ANNALES D'ÉCONOMIE ET DE STATISTIQUE. – N° 15/16 — 1989

Good and Bad Competition in Spatial Markets for Search Goods:

The Case of Linear Utility Functions

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ABSTRACT. – This paper compares the decisions of *s* single product oligopolists and of a multiproduct monopolist with respect to pricing, product choice, location and entry, within a 3 stage game on the part of the oligopolists. It turns out that from the consumer's point of view a monopolistic versus competitive market structure may be good or bad depending on their choosiness.

Bonne et mauvaise concurrence dans des marchés spatiaux avec recherche entre divers biens : le cas des fonctions d'utilité linéaires

RÉSUMÉ. – Cet article compare les décisions de *s* oligopoles produisant un seul produit à celles d'un monopole produisant plusieurs produits en ce qui concerne prix, choix des produits, localisation, et entrée. Le comportement des oligopoles est représenté par un jeu en trois étapes. On démontre que les consommateurs peuvent préférer la structure de marché monopoliste à la concurrence selon leur facilité à rechercher le produit le plus avantageux.

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1 Introduction

Consumer's search for the preferred alternative amongst several substitutes in a differentiated market almost invariably involves transactions costs of overcoming space. The consumers save on these transactions costs (*i.e.* incur economies of scope in searching), if several of these substitutes are offered at one location. Furthermore, the consumer's expected utility increases if he is so enabled to select from a larger set of alternatives. This in turn may induce him to travel over a long distance towards a larger market place – a location at which more of these alternatives are offered – relative to a small one.

The sellers of these substitutable commodities eventually react to this consumer behavior by agglomerating in space, or by selling at one location several of the substitutable commodities. It is intuitive that their pricing behavior is influenced by the set of commodities sold at such a market place, and the industrial structure prevailing therein. While we expect that the multiproduct seller will to some extent absorb, via higher prices, consumers' transaction costs advantages and utility gains form several offers at one location, we also expect that this absorption process is diluted by competition amongst single product sellers. Surprisingly this is not so: indeed, depending on the primitives of the model, single product competitive sellers may end up charging higher prices for their commodity variants than a multiproduct monopolist. Further interesting outcomes of this model are that such single product seller may in equilibrium prefer a competitive agglomeration to an unrestricted local monopoly; that despite competition they may end up selling at higher prices in this agglomeration than under local monopoly; and that they may offer fewer products upon agglomeration than a monopolistic multiproduct seller would do.

All these results are shown within a highly stylized and simple model introduced in section 2 of this paper. Throughout, we restrict our discussion to activities at one market place. The results are developed on one hand within a 3-stage-game involving the entry, location and joint choice of product variant and price of single product firms; and on the other hand within a multiproduct monopolist's optimization involving the number and price of product variants sold. Involving subgame perfectness as the equilibrium concept, we first discuss results pertaining to the last (pricing and product varying) stage of the game and the monopolist's corresponding decisions, in section 3. We also develop on the single product firms' locational equilibrium. Section 5 discusses the monopolist's optimal choice of number of product variants, and the corresponding equilibrium involving the entry of single product firms. We finish with concluding remarks, in relating our paper to the literature and discussing possible further extensions.

2 The Model

2.1. Consumers

The consumers in our model are endowed with preferences of the type

$$u(x, \alpha; \varepsilon) = a - b|\varepsilon - \alpha| + x$$

where

x = the numeraire commodity.

α = an index of a horizontally differentiated [1] variant of the differentiated product, $\alpha \in [0, 2\pi]$.

ε = the consumer's ideal variant, $\varepsilon \in [0, 2\pi]$.

Our consumer is endowed with income R, which he disposes of by locally consuming nonnegative quantities x of the numeraire commodity, and one indivisible unit of our differentiated commodity available at a market place z kilometers away, at a transport cost c per kilometer. Before patronising this market place, the consumer knows that s variants of the differentiated commodity are offered there, where each variant i is characterized by a position on the circumference of a circle with radius 1. Thus, $\alpha_i \in [0, 2\pi]$. The consumer also has a point expectation about the price p^e at which the individual variants are offered. In terms of the variants, our consumer assumes that $y_i: = |y - \alpha_i|$ is uniformly and independently distributed on $[0, \pi]$ with a distribution function $F(y) = y/\pi$. In all, the consumer's expected utility from patronising a market place of size s is given by

$$w(s) = -\int_0^{\pi} (a - by)\, d(G(y))^s$$

where

$$G(y) = 1 - F(y) = 1 - \frac{y}{\pi}.$$

This yields

$$w(s) = a - \frac{b\pi}{s+1}. \tag{1}$$

Our consumer decides to visit the market place if the expected utility from such a visit net of the payment for the commodity and of transactions

1. A standard reference is Gabszewicz and Thisse [1986].

costs exceeds an opportunity utility obtained from locally consuming the numeraire; in formal terms, if

$$w(s)+\mathrm{R}-p^e-cz \geqslant \mathrm{R}.$$

This induces a critical distance $z=z(p^e; s)$ defined by the equality of the above magnitudes. Trivially, z increases in s and a, and decreases in b and p^e. Upon a visit of the market place, the consumer purchases the commodity variant i maximizing $a-by_i-p_i$. So much for the typical consumer.

We now suppose that at each geographical location consumers are uniformly distributed over $[0, 2\pi]$ with respect to their ideal variant ε. Furthermore, the space geographical locations is a long line whose bounderies should not matter within the context of this model. Consumers identical up to ε are also uniformly distributed along this line. Both densities are normalized to unity.

2.2. Firms

We consider two types of firms, namely the classical single product firm, and a multiproduct firm selling monopolistically one or more product variants at one location. Both firms offer each variant at a fixed cost K and constant marginal cost k. [2]

Each firm perceives correctly the demand it can fetch conditional on the number of variants sold, and prices charged in the market place. Most specificly, if

$$p^e=\frac{1}{s}\sum_{i=1}^{s} p_i,$$

then consumers are attracted to the market place up to a distance $z(p^e; s)$ from the market place. This defines the total demand fetched by the market place, called *market demand.* In turn, the share of this market demand, called *market share* fetched for variant i is determined by

$$\beta_i(p, \alpha)=\frac{p_{i+1}+p_{i-1}-2p_i}{2b}+\frac{1}{2}[\alpha_{i+1}-\alpha_{i-1}],$$

where the variants $\alpha_1, \ldots, \alpha_s$ offered in the market place are ordered on the unit circle. [3]

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2. Thus, we abstain from considering economies of scope the monopolist undoubtedly enjoys in selling several of the commodity variants. We do this in order to isolate the effects of consumers' economies of scope in searching at one location.

The fixed costs of bringing a variant into the market are considered only later. After the entry and (costless) location decision, it is the single product firm's objective to use p_i and α_i to maximize

$$\Pi_i^c(p, \alpha) = 2(p_i - k)\,\beta_i(p, \alpha)\,z(p, s). \tag{2}$$

Similarly, it is the objective of the multi-product spatial monopolist to use the vectors of prices and variants p and α, respectively, to maximize

$$\Pi^M(p, \alpha) = 2\sum_{i=1}^{s}(p_i - k)\,\beta_i(p, \alpha)\,z(p, s).$$

In turn, an equilibrium in this (third) stage of the game involving single-product-firms is a collection of variants and prices such that no firm considers it profitable to change its strategy given the strategic choices made by the other firms, and the consumers' price expectations are confirmed.

3 Choice of Prices and Variants

Before characterizing the single-product firms' and the multi-product monopolist's choices of prices and variants, we first wish to demonstrate the existence of equilibria and the corresponding optimum in the following two theorems:

THEOREM 1 M (monopoly) [4]: Let

$$a - k \geqslant \frac{2b\pi}{s}.$$

Then there exists a symmetric solution of

$$\max_{p,a} 2z(p; s)\sum_{j=1}^{s}(p_j - k)\,\beta_j(p, \alpha).$$

3. Observe that this formulation is consistent with the assumed consumer behavior, as long as all consumers buy one unit of the differentiated commodity upon a visit of the market place and as long as firms don't undercut each other. Indeed, we invoke the by now standard no-undercutting-condition NOSHEK [1980]. In another interpretation, β_i is the perceived market share, which is locally consistent with consumer behavior in equilibrium (BONANO [1988]) We have gone through a tedious check of all cases such that consumers decide not to buy upon observation of all available varieties. Under our assumptions on the parameters of the model, made explicit in the statements of our results, these are not at all affected by this potential outcome.
4. All proofs are relegated to the Appendix.

THEOREM 1 C (competition): Under the condition of Theorem 1 M there exists a symmetric equilibrium involving s single-product sellers. Furthermore, the symmetric equilibrium price is unique given the symmetric choice of variants.

We now turn to a characterization of the multi-product monopolist's profit maximum and the corresponding single-product sellers' equilibrium in prices and product variants. It will be helpful to investigate first the properties of the function $w(s, a, b)$. Lemma 1 shows that we cannot vary s, a, b independently. Otherwise w can become negative, whereas we need $w > k$ for our analysis.

LEMMA 1: $w(s; a, b) = a - b\,\pi/(s+1)$

(*a*) $w(s; a, b)$ increases in s and

$$\lim_{s \to \infty} w(s; a, b) = a$$

$$\lim_{s \to (b\pi/a)-1} w(s; a, b) = 0$$

(*b*) $w(s; a, b)$ decreases in b and

$$\lim_{b \to 0} w(s; a, b) = a$$

$$\lim_{b \to a(s+1)/\pi} w(s; a, b) = 0$$

(*c*) $w(s; a, b)$ increases in a and

$$\lim_{a \to \infty} w(s; a, b) = \infty$$

$$\lim_{a \to b\pi/(s+1)} w(s; a,b) = 0.$$

These properties follow immediately from the specification of w. For the ensuing analysis, we wish to vary the parameter b reflecting consumers' choosiness independently of s. We therefore consider the joint variation of a and b, by defining

$$a_D(b) := D + b\pi/2 \qquad \text{with} \quad D > k.$$

This implies

$$w_D(s; b) = D + \frac{b\pi}{2}\,\frac{s-1}{s+1}$$

implying in particular that $w_D(1; b) = D$, $\forall b$ and $w_D(s; b) > k$.

The properties of w_D are summarized in

LEMMA 2: (*a*) $w_D(s; b)$ increases in s and

$$\lim_{s \to \infty} w_D(s; b) = D + b\pi/2$$

$$\lim_{s \to 1} w_D(s; b) = D$$

(*b*) $w_D(s; b)$ increases in b and

$$\lim_{b \to 0} w_D(s; b) = D$$

$$\lim_{b \to \infty} w_D(s; b) = \infty.$$

We now are prepared to investigate the comparative statics of the monopolistic profit maximum. As a preliminary, we state the explicit solution to that in

LEMMA 3: The monopolistic price and profit is computed as

(*a*)

$$p^M(s; a, b) = \frac{w(s; a, b) + k}{2}$$

(*b*)

$$\Pi^M(s; a, b) = \frac{\pi}{c}[w(s; a, b) - k]^2.$$

The following theorem follows immediately from lemmata 1-3:

THEOREM 2 M: The monopolistic optimum has the following properties:

(*a*) $p^M(s; a, b)$ is increasing in s;

(*b*) $z(p^M(s; a, b); s, a, b)$ is increasing in s;

(*c*) $\Pi^M(s; a, b)$ is increasing in s.

Not unexpectedly, the monopolist absorbs some of the increasing consumer surplus from an increase in s by charging a higher price. However, that increase is not as strong as to absorb completely that surplus, as reflected in an increase in the market area z. Finally, his profits before setup costs are also monotonicly increasing in s.

Let $\Pi_D^M(s; b)$ be the monopolistic profit maximum if w is replaced by w_D. This specification enables us to show

THEOREM 3 M:

$$\forall s \geq 3, \qquad \exists b_D(s), \qquad \forall b \geq b_D(s)$$
$$\Pi_D^M(1; b) < 1/s\, \Pi_D^M(s; b).$$

Theorem 3 M tells us that if consumers are sufficiently choosy, the monopolist's average profits from selling $s \geq 3$ commodity variants will be higher than those obtained from selling a single variant! This is a clear indication of the increasing returns associated with selling many commodity variants at one location despite the fact that these variants are substitutable in the consumers' eyes. This concludes our discussion of the comparative statics associated with the monopolistic choice of prices and variants.

We now turn to the same class of comparative statics for the single-product sellers if assembled at one market place. Let $p_D^C(s; b)$ and $\Pi_C^D(s; b)$ denote the equilibrium prices charged, and profits (before setup costs) obtained by the single-product sellers, if w_D is used. The following theorem summarizes characteristics of these functions:

LEMMA 3 C: Let $s \geqslant 4$. Then

(*a*) $p_D^C(s; b)$ increases in b.

(*b*) $\lim_{b \to 0} p_D^C = k$.

(*c*) $\lim_{b \to 0} \Pi_D^C(s; b) = 0$.

(*d*) $\lim_{b \to \infty} \Pi_D^C(s; b) = \infty$.

(*e*) $\lim_{s \to \infty} p_D^C(s; b) = k$ for $b > 0$.

(*f*) $\lim_{s \to \infty} \Pi_D^C(s; b) = 0$ for $b > 0$.

The results of Lemma 3 C confirm our intuition: equilibrium prices and profits increase as consumers become more choosy, and conversely decrease towards marginal costs and zero respectively, as consumers consider the variants as perfect substitutes. Furthermore, prices converge towards marginal costs and profits converge towards zero if the number of variants offered in the market place increases beyond of bounds. While these results by themselves should increase our confidence in the present model setup, they are also very helpful in demonstrating

THEOREM 2 C: For all $s \geqslant 4$, $\exists\, b(s)$, $\forall\, b > b(s)\, p_D^C(s; b) > p_D^M(1; b)$.

The following estimates are helpful in demonstrating one of our key results, that if consumers are sufficiently choosy, competitive profits may increase in s:

LEMMA 4 C: (*a*)

$$p^C(s) < k + \frac{2b\pi}{s}.$$

(*b*) For $s \geqslant 4$

$$p_D^C(s) > k + \frac{2b\pi}{s^2}.$$

(*c*) For $s \geqslant 4$

$$p_D^C(s) > k + \frac{b\pi}{2s}.$$

THEOREM 3 C: For all $s \geqslant 5$, $\exists\, b(s)$, $\forall\, b > b(s)$

$$\Pi_D^C(s;\, b) > \Pi_D^M(1;\, b).$$

Thus the result already obtained for the monopolist that multivariant selling at one market place maybe profitable turns out to hold for competitive single-product sellers as well! Of course, the statement made in theorem 2 and 3 for the multi-product monopolist and the single-product sellers invite a comparison. A first run is conducted in the following

THEOREM 2 M-C:

$$\forall\, s \geqslant 2, \exists\, b(s), \forall\, b > b(s), \exists\, a \text{ such that } p^C(s;\, a,\, b) > p^M(s;\, a,\, b).$$

This means that for suitable parameters of the consumers' utilities, competitive prices are always larger than monopolistic ones in market places offering the same range of commodities. Thus, competitors excessively exploit consumers' surpluses if the consumers are sufficiently choosy. The excessiveness of this is demonstrated in

COROLLARY 3 M-C:

$$\frac{1}{s}\Pi^M(s;\, a,\, b) \geqslant \Pi^C(s;\, a,\, b).$$

Indeed, the multi-product monopolist's average profits are strictly higher than single-product competitors profits except for the combinations $(a(s), b(s))$ as characterized in the proof of Theorem 2 M-C and the uniqueness of $p^M(s;\, a,\, b)$.

This concludes our discussion of monopolistic and competitive choices of prices and variants in our model. These results are not only of interest on their own. In section 4, we will see that they also have a strong impact on the monopolist's choice of the product bundle offered, as well as on competitive entry. However, before discussing that, we wish to state briefly conditions for the existence of an equilibrium involving the joint location of single-product sellers. Here we analyze exclusively the situation, where the choice of a specific firm is to join the market place location or to establish at a distant location where consumers of the market place are not affected by actions of this firm. These conditions are specified in

THEOREM 4 C: For all $s \geqslant 5$, $\exists\, b(s)$, $\forall\, b > b(s)$, such that $b > b(s)$ implies $l_i = l_j$, $\forall\, i, j = 1, \ldots, s$, where l_i denotes the location of firm i.

The proof follows directly from Theorem 3 C.

4 Entry Equilibrium and Monopolistic Optimum in Product Variants

Once again, we consider first the monopolist's optimal choice. Given he is faced with entry costs K per variant, his optimal choice is specified in

THEOREM 5 M: Let $\pi(a-(b\pi/2)-k)^2 \neq c\mathrm{K}$ then

(*a*) $\forall \mathrm{K}>0, \forall b>0$ with $\mathrm{D}-k \geqslant \pi b$, $\exists$ a unique $s_D^{\mathrm{M}} \in \mathbb{R}_+$.

(*b*) $\forall \mathrm{K}>0, \forall a, b$ with $a-k>\pi b$, $\exists$ unique $s^{\mathrm{M}} \in \mathbb{R}_+$.

After having thus shown existence and uniqueness of an optimal number of product variants introduced by the monopolist, we wish to ensure that this number is strictly larger than 1, by

THEOREM 6 M: Let $(\pi/c)[\mathrm{D}-k]^2>\mathrm{K}$. Then $\forall \mathrm{K}>0$ and $\forall s \geqslant 3$, $\exists b_{\mathrm{K}}(s)$ such that $\forall b \geqslant b_{\mathrm{K}}(s)$

$$s_D^{\mathrm{M}}(b_{\mathrm{K}}(s)) \geqslant s.$$

Theorem 6 M states quite naturally that s^{M} can become large, if consumers are sufficiently choosy.

We now turn to a discussion of competitive entry in one market place. Consider first

THEOREM 5 C: $\forall \mathrm{K}>0, \forall b>0, \exists s^{\mathrm{C}}$ such that $\Pi_{\mathrm{D}}^{\mathrm{C}}(s;\, b)<\mathrm{K}, \forall s>s_{\mathrm{D}}^{\mathrm{C}} \geqslant 0$.

This demonstrates existence of an entry equilibrium involving single-product firms. The following theorem gives conditions under which certain numbers of sellers enter the market:

THEOREM 6 C:

(*a*) Let $(\pi/c)[\mathrm{D}-k]^2>\mathrm{K}$. Let $s \geqslant 5$. $\exists b(s), \forall b \geqslant b(s) : s_{\mathrm{D}}^{\mathrm{C}} \geqslant s$.

(*b*) Let $(\pi/c)[\mathrm{D}-k]^2<\mathrm{K}$. Then $s^{\mathrm{C}}=0$ is an equilibrium number of firms.

Part (*a*) of this theorem follows directly from the Theorem 3 C. Part (*b*) then follows trivially.

Quite similar to the monopoly situation, an entry equilibrium involving many sellers obtains if consumers are sufficiently choosy. However, the returns to selling from the agglomeration of single-product firms may be such that no individual firm considers profitable an entry into the market, and therefore no equilibrium involving positive entry obtains despite the possibility that the joint entry of firms involves positive profits in equilibrium! Thus, no firm will enter without pre-play communication.

Once again, we are now prepared to compare the monopolistic optimum with the equilibrium entry involving single-product firms. A first comparison is prepared in

THEOREM 5 M-C: $\exists$ K and b such that $s_D^M < s_D^C$ and $p_D^M(s_D^M, b) > p_D^C(s_D^C; b)$.

Theorem 5 M-C tells us that for certain (sufficiently small) b both the choices provided and prices at which these are offered by competitive one-product sellers are unequivocally preferable to those offered by the multi-product monopolist. While this confirms standard expectations, it is by nomeans the only possibility arising in this comparison. One interesting case that signals the contrary is given in

THEOREM 6 M-C: There exists K, a, b such that $s^M > 1$ and $s^C = 0$.

Thus we are faced with the dilemma of not being able to evaluate unequivocally the relative performance of the multi-product monopolist versus the competitive single-product sellers. This comparative evaluation is difficult even without the market area, or more generally, extensive margin effect driving our results. In the former case, TIROLE [1988] demonstrates by example that while the monopolist in general charges higher prices than competitors do, he by doing so is induced to introduce more commodities than the competitive sellers. Within our model, we should be aware of the fact, however, that the externality introduced by the market area effect ceteris paribus speaks for the superiority of monopolistic choices as seen from an efficiency point of view: Only the monopolist will internalize fully the externality so generated.

5 Concluding Remarks

We have shown in our model that economies of scope in searching within a geographical market induce agglomeration, product selection, and pricing effects that are left unexplained otherwise. While these economies of scope are discussed in earlier work [cf. e.g. STAHL [1982], STURART [1979] or WOLINSKY [1983]], only the spatial concentration of single-product sellers is derived there. What is novel in the present paper is the generalization towards including explicitly sellers' pricing decisions and ensuing impacts on location, together with a characterization of multi-product monopoly and with its comparison to the competitive case.

While our model is thus considerably more general than earlier work, it also opens avenues for substantial further extensions: A primary one is that consumers search may be conducted across several market places, thus inducing competition across those market places. And an obvious second one is to introduce in the possibility of selling several commodity variants, and thus endogenizing the market structure observed at one market place.

However, both extensions require efforts that are beyond the scope of this paper.

APPENDIX

THEOREM 1 M (Monopoly): Let

$$a-k \geqslant \frac{2b\pi}{s}.$$

Then there exists a symmetric solution of

$$\max_{p,\,a} 2z(p;\, s) \sum_{j=1}^{s} (p_j-k)\,\beta_j(p,\, a).$$

Proof:

$$\frac{1}{2}\frac{\partial \Pi^M}{\partial p_i} = \beta_i z + \sum_{j=1}^{s} (p_j-k)\left[\frac{\partial \beta_j}{\partial p_i} z + \beta_j \frac{\partial z}{\partial p_i}\right] = 0$$

$$\frac{1}{2}\frac{\partial \Pi^M}{\partial \alpha_i} = \frac{1}{2}\left[(p_{i-1}-k)-(p_{i+1}-k)\right] z = 0.$$

The latter equation implies $p_{i-1}=p_{i+1}$, $\forall i$. For odd s, this implies in turn $p_i = p\,\forall i$. If s is even, let $p_{2i} =: p_2$ and $p_{2i+1} =: p_1$, $\forall i$. The first equation then implies

$$\frac{1}{2}\frac{\partial \Pi^M}{\partial p_1} = \left(\frac{p_2-p_1}{b} + \frac{1}{2}|a_2-a_s|\right) z$$

$$+\left[\frac{(p_2-k)}{b} - \frac{(p_1-k)}{b}\right] z - \frac{1}{cs}\Sigma (p_j-k)\,\beta_j = 0$$

$$(*) \qquad = \left(\frac{2(p_2-p_1)}{b} + \frac{1}{2}|a_2-a_s|\right) z - \frac{1}{cs}\Sigma (p_j-k)\,\beta_j = 0$$

From this follows immediately that $|a_2-a_s| = |a_4-a_2| = |a_{2i}-a_{2(i-1)}|$. Since $|a_2-a_s| + |a_4-a_2| + \ldots + |a_s-a_{2(s-1)}| = 2\pi$, we obtain $|a_{2(i+1)}-a_{2i}| = 4\pi/s$. Analogously, it follows that $|a_{2i+3}-a_{2i+1}| = 4\pi/s$. From this, together with (*) it follows by comparing $\partial\Pi^M/\partial p_1$ and $\partial\Pi^M/\partial p_2$ that $p_i = p$, $\forall i$ and this implies $\beta_i(p,\, a) = 2\pi/s$. □

THEOREM 1 C (Competition): Under the condition of Theorem 1 M there exists a symmetric equilibrium involving s single-product sellers. Furthermore, the symmetric equilibrium price is unique given the symmetric choice of variants.

Proof:

$$\Pi_i(p,\, a) = \frac{2}{c}(p_i-k)\left[\frac{p_{i+1}-p_{i-1}-2p_i}{2b} + \frac{1}{2}(a_{i+1}-a_{i-1})\right]\left(w(s) - \frac{1}{s}\sum_{j=1}^{s} p_j\right)$$

This function is continuous in p_i and $\alpha_i . p_i$ is bounded below by k and bounded above by

$$w(s)-\frac{1}{s}\sum_j p_j \geqslant 0.$$

Thus, there exists a maximum (p_i, α_i) of this function. Necessarily,

$$\frac{1}{2}\frac{\partial \Pi_i}{\partial p_i}=\beta_i z+(p_i-k)\left[\frac{\partial \beta_i}{\partial p_i}z+\beta_i\frac{\partial z}{\partial p_i}\right]=0$$
$$=\beta_i z-(p_i-k)\left[\frac{z}{b}+\frac{\beta_i}{cs}\right]=0.$$

It remains to show that this system has a symmetric solution in the sense that $p_j=p$, $\forall j$ and $a_{i+1}-a_{i-1}=4\pi/s$ implying $\beta_i=2\pi/s$. The system then reduces to the equation

$$(**)\qquad \frac{2\pi}{s}(w(s)-p)=(p-k)\left(\frac{w(s)-p}{b}+\frac{2\pi}{s^2}\right).$$

The left hand side is linearly falling in p. It equals 0 at $p=\mathrm{w(s)}$ and is positive at $p=k$. The right hand side is concave in p; it is 0 at $p=k$ and positive at $p=w(s)$. Thus, there exists a unique p solving equation (**). □

THEOREM 3 M:

$$\forall s\geqslant 3,\qquad \exists\, b_D(s),\qquad \forall b\geqslant b_D(s)$$
$$\Pi_D^M(1;\, b)<1/s\,\Pi_D^M(s;\, b).$$

Proof: Choose b such that $a-p_D^M(s;\, b)\geqslant b\pi/s$, $s\geqslant 1$. Using Lemma 3, we need to show that

$$\sqrt{s}(w_D(1;\, b)-k)<w_D(s;\, b)-k$$

$$\Leftrightarrow\quad \sqrt{s}(\mathrm{D}-k)<\mathrm{D}-k+\frac{b\pi}{2}\frac{s-1}{s+1}$$

$$\Leftrightarrow\quad (\sqrt{s}-1)(\mathrm{D}-k)<\frac{b\pi}{2}\frac{s-1}{s+1}.$$

Choose $b_D(s)$ as the solution of

$$(\sqrt{s}-1)(\mathrm{D}-k)=\frac{b\pi}{2}\frac{s-1}{s+1}.$$

It remains to show whether

$$a-p_D^M(s;\, b)\geqslant\frac{b\pi}{s}$$

holds for all b:

$$a-p_D^M(s;\, b)=a-\frac{w_D+k}{2}=D+\frac{b\pi}{2}-\frac{D+k}{2}-\frac{b\pi}{4}\frac{s-1}{s+1}$$

$$=\frac{D-k}{2}+b\pi\left(\frac{1}{2}-\frac{s-1}{4(s+1)}\right)\geqslant\frac{b\pi}{s}$$

$$\Leftrightarrow\quad \frac{D-k}{2}\geqslant b\pi\left(\frac{1}{s}-\frac{s-1}{4(s+1)}-\frac{1}{2}\right).$$

The expression in brackets is negative for $s\geqslant 3$. Therefore

$$a-p_D^M(s;\, b)\geqslant\frac{b\pi}{s}$$

for $s\geqslant 3$ and all $b\in\mathbb{R}_+$.

LEMMA 3C: Let $s\geqslant 4$. Then

(*a*) $p_D^C(s;\, b)$ increases in b.

(*b*) $\lim\limits_{b\to 0} p_D^C=k$.

(c) $\lim\limits_{b\to 0} \Pi_D^C(s;\, b)=0$.

(*d*) $\lim\limits_{b\to\infty} \Pi_D^C(s;\, b)=\infty$.

(*e*) $\lim\limits_{s\to\infty} p_D^C(s;\, b)=k$ for $b>0$.

(*f*) $\lim\limits_{s\to\infty} \Pi_D^C(s;b)=0$ for $b>0$.

Proof:

$$(***)\quad \frac{2\pi}{s}(w(s;\, b)-p)=(p-k)\left(\frac{w(s,\, b)-p}{b}+\frac{2\pi}{s^2}\right)$$

$$\Leftrightarrow\quad \frac{2\pi}{s(p-k)}=\frac{1}{b}+\frac{2\pi}{s^2(w(s;\, b)-p)}$$

$$\Leftrightarrow\quad \frac{2\pi}{p-k}=\frac{s}{b}+\frac{2\pi}{s(\omega(s;\, b)-p)}$$

Suppose p^C decreases in b. Then the left hand side of expression (***) increases and its right hand side decreases. Contradiction.

(*b*) If b converges towards 0, then s/b converges towards ∞ and the second term of (***) remains positive. Therefore $p-k$ must converge towards 0.

(*c*)

$$\Pi^C(s;\, b)=\frac{4\pi}{cs}(p-k)(w(s;\, b)-p)$$

since by definition

$$\lim_{b \to 0} w_{\mathrm{D}}(s;\, b) = \mathrm{D},$$

the assertion follows by making use of (*b*).

(*d*) Lemma 2 implies

$$\lim_{b \to \infty} w_{\mathrm{D}}(s;\, b) = \infty.$$

Consider (***). s/b converges towards 0 for $b \to \infty$. Suppose that

$$\lim_{b \to \infty} (w(s;\, b) - \mathrm{p}) < \infty.$$

Then

$$\lim_{b \to \infty} p^{\mathrm{C}}(s;\, b) = \infty$$

and therefore

$$\lim_{b \to \infty} (p^{\mathrm{C}}(s;\, b) - k) = \infty.$$

This contradicts (***). Therefore

$$\lim_{b \to \infty} (w(s;\, b) - p) = \infty$$

and therefore

$$\lim_{b \to \infty} (p^{\mathrm{C}}(s;\, b) - k) = \infty$$

which proves the assertion.

(*e*) Consider again (***). $s \to \infty$ *implies* RHS $\to \infty$ *and therefore* $p(s;\, b) - k \Upsilon 0$.

(*f*) (***) is equivalent to

$$\frac{2\pi s}{p-k} = \frac{s^2}{b} + \frac{2\pi}{w(s;\, b) - p}.$$

RHS $\to \infty$ for $s \to \infty$ implying $(p-k)/s \to 0$. Since

$$\lim_{s \to \infty} p^{\mathrm{C}}(s;\, b) = k,$$

it follows that

$$\lim_{s \to \infty} w(s;\, b) - p^{\mathrm{C}}(s;\, b) = \lim_{s \to \infty} w(s;\, b) - k > 0. \quad \square$$

THEOREM 2C: For all $s \geqslant 4$, $\exists\, b(s)$, $\forall\, b > b(s)\, p_{\mathrm{D}}^{\mathrm{C}}(s;\, b) > p_{D}^{\mathrm{M}}(1;\, b)$.

Proof: Consider equation (***): It implies for $s > 1$:

$$(+) \qquad s + \frac{2b\pi}{s(w_D(s;b) - p_D^C(s;b))} = \frac{2b\pi}{p_D^C(s;b) - k}$$

for $s = 1$

$$(++) \qquad \frac{2b\pi}{w_D(1;b) - p_D^M(1;b)} = \frac{2b\pi}{p_D^M(1;b) - k}.$$

Suppose now that $p_D^C(s;b) = p_D^M(1;b) = (D+k)/2$ substracting (+ +) from (+),

$$s^2 + \frac{4b\pi}{(2w_D(s;b) - (D+k))} = \frac{4b\pi s}{D-k}$$

⇔

$$s^2 = 4b\pi \frac{[s(2wD(s;b) - (D+k)) - (D-k)]}{(2w_D(s;b) - (D+k))(D-k)}$$

⇔

$$s^2\left[2\left(D + \frac{b\pi}{2}\frac{s-1}{s+1}\right) - (D+k)\right](D-k)$$
$$= 4b\pi\left[2sD + b\pi s\frac{s-1}{s+1} - s(D+k) - (D-k)\right]$$

⇔

$$s^2(D-k)^2 = 4b\pi\left[b\pi s\frac{s-1}{s+1} - (s-1)(D-k)\right] - b\pi s^2\frac{s-1}{s+1}(D-k).$$

Observe that $0 < \mathrm{LHS} < \infty$. Also, RHS is convex in b and

$$\lim_{b \to 0} \mathrm{RHS} = 0, \qquad \lim_{b \to \infty} \mathrm{RHS} = \infty.$$

Thus, exists an unique $b(s)$ solving the equation. Furthermore, $\mathrm{LHS} < \mathrm{RHS}$ for $b > b(s)$ and therefore

$$s + \frac{2b\pi}{s(w_D(s;b) - p_D^M(1;b))} < \frac{2b\pi}{p_D^M(1;b) - k}.$$

Since LHS increases and RHS decreases in p, we obtain the result. □

LEMMA 4 C: (*a*)

$$p^C(s) < k + \frac{2b\pi}{s}.$$

(*b*) for $s \geqslant 4$

$$p_D^C(s) > k + \frac{2b\pi}{s^2}.$$

(*c*) For $s \geqslant 4$

$$p_D^C(s) > k + \frac{b\pi}{2s}.$$

Proof follows directly from a manipulation of (***).

THEOREM 3 C: For all $s \geqslant 5$, $\exists\, b(s)$, $\forall\, b > b(s)$

$$\Pi_D^C(s;\, b) > \Pi_D^M(1;\, b).$$

Proof:

$$\Pi_D^C(s;\, b) = \frac{4\pi}{cs}(p_D^C(s;\, b) - k)(w_D(s;\, b) - p_D^C(s;\, b))$$

$$> \frac{8\,b\pi^2}{cs^3}(w_D(s;\, b) - p_D^C(s;\, b)) \quad \text{because of Lemma 4 C } (b)$$

$$> \frac{8\,b\,\pi^2}{cs^3}\left(D - k + \frac{b\pi}{2}\,\frac{s-1}{s+1} - \frac{2\,b\,\pi}{s}\right) \quad \text{because of Lemma 4C } (a).$$

For $s \geqslant 5$, the expression in brackets is positive for all b. Consider now

$$\tilde{\Pi}_D^C(s;\, b) = \frac{8\,b\,\pi^2}{cs^3}\left[D - k + \frac{b\,\pi}{2}\left(\frac{s-1}{s+1} - \frac{4}{s}\right)\right].$$

Obviously

$$\lim_{b \to 0} \tilde{\Pi}_D^C(s;\, b) = 0 \qquad \text{and} \qquad \lim_{b \to \infty} \tilde{\Pi}_D^C(s;\, b) = \infty.$$

However, $\Pi_D^M(1;\, b) = (\pi/c)(D-k)^2 > 0$ and thus independent of b. □

THEOREM 2 M-C: $\forall\, s \geqslant 2$, $\exists\, b(s)$, $\forall\, b > b(s)$, $\exists\, a$ such that

$$p^C(s;\, a,\, b) > p^M(s;\, a,\, b)$$

Proof: Consider equation (***). Then

(3) $$\frac{s}{b} + \frac{2\pi}{s(w(s) - p^C)} = \frac{2\pi}{p^C - k}$$

(4) $$\frac{2\pi}{w(s) - p^M} = \frac{2\pi}{p^M - k}.$$

Suppose now that $p^C = p^M = p = (w(s)+k)/2$: Subtraction of (4) from (3) yields

$$\frac{s}{b} = \frac{2\pi}{w(s)-p}\left(\frac{s-1}{s}\right)$$

⇔

(5) $$w(s) - k = 4b\pi\,\frac{s-1}{s^2}.$$

For $w(s;\,a,\,b) = a - b\pi/(s+1)$ we obtain

(6) $$a - k = b\pi\left(4\,\frac{s-1}{s^2} + \frac{1}{s+1}\right).$$

However, $4(s-1)/s + s/(s+1) > 2$ for $s \geqslant 2$.

Now, choose $(a-k)/b\pi = \alpha$ with $\alpha := 4(s-1)/s^2 + 1/(s+1) > 2/s$. Let $a(s)$ and $b(s)$ be its solution. Hence

$$a(s) - k = \alpha\, b(s)\,\pi \geqslant \frac{2\,b(s)\,\pi}{s}.$$

Also, $w(s;\,a(s),\,b(s)) > k$ because of (5). Holding $b(s)$ fixed and reducing a below $a(s)$, we get

$$a - k < b(s)\,\pi\left(4\,\frac{s-1}{s^2} + \frac{1}{s+1}\right).$$

and therefore

$$\frac{s}{b(s)} + \frac{2\pi}{s\,(w(s;\,a,\,b(s)) - p^M(s;\,a,\,b(s)))} < \frac{2\pi}{p^M(s;\,a,\,b(s))}.$$

Since LHS increases and RHS decreases in p, we obtain

$$p^C(s;\,a,\,b(s)) > p^M(s;\,a,\,b(s)).\quad \square$$

Remark: Equation (5) in the proof above cannot hold for $w_D(s;\,b)$ for any $s \geqslant 2$ and $b > 0$. Hence, we have $p_D^C(s;\,b) < p_D^M(s;\,b)$.

THEOREM5 M: Let $(\pi/c)[a - (b\pi/2) - k]^2 \neq K$ then

(*a*) $\forall K > 0$, $\forall b > 0$ with $D - k \geqslant \pi b$, $\exists\, a$ unique $s_D^M \in \mathbb{R}_+$.

(*b*) $\forall K > 0$, $\forall a, b$ with $a - k > \pi b$, $\exists$ unique $s^M \in \mathbb{R}_+$.

Proof:

LEMMA: Π_D^M is continuously differentiable in $s. > 0\, \Pi_D^M$ is concave in s. for $s \geqslant 1$.

Proof of the Lemma:

$$\frac{\partial \Pi^M}{\partial s} = \frac{2\pi}{c}\,(w(s) - k)\,\frac{\partial w}{\partial s}$$

$$\frac{\partial^2 \Pi^M}{\partial s^2}=\frac{2\pi}{c}\left[\left(\frac{\partial w}{\partial s}\right)^2+(w(s)-k)\,\frac{\partial^2 w}{\partial s^2}\right]$$

$$\frac{\partial w_D}{\partial s}=\frac{b\pi}{(s+1)^2}$$

$$\frac{\partial^2 w_D}{\partial s^2}=-\frac{2b\pi}{(s+1)^3}$$

$$\Rightarrow \quad \frac{\partial^2 \Pi^M}{\partial s^2}=\frac{2\pi}{c}\left[\frac{b^2\pi^2}{(s+1)^4}-(w(s)-k)\,\frac{2b\pi}{(s+1)^3}\right]$$

$$\frac{\partial^2 \Pi_D^M}{\partial s^2}=\frac{2\pi}{c}\left[\frac{b^2\pi^2}{(s+1)^4}-(D-k)\,\frac{2b\pi}{(s+1)^3}-\frac{b^2\pi^2}{(s+1)^4}\,(s-1)\right]$$

$$=\frac{2\pi}{c}\left[-\frac{b^2\pi^2}{(s+1)^4}\,(s-2)-(D-k)\,\frac{2b\pi}{(s+1)^3}\right]<0$$

for $s\geqslant 1$ and $D-k\geqslant b\pi$. □

Proof of proposition 5M: Concavity of Π_D^M implies at most one $s\geqslant 0$ maximizing monopoly profits. However,

$$\frac{\partial^2 \Pi^M}{\partial s^2}=\frac{2\pi}{c}\left[-(a-k)\,\frac{2b\pi}{(s+1)^3}+\frac{b^2\pi^2}{(s+1)^4}+\frac{2b^2\pi^2}{(s+1)^4}\right]$$

$$=\frac{2b\pi^2}{c(s+1)^4}\left[3b\pi-2(a-k)(s+1)\right]<0,$$

since by assumption $2(a-k)(s+1)>3(a-k)\geqslant 3b\pi$. □

THEOREM 6M: Let $(\pi/c)\,[D-k]^2>K$. Then $\forall K>0$ and $\forall s\geqslant 3$, $\exists\, b_K(s)$ such that $\forall b\geqslant b_k(s)$

$$s_D^M(b_K(s))\geqslant s.$$

Proof: Since by the above Lemma Π_D^M is concave in s, it suffices to examine the first order condition

$$\frac{\partial \Pi^M}{\partial s}=\frac{2b\pi^2}{c(s+1)^2}\left(D+\frac{b\pi}{2}\,\frac{s-1}{s+1}-k\right)=K.$$

Since the left hand side increases in b from 0 to ∞, we obtain exactly one b such that the first order necessary condition is satisfied. □

THEOREM 5C: $\forall K>0$, $\forall b>0$, $\exists\, s^C$ such that $\Pi_D^C(s;b)<K$, $\forall s>s_D^C\geqslant 0$.

Proof:

$$\Pi_D^C(s;b)=\frac{4\pi}{cs}\,(p_D^C(s)-k)(w_D(s)-p_D^s(s))$$

$$<\frac{8b\pi^2}{cs^2}\,(w_D(s)-p_D^s(s)) \quad \text{because of Lemma 4C}(a)$$

$$<\frac{8b\pi^2}{cs^2}\left(D-k+\frac{b\pi}{2}\,\frac{s-1}{s+1}-\frac{2b\pi}{s^2}\right) \quad \text{because of Lemma 4C}(b). \quad \square$$

THEOREM 5 M-C: $\exists$ K and b such that $s_D^M \leqslant s_D^C$ and $p_D^M(s_D^M, b) > p_D^C(s_D^C; b)$.

Proof: $s \geqslant 4$ implies $p_D^M(s; b) > p_D^C(s; b)$. Let $s=4$. Choose b and K such that $s_D^M = 4$. This implies a choice of b small enough such that

$$\frac{\Pi}{c}(\mathrm{D}-k)^2 > \frac{2b\pi^2}{c(s_M+1)^2}(w_D(s; b)-k) = \mathrm{K}.$$

Now

$$\Pi_D^C(s; b) = \frac{4\pi}{cs}(p_D^C(s; b)-k)(w_D(s; b)-p_D^s(s; b))$$

$$> \frac{2\pi^2 b}{cs^2}(w_D(s; b)-p_D^s(s; b)) \quad \text{because of Lemma 4C}(c)$$

$$> \frac{2\pi^2 b}{cs^2}\left(w_D(s; b)-k-\frac{2b\pi}{s}\right) \quad \text{because of Lemma 4C}(a).$$

Also

$$\frac{1}{s^2}(w_D(s)-k)-\frac{2b\pi}{s^3} > \mathrm{K} = \frac{1}{(s+1)^2}(w_D(s)-k)$$

$$\Leftrightarrow \quad (w_D(s)-k)\left(\frac{1}{s^2}-\frac{1}{(s+1)^2}\right) > \frac{2b\pi}{s^3}.$$

This inequality must satisfied by a choice of sufficiently small b. Thus there exists a b_{MC} such that $\forall b \leqslant b_{MC}$, $\exists$ K such that $4 = s_D^M \leqslant s_D^C$. Then

$$p_D^M(4; b) = \frac{\mathrm{D}+(b\pi/2)(3/5)+k}{2} = \frac{\mathrm{D}+k}{2}+\frac{3}{20}b\pi$$

$$\frac{\mathrm{D}+k}{2}+\frac{3}{20}b\pi > k+\frac{2b\pi}{4} \quad \Leftrightarrow \quad \mathrm{D}-k > b\pi(1-0{,}3).$$

This inequality is also satisfied for all sufficiently small b. Thus

$$p_D^M(4; b) > k+\frac{2b\pi}{4} \geqslant k+\frac{2b\pi}{s} \quad \textit{for all } s \geqslant 4.$$

But $k+(2b\pi/s) > p_D^C(s; b)$ because of Lemma 4C (a). Therefore $p_D^M(s^M; b) > p_D^C(s^C; b)$ for sufficiently small b. □

THEOREM 6 M-C: There exists K, a, b such that $s^M > 1$ and $s^C = 0$.

Proof: 1. For fixed $s \geqslant 2$, choose a, b, k such that

$$a-\frac{b\pi}{s+1}-k = w(s; a, b)-k = \frac{b\pi}{6}\frac{(4+3s)}{s^2}.$$

Manipulating the necessary condition (in price) for the single-product seller's profit yields $p^C(s; a, b) = k+(b\pi/2s)$.

2. Choose $s=2$. Then for an appropriate constant h

$$\Pi^{C}(2)=\frac{h}{2}\left(w(2)-k-\frac{b\pi}{4}\right)<K=\frac{2h}{9}(w(2)-k)$$

if and only if

$$(\omega(2)-k)\left(1-\frac{4}{9}\right)<\frac{b\pi}{4}$$

$$\Leftrightarrow \quad 50<54.$$

Thus $\Pi^{C}(2)<K$ if K is choosen such that the necessary condition for $s^{M}=2$ is satisfied.

3. Choose $a-k=(3/4)b\pi$. In the proof of the above lemma, we have shown that $\Pi^{M}(s)$ is concave in s if

$$2(a-k)(s+1)-3b\pi>0$$

$$\Leftrightarrow \quad \frac{3}{2}b\pi(s+1)>3b\pi \quad \Leftrightarrow \quad s+1\geqslant 2 \quad \Leftrightarrow \quad s\geqslant 1.$$

Thus Π^{M} is concave in s.

4.

$$\Pi^{M}(2)=\frac{\pi}{c}\left[a-k-\frac{b\pi}{3}\right]^{2}>K=\frac{2b\pi^{2}}{c9}\left[a-k-\frac{b\pi}{3}\right]$$

If and only if

$$\left[a-k-\frac{b\pi}{3}\right]>\frac{2}{9}b\pi$$

if and only if

$$\frac{3}{4}-\frac{1}{3}>\frac{2}{9} \quad \Leftrightarrow \quad \frac{5}{12}>\frac{2}{9} \quad \Leftrightarrow \quad 45>24.$$

Thus $s^{M}=2$.

5.

$$\Pi^{M}(1)=\frac{\pi}{c}\left[\frac{3}{4}b\pi-\frac{b\pi}{3}\right]^{2}<K=\frac{2b\pi^{2}}{c9}\left[\frac{3}{4}-\frac{1}{3}\right]b\pi$$

if and only if

$$\frac{1}{16}<\frac{1}{9}\,\frac{5}{6}=\frac{5}{54}.$$

This excludes $s^{C}=1$.

6.

$$\Pi^M(3) = \frac{\pi}{c}\left[\frac{3}{4}\,b\pi - \frac{1}{4}\,b\pi\right]^2 = \frac{b^2\pi^3}{c4} < 3\,K = 3\,\frac{5}{54}\,\frac{b^2\pi^3}{c}$$

$$\Leftrightarrow\quad \frac{1}{4} < \frac{5}{18} \quad\Leftrightarrow\quad 18 < 20.$$

Thus $(1/s)\,\Pi^M(s) < K$ for $s \geqslant 3$. Corollary 4 M-C ensures that $\Pi^C(s) < K$ for all $s \geqslant 1$. □

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