}{\partial\gamma}+2x\right] f(\alpha)\,d\alpha - \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \frac{1}{4(\beta+\gamma)^2}[\alpha-(a(\gamma,x)-2\gamma x)]^2 f(\alpha)\,d\alpha - \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} x^2 f(\alpha)\,d\alpha = 0$$

$$\text{(24')}\quad \partial E\pi/\partial x = -\frac{\partial K}{\partial x} + \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \frac{1}{2(\beta+\gamma)}[\alpha-(a(\gamma,x)-2\gamma x)] \times\left[-\frac{\partial a(\gamma,x)}{\partial x}+2\gamma\right] f(\alpha)\,d\alpha - \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} 2\gamma x f(\alpha)\,d\alpha = 0$$

$$\text{(26)}\quad \alpha(\beta,\gamma,x) = a(\gamma,x) - 2x\gamma^{1/2}(\gamma^{1/2}-(\beta+\gamma)^{1/2}).$$

We will introduce an increase in uncertainty by adding to $f(\alpha)$ a MPS $h(\alpha)$ with CDF $H(\alpha)$. Let

$$w(\alpha) = f(\alpha) + \varepsilon h(\alpha)$$

where $h(\alpha)$ is a MPS, that is $h(\alpha)$ satisfies the following properties:

$$\text{(27)}\quad \begin{cases} \int_{\underline{\alpha}}^{\bar{\alpha}} \alpha h(\alpha)\,d\alpha = 0; \quad H(\bar{\alpha}) = 0; \quad \int_{\underline{\alpha}}^{\bar{\alpha}} H(\alpha)\,d\alpha = 0; \\ \exists z: \quad H(\alpha) \geqslant 0 \text{ for } \alpha \leqslant z \text{ and } H(\alpha) \leqslant 0 \text{ for } \alpha > z; \\ z(\alpha) \text{ concave} \Rightarrow \int_{\underline{\alpha}}^{\bar{\alpha}} z(\alpha) h(\alpha)\,d\alpha < 0. \end{cases}$$

A marginal increase in uncertainty from $f(\alpha)$ can be modelled as an increase in ε evaluated at $\varepsilon = 0$. Let $E\pi(\varepsilon)$ be the expected profit obtained by using $w(\alpha)$ rather than $f(\alpha)$. Totally differentiating the first-order conditions with respect to x, γ and ε leads to

$$\text{(28)}\quad \begin{cases} \frac{\partial^2 E\pi(\varepsilon)}{\partial\gamma^2} d\gamma + \frac{\partial^2 E\pi(\varepsilon)}{\partial\gamma\,\partial x} dx = -\frac{\partial^2 E\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon} d\varepsilon \\ \frac{\partial^2 E\pi(\varepsilon)}{\partial x\,\partial\gamma} d\gamma + \frac{\partial^2 E\pi(\varepsilon)}{\partial x^2} dx = -\frac{\partial^2 E\pi(\varepsilon)}{\partial x\,\partial\varepsilon} d\varepsilon. \end{cases}$$

With all the derivatives evaluated at the optimal x^*, γ^* and $\varepsilon = 0$. Let D be the determinant of the system: $D > 0$ with $d_{11} < 0$, $d_{22} < 0$ and $d_{12} \gtrless 0$.

We must evaluate $\left.\frac{\partial^2 \mathrm{E}\pi}{\partial\gamma\,\partial\varepsilon}\right|_{\varepsilon=0}$ and $\left.\frac{\partial^2 \mathrm{E}\pi}{\partial x\,\partial\varepsilon}\right|_{\varepsilon=0}$.

Consider first $\left.\frac{\partial^2 \mathrm{E}\pi}{\partial\gamma\,\partial\varepsilon}\right|_{\varepsilon=0}$:

$$(29)\quad \frac{\partial^2 \mathrm{E}\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon} = -x^2 \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha$$
$$- \frac{((\partial a(\gamma,x)/\partial\gamma)-2x)}{2(\beta+\gamma)} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \alpha\, h(\alpha)\,d\alpha$$
$$+ \frac{((\partial a(\gamma,x)/\partial\gamma)-2x)(a(\gamma,x)-2\gamma x)}{2(\beta+\gamma)} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha$$
$$- \frac{(a(\gamma,x)-2\gamma x)^2}{4(\beta+\gamma)^2} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha$$
$$+ \frac{(a(\gamma,x)-2\gamma x)}{2(\beta+\gamma)^2} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \alpha\, h(\alpha)\,d\alpha$$
$$+ \frac{1}{4(\beta+\gamma)^2} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} (-\alpha^2)\, h(\alpha)\,d\alpha.$$

We must consider two cases:

$$\alpha(\beta,\gamma,x) \leqslant \underline{\alpha} \qquad \text{and} \qquad \alpha(\beta,\gamma,x) > \underline{\alpha}.$$

In the first case, whatever be the value in $[\underline{\alpha}, \bar{\alpha}]$ of the intercept of the demand function, production takes place in the second stage and $\alpha(\beta, \gamma, x)$ may be set at $\underline{\alpha}$. Then, by the properties (27) of a MPS, we have

$$(30)\qquad \frac{\partial^2 \mathrm{E}\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon} = \frac{1}{4(\beta+\gamma)^2} (-\alpha^2)\, h(\alpha)\,d\alpha < 0$$

which is negative since $-\alpha^2$ is concave. In the second case, $\alpha(\beta, \gamma, x) > \underline{\alpha}$, we could consider a MPS involving only values of α above $\alpha(\beta, \gamma, x)$, that is such that $h(\alpha)=0$ for $\alpha \leqslant \alpha(\beta, \gamma, x)$. Then $\frac{\partial^2 \mathrm{E}\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon} < 0$ as before and therefore for these two forms of increased uncertainty, $\left.\frac{\partial\gamma}{\partial\varepsilon}\right|_{x \text{ given}} < 0$ indicating that such increases in uncertainty increase the technological flexibility chosen by the firm. [5]

In the case where the MPS involves value of α below $\alpha(\beta, \gamma, x)$ it is no more true that $\int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha = 0$ or that $\int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \alpha\, h(\alpha)\,d\alpha = 0$ and the sign

5. This case is the only case considered in the literature. Hence the positive relation usually found in the Industrial Organization literature. See for example MILLS [1984] and VIVES [1986 *a*], [1986 *b*].

of $\frac{\partial^2 E\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon}$ is indeterminate. Hence the possibility that such an increase in uncertainty will reduce the technological flexibility chosen by the firm. The example presented in the next section clearly shows the existence of this phenomenon.

Consider now $\frac{\partial^2 E\pi(\varepsilon)}{\partial x\,\partial\varepsilon}$:

$$(31)\quad \frac{\partial^2 E\pi(\varepsilon)}{\partial x\,\partial\varepsilon} = -2\gamma x \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha - \frac{((\partial a(\gamma,x)/\partial x)-2\gamma)}{2(\beta+\gamma)} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \alpha h(\alpha)\,d\alpha + \frac{((\partial a(\gamma,x)/\partial x)-2\gamma)(a(\gamma,x)-2\gamma x)}{2(\beta+\gamma)} \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} h(\alpha)\,d\alpha$$

Again, we must consider two cases: $\alpha(\beta,\gamma,x) \leqslant \underline{\alpha}$ and $\alpha(\beta,\gamma,x) > \underline{\alpha}$. In the former case, we can use the properties of an MPS to obtain $\frac{\partial^2 E\pi(\varepsilon)}{\partial x\,\partial\varepsilon} = 0$. In the second case, $\alpha(\gamma,x) > \underline{\alpha}$, we could consider first a MPS on the interval $[\alpha(\gamma,x),\bar{\alpha}]$ which is also a MPS on $[\underline{\alpha},\bar{\alpha}]$ to obtain as above $\frac{\partial^2 E\pi^*}{\partial x\,\partial\varepsilon} = 0$ and therefore for these two forms of increased uncertainty, $\left.\frac{\partial x}{\partial\varepsilon}\right|_{\gamma\text{ given}} = 0$, indicating that an increase in uncertainty has no effect on the technological efficiency level (capacity) chosen by the firm, at least for the given optimal flexibility level γ^*. However, if the MPS involves values of α below $\alpha(\beta,\gamma,x)$ in the second case, we cannot determine the sign of the effect of an increase in uncertainty on x.

If we allow both x and γ to vary with ε, then again we must consider two cases. In the first case, $\alpha(\beta,\gamma,x) \leqslant \underline{\alpha}$, we obtain:

$$(32)\quad \frac{d\gamma}{d\varepsilon} = \frac{\begin{vmatrix} -\frac{\partial^2 E\pi}{\partial\gamma\,\partial\varepsilon} & \frac{\partial^2 E\pi}{\partial\gamma\,\partial x} \\ -\frac{\partial^2 E\pi}{\partial x\,\partial\varepsilon} & \frac{\partial^2 E\pi}{\partial x^2} \end{vmatrix}}{D} < 0$$

which is negative since $D > 0$, $\partial^2 E\pi/\partial\gamma\,\partial\varepsilon < 0$, $\partial^2 E\pi/\partial x\,\partial\varepsilon = 0$ and $\partial^2 E\pi/\partial x^2 < 0$. Hence inflexibility γ decreases with the increase in uncertainty. Also

$$(33)\quad \frac{dx}{d\varepsilon} = \frac{\begin{vmatrix} \frac{\partial^2 E\pi}{\partial\gamma^2} & -\frac{\partial^2 E\pi}{\partial\gamma\,\partial\varepsilon} \\ \frac{\partial^2 E\pi}{\partial x\,\partial\gamma} & -\frac{\partial^2 E\pi}{\partial x\,\partial\varepsilon} \end{vmatrix}}{D} \gtrless 0,$$

the sign of which cannot be determined since the sign of $\frac{\partial^2 \mathrm{E}\,\pi}{\partial x\,\partial\gamma}$ is not known. MILLS and SCHUMANN [1985] offer empirical evidence that firm size (capacity) and flexibility vary inversely within industries. Such an inverse relationships could be obtained here if the marginal profitability of capacity increases with inflexibility or decreases with flexibility, that is if $\frac{\partial^2 \mathrm{E}\,\pi^*}{\partial x\,\partial\gamma} > 0$. In such a case $\partial x/\partial\varepsilon < 0$, since $\partial^2 \mathrm{E}\,\pi/\partial x\,\partial\varepsilon = 0$, $\partial^2 \mathrm{E}\,\pi/\partial\gamma\,\partial\varepsilon < 0$ (30) and $\partial^2 \mathrm{E}\,\pi/\partial x\,\partial\gamma > 0$ (by assumption). Hence capacity then decreases with an increase in uncertainty. Those results can also be derived when $\alpha(\beta, \gamma, x) > \underline{\alpha}$ if the MPS involves only values of α above $\alpha(\beta, \gamma, x)$. Otherwise, both effects are of indeterminate sign.

In order to get some feeling about how increased uncertainty can reduce flexibility, consider the following example illustrated in Figure 2, where we compare two techniques with different flexibility but the same capacity level. Switching from γ to γ', with $\gamma < \gamma'$, increases the benchmark of the demand level under which the plant is idle: $\alpha(.,\gamma,.) < \alpha(.,\gamma',.)$. Suppose that $\underline{\alpha} < \alpha(.,\gamma,.)$ and that the initial distribution is concentrated around $\alpha(.,\gamma,.)$ so that technique γ is more profitable than γ'. Let us consider a MPS reducing the probability of α in the neighborhood of $\alpha(.,\gamma,.)$ and putting additional weight in the vicinity of $\underline{\alpha}$ and $\bar{\alpha}$. Hence the demand realizations for which γ is more profitable than γ' are less frequent so that the increased uncertainty would result in the adoption of the less flexible technique γ'.

5 An Example

The object of this section is to show through an example that an increase in uncertainty may very well induce a firm to move towards a less flexible technology when the increase in uncertainty does involve the probability density function over states of the world in which the firm would not produce in the second stage, that is once the uncertainty is levied. We saw in the previous section that if the increase in uncertainty, measured by a mean preserving spread applied to the original distribution, involves only states for which the firm does produce in the second stage, then it induces the firm to move towards a more flexible position or technology even when both the flexibility parameter and the efficient scale parameter of the technology are allowed to change. However, there is no reason to believe that situations in which a firm does not produce for some realizations or states in the support of the distribution are atypical or exceptional. Hence we believe that the somewhat counterintuitive result that flexibility is reduced in the presence of increased uncertainty may be an important phenomenon.

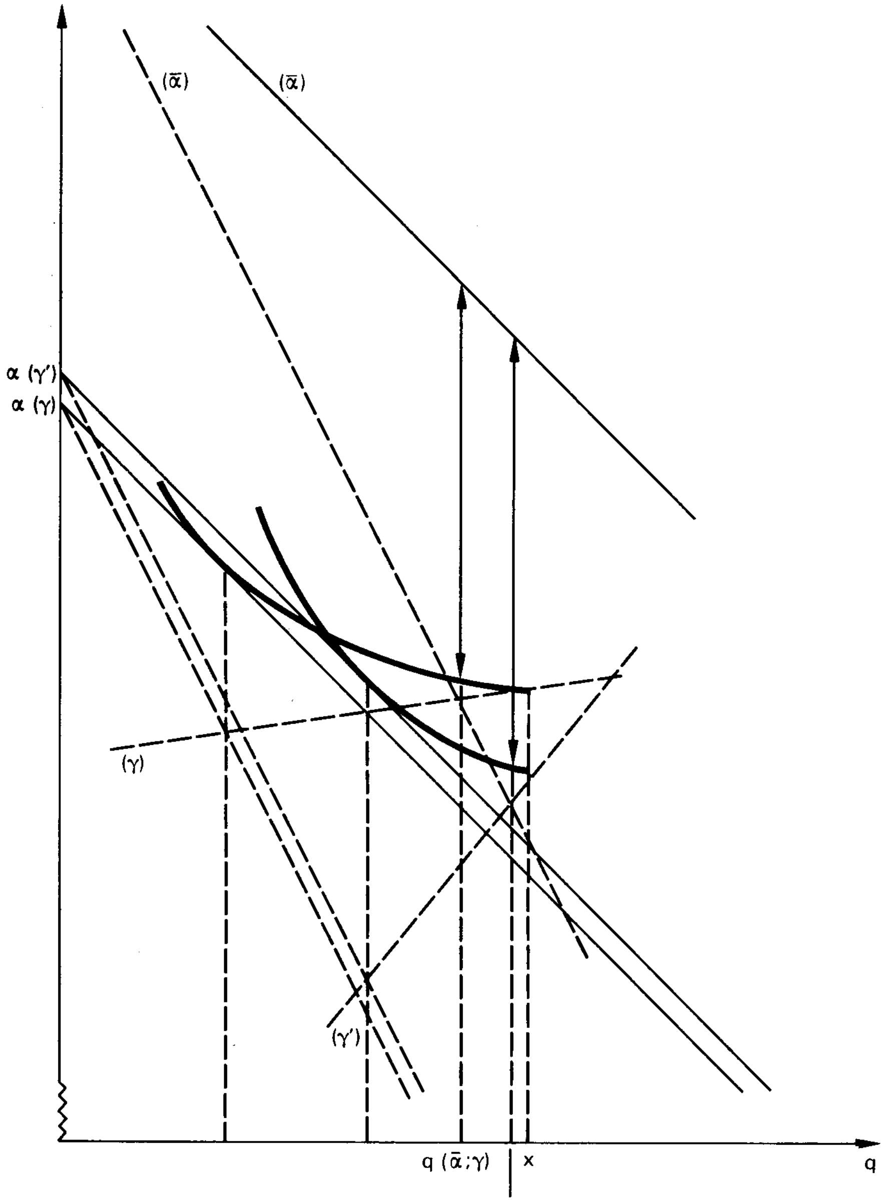

FIGURE 2

The example is as follows:

$$a(\gamma, x) = \gamma^{-1}(x-2.4)^2 + 12$$

$$[\underline{\alpha}, \bar{\alpha}] = [12.2, 15.2]$$

$$f(\alpha) \quad \text{is uniform over } [\underline{\alpha}, \bar{\alpha}]$$

$$\beta = 0.5$$

$$\mathrm{K}(a(\gamma, x), \gamma, x) = \begin{cases} (\gamma-2.5)^2 + (x-2)^3 & \text{for} \quad x \geqslant 2 \\ \infty & \text{for} \quad x < 2 \end{cases}$$

The function $a(\gamma, x)$ is decreasing and convex in γ as it should be. The investment cost function $\mathrm{K}(a(\gamma, x), \gamma, x)$ indicates that inflexibility (or flexibility) is cheap close to $\gamma = 2.5$ and costly when one moves away in either direction from that value. As for capacity, the cost is increasing and convex in x for $x \geqslant 2$.

The expected profit maximizing choice (x^*, γ^*) of size and flexibility was obtained through successive approximations first by a complete search over a fine grid of values for $(x, \gamma) \in [(2, 1), (4, 4)]$ to locate the region of the maximum expected profit and then by evaluating the first-order derivatives $\partial \mathrm{E}\pi/\partial x$ and $\partial \mathrm{E}\pi/\partial \gamma$ in that region to obtain a better approximation. The first-order and second-order derivatives (very cumbersome expressions) were obtained analytically by the symbolic manipulation program MACSYMA of SYMBOLICS INC. [1987]. The reader is referred to BOYER and MOREAUX [1989] for all the details. The results are as follows:

$$\gamma^* = 2.49991365$$

$$x^* = 2.22771845$$

$$\partial \mathrm{E}\pi/\partial x = 0.0000000152 \approx 0$$

$$\partial \mathrm{E}\pi/\partial \gamma = 0.0000000143 \approx 0$$

$$\partial^2 \mathrm{E}\pi/\partial x^2 = -2.9663222956 < 0$$

$$\partial^2 \mathrm{E}\pi/\partial \gamma^2 = -2.0007070294 < 0$$

$$\partial^2 \mathrm{E}\pi/\partial x\, \partial \gamma = -0.0837487413 < 0$$

$$\mathrm{E}\pi(x^*, \gamma^*) = 1.6075563926$$

$$\alpha(\beta, \gamma^*, x^*) = 13.0749954017 > \underline{\alpha} = 12.2.$$

Moreover from (26), we have

$$\partial \alpha(\beta, \gamma^*, x^*)/\partial \gamma = 0.0137776985 > 0$$

indicating that this case corresponds to the second possibility mentioned in the introduction and illustrated in Figure 2.

The particular MPS $h(\alpha)$ we consider to represent an increase in uncertainty is as follows, where $\tilde{\alpha} = \frac{1}{2}(\bar{\alpha} + \underline{\alpha})$:

$$h(\alpha) = \begin{cases} s & \text{for } \underline{\alpha} \leqslant \alpha \leqslant \alpha(\beta, \gamma, x) \\ -s[\alpha(\beta, \gamma, x) - \underline{\alpha}]/[\tilde{\alpha} - \alpha(\beta, \gamma, x)] & \text{for } \alpha(\beta, \gamma, x) \leqslant \alpha \leqslant \tilde{\alpha} \\ -s[\alpha(\beta, \gamma, x) - \underline{\alpha}]/[\tilde{\alpha} - \alpha(\beta, \gamma, x)] & \text{for } \tilde{\alpha} \leqslant \alpha \leqslant \underline{\alpha} + \bar{\alpha} - \alpha(\beta, \gamma, x) \\ s & \text{for } \underline{\alpha} + \bar{\alpha} - \alpha(\beta, \gamma, x) \leqslant \alpha \leqslant \bar{\alpha} \end{cases}$$

It is straightforward to verify that $h(\alpha)$ is a MPS; hence $\varepsilon h(\alpha)$ is also a MPS for $\varepsilon > 0$. It can be represented as in Figure 3.

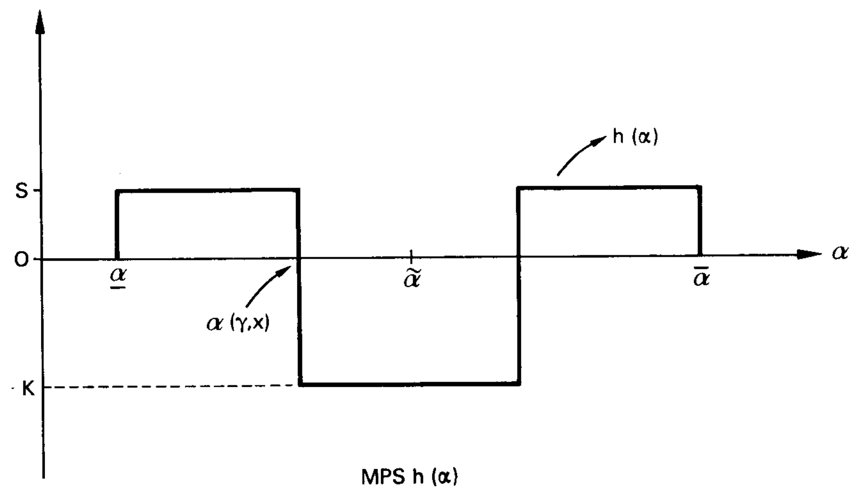

FIGURE 3

We can then evaluate the expression for

$$\frac{\partial^2 E\pi^*(\varepsilon)}{\partial\gamma\,\partial\varepsilon}$$

at the optimal point (γ^*, x^*). The sign of this expression is also the sign of $\left.\frac{\partial\gamma}{\partial\varepsilon}\right|_{x=x*}$ which we are interested in. Regrouping the terms of (29) we can write

$$(29') \quad \frac{\partial^2 \mathrm{E}\pi(\varepsilon)}{\partial\gamma\,\partial\varepsilon} = \left[-x^2 + \frac{((\partial a(\gamma, x)/\partial\gamma) - 2x)(a(\gamma, x) - 2\gamma x)}{2(\beta+\gamma)} - \frac{(a(\gamma, x) - 2\gamma x)^2}{4(\beta+\gamma)^2} \right] \int_{\alpha(\beta, \gamma, x)}^{\bar{\alpha}} h(\alpha)\, d\alpha$$

$$+ \left[- \frac{((\partial a(\gamma, x)/\partial\gamma) - 2x)}{2(\beta+\gamma)} + \frac{(a(\gamma, x) - 2\gamma x)}{2(\beta+\gamma)^2} \right] \times \int_{\alpha(\beta, \gamma, x)}^{\bar{\alpha}} \alpha\, h(\alpha)\, d\alpha$$

$$+ \frac{1}{4(\beta+\gamma)^2} \int_{\alpha(\beta, \gamma, x)}^{\bar{\alpha}} (-\alpha^2)\, h(\alpha)\, d\alpha$$

For the particular case considered, we find the value of this expression to be $0.0041\,s > 0$. Hence, the *increase in uncertainty* as represented by the marginal mean preserving spread $\varepsilon h(\alpha)$ generates an increase in the inflexibility parameter γ, hence a *reduction in flexibility.*

6 Concluding Comments

Jones and Ostroy [1984] provide a condition (their proposition 0) under which an increase in uncertainty should imply an increase in flexibility. We can now give another intuitive explanation for the occurrence of the reverse relation in our model. Jones and Ostroy state that "... *for an increase in variability to cause a shift toward a more flexible (strictly speaking not less flexible) position, this gain in value must be greater the more flexible the position*" (p. 19). In our context, this condition is satisfied if the following holds for $\gamma' > \gamma$:

$$\mathrm{E}\pi(\alpha; \gamma, x \mid \varepsilon) - \mathrm{E}\pi(\alpha; \gamma, x \mid \varepsilon = 0) \geqslant \mathrm{E}\pi(\alpha; \gamma', x \mid \varepsilon) - \mathrm{E}\pi(\alpha; \gamma', x \mid \varepsilon = 0)$$

which is equivalent to

$$(34) \quad \int_{\alpha(\beta, \gamma, x)}^{\bar{\alpha}} \mathrm{V}\pi(\alpha; \gamma, x)\, h(\alpha)\, d\alpha \geqslant \int_{\alpha(\beta, \gamma, x)}^{\bar{\alpha}} \mathrm{V}\pi(\alpha; \gamma', x)\, h(\alpha)\, d\alpha$$

where $h(\alpha)$ being a MPS satisfies (27). When $\alpha(\beta, \gamma, x)$ and $\alpha(\beta, \gamma', x)$ are both $\leqslant \underline{\alpha}$ or when $h(\alpha) = 0$ for $\alpha \leqslant \max\{\alpha(\gamma, x), \alpha(\gamma', x)\}$, the inequality (34) can be written as

$$(35) \quad \frac{1}{4(\beta+\gamma)} \int_{\underline{\alpha}}^{\bar{\alpha}} \alpha^2 h(\alpha)\, d\alpha \geqslant \frac{1}{4(\beta+\gamma')} \int_{\underline{\alpha}}^{\bar{\alpha}} \alpha^2 h(\alpha)\, d\alpha$$

which is satisfied for $\gamma > \gamma'$ since α^2 is a convex function and $h(\alpha)$ is a MPS. However, when the MPS involves values of α for which the firm does not produce in the second stage, the inequality (34) cannot be reduced to (35) and the inequality need not hold; hence the possibility that an increase in uncertainty will generate a reduction in flexibility.

We mentioned in section 3 that the notion of flexibility is related also to the notion of irreversibility: a more irreversible decision is in a sense a less flexible decision. FREIXAS and LAFFONT [1984] provide a condition under which more uncertainty – in the sense of a finer information structure to be available in the future – leads to less irreversible decisions. They state that "*... the irreversibility effect ... is equivalent to ... complementarity in the second period* ex ante *valuation function between the feasible set and information.*" (p. 107). In our context, this condition is satisfied if the following holds (remember that γ stands for inflexibility):

$$(36)\quad \frac{\partial}{\partial \gamma}\left[\int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} V\pi(\alpha;\gamma^*,x^*)\,w(\alpha)\,d\alpha - \int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} V\pi(\alpha;\gamma^*,x^*)\,f(\alpha)\,d\alpha\right] \leqslant 0.$$

Since $w(\alpha) = f(\alpha) + \varepsilon h(\alpha)$, we can rewrite (36) as follows:

$$(37)\quad \frac{\partial}{\partial \gamma}\int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} V\pi(\alpha;\gamma^*,x^*)\,h(\alpha)\,d\alpha \leqslant 0.$$

When $\alpha(\beta, \gamma, x)$ and $\alpha(\beta, \gamma', x)$ are both $\leqslant \underline{\alpha}$ or when $h(\alpha) = 0$ for $\alpha \leqslant \max\{\alpha(\gamma, x), \alpha(\gamma', x)\}$, the inequality (37) can be written as

$$(38)\quad \frac{\partial}{\partial \gamma}\left[\frac{1}{4(\beta+\gamma)}\int_{\alpha(\beta,\gamma,x)}^{\bar{\alpha}} \alpha^2 h(\alpha)\,d\alpha\right] \leqslant 0.$$

This condition is satisfied since α^2 is convex and $h(\alpha)$ is a MPS. Again, when the MPS involves values of α for which the firm does not produce in the second stage, the inequality (37) cannot be reduced to (38) and the inequality need not be satisfied; hence the possibility that an increase in uncertainty will generate a reduction in flexibility.

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