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Measuring Marginal Effective Tax Rates: Theory and Application to Canada

Robin BOADWAY *

ABSTRACT. – This paper summarizes the various methodologies for estimating marginal effective tax rates applying to investment decisions. The conceptual framework is developed for several types of investment decisions, including depreciable capital, inventories, and depletable assets. The analysis involves incorporating personal and corporate taxes into the neo-classical theory of the firm. Some of the theoretical issues are discussed including the treatment of adjustment costs, non-price-taking behaviour, and the implications of an open economy. The problems involved with implementing effective tax rates empirically are discussed. These include the arbitrage assumption, the treatment of risk, the treatment of firms temporarily in a non-taxable position, and data problems, including problems of aggregation. A number of illustrative calculations are presented using Canadian data.

Mesure des taux marginaux effectifs d'imposition. Théorie et application au Canada

RÉSUMÉ. – Cet article résume les différentes méthodologies utilisables pour mesurer les taux marginaux effectifs d'imposition s'appliquant aux décisions d'investissement. Le cadre théorique est développé pour une gamme de situations, et prend en compte aussi bien les stocks que la dépréciation du capital ou l'existence de ressources non renouvelables. L'analyse requiert en particulier l'introduction de la fiscalité sur les personnes et sur les sociétés dans la théorie néo-classique de l'entreprise. Quelques questions théoriques sont discutées à cette occasion : coûts d'ajustement, comportements non concurrentiels, ou spécificités des économies ouvertes. On étudie aussi les problèmes liés à la mise en place effective des taux considérés; en particulier, les hypothèses d'arbitrage, le traitement du risque, le cas des firmes temporairement non imposables, et les problèmes de données (notamment d'agrégation). De nombreux exemples chiffrés, portant sur des données canadiennes, illustrent ces aspects.

* R. W. BOADWAY: Department of Economics, Queen's University, Kingston, Ontario K7L 3N6, Canada.

1 Introduction

The concept of a marginal effective tax rate (hereafter, simply a marginal tax rate) on capital income has become widely used as its application has been extended to more and more countries. Perhaps the most well known methodology for devising and measuring marginal tax rates is that associated with the work of King and Fullerton [1984] who compute such rates for four countries under a variety of scenarios. Their work has since been extended to a number of other countries, including our own (Daly *et al.* [1986]).

Parallel to this work, a quite separate but related methodology has been developed by myself and other co-authors, particularly Neil Bruce and Jack Mintz, for measuring marginal tax rates. This methodology has been applied extensively to the case of Canada. It differs from the King-Fullerton approach in a variety of ways, many of which will be made clear in what follows. It attempts to measure actual marginal tax rates rather than hypothetical ones; it incorporates an open capital market and uses different arbitrage assumptions; it uses actual data on rates of return and expected inflation; and, most important, it extends the analysis to investment decisions other than those on depreciable capital.

The purpose of this paper is to review this alternative methodology and its application to various investment decisions. A summary of the sorts of results that have been obtained will be presented. The methodology seems only to have been applied to the Canadian case to date. However, it is clear that it would be straightforward to apply it to other countries as well. Our discussion will concentrate mainly on our own methodology, with references to the King-Fullerton methods for contrast only. A fuller discussion and critique of the latter may be found elsewhere. (See Boadway [1987].)

2 Effective Tax Rates in Theory. The Case of Depreciable Capital

Much of the theory of taxation and investment has been developed in the context of depreciable capital, so we begin with that case. The marginal tax rate represents the difference between the pre-tax rate of return on the marginal investment and the after-tax return to those who finance it. To derive an expression for the pre-tax rate of return, we use the conventional dynamic neo-classical theory of the firm.

Consider a firm which produces output according to the strictly concave production function $F(K_t)$ where K_t is the capital stock. All other arguments are suppressed for simplicity. In the absence of new share issues, the dividend stream D_t of the firm may be written:

$$D_t = (1-u) P_t F(K_t) - (1-\varphi) Q_t (\dot{K}_t + \delta K_t) + u \alpha A_t + \dot{B}_t - i(1-u) B_t \tag{1}$$

where P_t = price of output;

Q_t = price of investment goods;

u = corporate tax rate;

φ = investment tax credit rate;

δ = true depreciation rate on capital;

α = depreciation rate on capital for tax purposes;

i = interest rate;

A_t = undepreciated value of capital for tax purposes;

B_t = debt of the firm.

This formulation makes particular assumptions about the tax structure which could easily be revised. [1] All rates of return and tax rates are treated as constant for simplicity.

It is convenient to write (1) in the following form:

$$D_t = X_t + \dot{B}_t - i(1-u) B_t \tag{2}$$

where X_t is called the cash flow of the firm. The latter two terms capture the financial flows of the firm with non-shareholders.

Assuming competitive capital markets, capital market equilibrium requires:

$$\rho E_t = (1-c) \dot{E}_t + (1-\theta) D_t \tag{3}$$

where ρ is the nominal after-tax rate of return on equity to existing shareholders, E_t is the value of equity in the firm, c is the shareholders' personal tax rate on capital gains (converted to an effective rate on accruals) and θ is the shareholders' tax rate on dividends. Solving (3) for E_t gives:

$$E_t = \int_t^{\infty} e^{-\rho (s-t)/(1-c)} D_s (1-\theta)/(1-c)\, ds. \tag{4}$$

1. The tax base is revenue less nominal interest ($i B_t$) less tax depreciation (αA_t) where $A_t = e^{-\alpha t} A_0 + \int_0^t e^{-\alpha (t-s)} (1-\varphi) Q_s I_s ds$ and $I_s = \dot{K}_t + \delta K_t$ is gross investment. The base for tax depreciation is reduced by the investment tax credit, which corresponds with the Canadian case.

Thus, the equity value of the firm is the tax-adjusted dividend stream discounted by $\rho/(1-c)$, which is the pre-tax return on equity (retained earnings) held in the firm.

The equity value defined by (4) would be a suitable objective function for the firm. However, as it stands it will not yield an internal solution. Both an investment policy ($\dot{K}$) and a financial policy ($\dot{B}$) must be determined. However, as is obvious from inspection, the objective function is monotonic in B_t.[2] To avoid this problem, we treat the financial structure as exogenously given, a procedure which is common in this literature. In particular, we assume that the debt-equity ratio is given as $b=B_t/E_t$.[3] Using this definition of b in (2), substituting the result in (3) and again solving for (4) yields:

$$E_t=(b+(1-c)/(1-\theta))^{-1}\int_t^{\infty} e^{-r(s-t)}X_s\,ds \tag{5}$$

where

$$r=\frac{\rho/(1-c)+i(1-u)b(1-\theta)/(1-c)}{1+b(1-\theta)/(1-c)}. \tag{6}$$

We can think of r as the nominal cost of financial capital to the firm. It is a weighted average of the cost of equity finance ($\rho/(1-c)$) and the cost of debt finance ($i(1-u)$). Furthermore, the weights can be shown to be the proportions in which additional investment is financed by new debt and retained earnings.[4]

The incorporation of new issues as a source of equity finance can readily be done. The nominal cost of new equity finance can be shown to equal $\sigma/(1-\theta)+\pi(1-(1-c)/(1-\theta))$ where π is the required return to new

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2. The financial part of the objective function may be written:

$$\int_t^{\infty} e^{-\rho(s-t)/(1-c)}(\dot{B}_s-i(1-u)B_s)(1-\theta)/(1-c)\,ds.$$

3. This procedure of assuming that a firm's optimization can be treated as a two-stage problem with the first stage representing the choice of a financial structure and the second stage a real capital structure can be justified under certain restrictive assumptions. For example, if the firm's costs of debt and/or equity capital are increasing functions of the debt-equity ratio, that will be the case. It will also be true if the firm is quantity constrained in debt.

4. From the capital market equilibrium condition (3), for a given value of E_t, reducing current dividends by \$ 1 causes share values to rise by $(1-\theta)/(1-c)$ dollars. Therefore, increasing retained earnings by $(1-\beta)$ will cause the value of equity to rise by $(1-\beta)(1-\theta)/(1-c)$. The fixed debt-equity ratio requires that $B=bE$. Therefore, to keep b fixed the debt increase β accompanying the increase in retained earnings is given by $\beta=b(1-\beta)(1-\theta)/(1-c)$. Solving this expression for β gives $\beta=b(1-\theta)/(1-c)/(1+b(1-\theta)/(1-c))$ as in the text.

shareholders.[5] If a proportion a of equity finance is from retained earnings, the cost of capital can be written as:

$$r = \beta i(1-u) + (1-\beta)[a\rho/(1-c) + (1-a)(\sigma/(1-\theta) + \pi(1-(1-c)/(1-\theta)))] \quad (7)$$

where $\beta = b(1-\theta)(1-c)/(1+b(1-\theta)/(1-c))$ is the proportion of new investment which is debt-financed.

By (5), the equity value of the firm is proportional to the present value of the cash flow X_s discounted by the cost of capital r. If we take this latter to be the objective function of the firm, the first-order condition on the real investment decision of the firm can be shown to equal:[6]

$$\frac{PF'(K)}{Q} = \frac{r+\delta-\dot{Q}/Q}{1-u}(1-\varphi)\left(1-\frac{u\alpha}{r+\alpha}\right) \quad (8)$$

where time subscripts have been suppressed for simplicity. Next, denoting p and q as real prices obtained by deflating P and Q by $e^{-\pi t}$ where π is the inflation rate, (8) becomes:

$$\frac{pF'(K)}{q} = \frac{r+\delta-\dot{q}/q-\pi}{1-u}(1-\varphi)\left(1-\frac{u\alpha}{r+\alpha}\right). \quad (9)$$

This is a standard user cost of capital expression incorporating taxes. It represents the gross-of-tax marginal product of capital. To convert it to a rate of return we subtract the economic depreciation rate. The gross rate of return r_g is defined as:

$$r_g = \frac{r+\delta-\dot{q}/q-\pi}{(1-u)}(1-\varphi)\left(1-\frac{u\alpha}{r+\alpha}\right)-(\delta-\dot{q}/q). \quad (10)$$

The measurement of r_g (i.e., the components of its right hand side) is an essential ingredient of the marginal effective tax calculation. The definition of the marginal effective tax rate is simply $t = r_g - r_n$ where r_n is the real after-tax rate of return to savers. In the context of this model, r_n is given by:

$$r_n = \beta i(1-m) + (1-\beta)(a\rho + (1-a)\alpha) - \pi \quad (11)$$

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5. The logic behind this is as follows. Treat (4) as referring to value per share. Let $d = De^{-\pi t}$ be the flow of real dividends per share. If d were constant over time, integration of (4) would yield $E = d(1-\theta)/(1-c)/(\rho/(1-c)-\pi)$. Consider now a new share issue of \$1. In itself, this will cause the value of existing equity to fall by \$1. Using the above expression for E, a change in E of \$1 is equivalent to a change in the perpetual flow of dividends of $d = \rho/(1-\theta) - \pi(1-c)/(1-\theta)$. This is the flow cost to existing shareholders of raising 1 \$ of new equity. See also AUERBACH [1979].
6. The actual problem of the firm is:

$$\max_{\dot{K},\dot{A}} \int_0^\infty e^{-rt}[P_t F(K_t) - (1-\varphi)Q_t(\dot{K}_t + \delta K_t) + u\alpha A_t]\,dt$$

$$\text{s.t.} \quad \dot{A}_t + \alpha A_t = (1-\varphi)Q_t(\dot{K}_t + \delta K_t).$$

where m is the personal tax rate on interest income. Equations (10) and (11) form the basis for measuring marginal tax rates, the details of which we return to below.

This basic formulation has made a number of rather restrictive assumptions. Before turning to other sorts of capital decisions, it is worth considering the implications of relaxing some of them.

a. *Non-Exponential Depreciation*

Neither the rate of real depreciation nor the rate of tax depreciation need be exponential. We could define a depreciation function $\Delta(K)$, for example, such that $I=\dot{K}+\Delta(K)$. In this case, the term δ in (9) and (10) would be replaced by $\Delta'(K)$. Similarly, vintage capital could also be incorporated. The tax depreciation rate could take on any arbitrary pattern as well. The term $u\alpha/(r+\alpha)$ is simply replaced by the present value expression for any depreciation schedule one desired. [7]

b. *Monopoly Behaviour*

If the firm is a monopolist, the left-hand side of (8) becomes the marginal value product per unit of capital, $(P+P'F(K))F'/Q$, or $(1-1/\eta)PF'/Q$ where η is the elasticity of demand. Calculating the marginal distortion using (10) would capture only the tax distortion, the difference between the private gross rate of return and the net return to savings. The social gross rate of return would have to include the monopoly distortion and would be given by:

$$r_g=\frac{r+\delta-\dot{q}/q-\pi}{(1-1/\eta)(1-u)}(1-\varphi)\left(1-\frac{u\alpha}{r+\alpha}\right)-(\delta-q/\dot{q}).$$

Monopsony power in the purchase of capital inputs is a special case of adjustment costs to which we now turn.

c. *Adjustment Costs*

The implications of adjustment costs for measuring the marginal tax rate depends upon the form of the adjustment costs and upon the extent to which they are tax deductible. Consider as an example the case in which adjustment costs are separable and are given by the function $\gamma(\dot{K}, K)$. If a proportion x are tax-deductible, the objective function of the firm must

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7. For example, indexing the book value of capital for depreciation would change the present value expression to $u\alpha/(r+\alpha-\pi)$. Alternatively, straightline depreciation over a length of life T would give a present value of tax savings of $u(1-e^{-rT})/rT$.

include as part of the cash flow the term $-(1-xu)\gamma(\dot{K}, K)$. The first-order conditions then simplify to:

$$\frac{PF'}{Q}-\frac{1-xu}{1-u}(\gamma_2+r\gamma_1-\dot{\gamma}_1)/Q$$
$$=\frac{r+\delta-\dot{Q}/Q}{1-u}(1-\varphi)\left(1-\frac{u\alpha}{r+\alpha}\right).$$

The lefthand side represents the gross marginal product of capital after adjustment costs. If adjustment costs were independent of tax, the procedure suggested above for measuring r_g would appropriately capture the social rate of return after adjustment costs. A sufficient condition for this would be that $x=1$ (so adjustment costs are tax-deductible) and r is independent of taxes. Failing this, the proper measurement of r_g would require terms involving the adjustment cost function which is not observable.

3 Other Investment Decisions

In principle, an effective tax rate could be derived and measured for any sort of decision for which taxes impinge at the margin. We have worked out effective tax rate expressions for some other decisions taken by firms, though the cases considered are by no means exhaustive. We present below the derivation of r_g for three different cases – non-depreciable capital, inventory capital and depletable resource properties. Other interesting cases which could be worked out include research and development, investment and harvesting of renewable resources, labour training, advertising and marketing, etc. In each case, what would be involved is a derivation of r_g. The computation of an effective tax rate as $t=r_g-r_n$ is as before.

a. *Non-Depreciable Capital*

The rate of return on non-depreciable capital (e.g. land) is simply the special case of depreciable capital where $\delta=\alpha=0$. Thus (10) reduces to:

(12) $$r_g=\frac{r-\dot{q}/q-\pi}{1-u}(1-\varphi)+\dot{q}/q.$$

Recall that taxes generally influence r as in (7).

b. *Inventory Capital*

A completely general theory of the holding of inventories can be very complicated indeed owing to the dynamic nature of the problem. We have made some reasonable simplifications to make the problem both manageable and intuitive. In particular, we have modelled the firm in the steady state. The firm produces an output X using as an input some raw material. An amount R of the raw material is held as inventory (or work in progress). The average holding period of a unit of inventory is $T = R/X$, chosen by the firm. The firm produces a unit of output using a unit of raw material drawn from inventory and incurs costs of $C(X, R)$ where $C_1 > 0$, $C_2 < 0$. The price of output is Q and the purchase price of raw material is P.

The corporate base includes total revenues (QX) less current costs (C) less interest costs less the FIFO value of raw materials taken out of inventory. We denote P_{-T} as the FIFO value of goods taken out of inventory after being held there a length of time T. [8] The problem of the firm is:

$$(13) \quad \max_{X, \dot{R}} \int_0^{\infty} e^{-rt}[(1-u)(QX - C(X, R)) - P(X+\dot{R}) + u P_{-T} X]\, dt$$

where $T = R/X$ and $P(X+\dot{R})$ represents the new acquisition of raw materials. The first order conditions for this problem reduce to

$$(14) \quad \frac{-C_2(X, R)}{P} = \frac{r - (1 - u e^{-\gamma T})\gamma}{1-u},$$

where $\gamma = \dot{P}/P$. This expression gives the marginal benefit of a unit of inventory holdings. To convert it to a rate of return, we subtract the real capital loss on holding a unit of inventory, so:

$$(15) \quad r_g = \frac{r - (1 - u e^{-\gamma T})\gamma}{1-u} + \frac{\dot{p}}{p}$$

where $\dot{p}/p = \dot{P}/P - \pi$.

c. *Depletable Resources*

As with inventories, we must make some simplifying assumption to render the problem manageable. We consider a firm which is simultaneously involved in exploration, investment in mining facilities, and extraction. Inventories are excluded so that sales equal extraction; it would be relatively straightforward to add inventories. The taxation of resources is notoriously complex in practice. For illustrative purposes we consider a relatively simple scheme which incorporates most of the key issues. [8]

In the exploration stage the firm hires current inputs L at a price W and produces a depletable asset according to the strictly concave function $S(L)$. It then invests in mining capital K at a price Q to make the asset ready for extraction. The production function is $Z(K, F)$ where F is the

8. The Canadian tax system also allows a deduction based on the FIFO value of the stock of inventories. For the way in which this is incorporated into the problem, see BOADWAY, BRUCE and MINTZ [1982].

9. A more detailed treatment of the Canadian resource tax system may be found in BOADWAY, BRUCE, MCKENZIE and MINTZ [1987].

current use of previously discovered asset. This is the only stage at which depreciable capital is used, though it would be straightforward to allow for it at either of the other two stages. Finally, the firm extracts an amount Y of the resource according to the strictly convex nominal cost function C(Y) and sells it at a price P. The dividend flow resulting from this three-stage process is:

$$D = PY - C(Y) - WL - Q(\dot{K} + \delta K) + \dot{B} - iB - T \tag{16}$$

where T is the tax liability.

The expression for tax liabilities depends both on the jurisdiction levying the tax and on the type of resource. Typically, firms will be liable both for a royalty (severance tax) and an income tax. The latter generally involves generous write-off provisions as well as some deduction for the use of the asset itself (a depletion allowance). We assume a royalty tax rate of g based on total revenues. The corporate tax liability will be written:

$$T_c = u[PY - C(Y) - WL - \alpha A - R - iB] + \varphi Q(\dot{K} + \delta K). \tag{17}$$

Here, R is the depletion allowance and is defined to be $R = t(PY - C(Y) - \alpha A)$, though most systems (including Canada's) are more complicated than that. All other variables in (17) are the same as defined earlier.

Proceeding as before, using the expression for taxes and the royalty rate, we define the cash flow of the firm to be:

$$\begin{aligned} X = {} & PY(1 - u(1-t) - g) - C(Y)(1 - u(1-t)) - WL(1-u) \\ & - Q(1-\varphi)(\dot{K} + \delta K) + \alpha A\, u(1-t). \end{aligned} \tag{18}$$

The firm maximizes the present value of its cash flow discounted by R in (7) and subject to the equation of motion on A and the following two resource constraints:

$$\begin{aligned} & \int_0^\infty (Y - Z(F, K))\, dt \leqslant 0 \\ & \int_0^\infty (F - S(L))\, dt \leqslant 0. \end{aligned} \tag{19}$$

The first states that the total resource extracted cannot exceed the total developed, while the second states that the total resource developed cannot exceed the total found. In a more general version of this problem, this constraint would have to hold at each point in time.

The first order conditions for this problem on $\dot{K}$, L and Y respectively reduce to:

$$\text{(20)} \qquad \frac{(p-c')}{q}\frac{\partial Z}{\partial K}=\frac{(r+\delta-\dot{q}/q-\pi)}{1-u(1-t)-gp/(p-c')} \times(1-\varphi)\left(1-\frac{\alpha u(1-t)}{r+\alpha}\right)$$

$$\text{(21)} \qquad \frac{(p-c')}{w}\frac{\partial Z}{\partial F}\frac{\partial S}{\partial L}=\frac{1-u}{1-u(1-t)-gp/(p-c')}$$

$$\text{(22)} \qquad \frac{\dot{p}-\dot{c}'}{p-c'}=r-\pi+\frac{\dot{p}}{p}-\frac{(r-\pi)g}{(1-u(1-t))(1-(c'/p))}.$$

The first of these is simply the before tax gross marginal product of capital. To convert it to r_g simply subtract $\delta-\dot{q}/q$. The second equation is the social value of marginal product per unit of input L. An effective tax rate can be obtained directly by subtracting unity from (21). The final equation is a form of Hotelling's rule. It gives the gross rate of return to society from not extracting the resource. It can be converted to an effective tax wedge by subtracting r_n.

4 From Theory to Measurement

Effective tax rate computations are based on calculating values for r_n as given by (11) and r_g as given by (10) or its analogue for other sorts of capital. The procedure we have followed is to attempt to evaluate all the parameters in, say, equations (10) and (11) for some level of aggregation. Thus, we attempt to measure actual marginal tax distortions for particular types of investment, rather than hypothetical ones (as in King and Fullerton [1984]). Before outlining the method used to obtain parameter values, it is worth mentioning some important conceptual issues and assumptions used as well as their limitations.

a. *The Level of Aggregation*

Given the specificity of most tax structures, there are in principle a large number of marginal distortions on investment in the economy. Some aggregation is inevitable. The minimum amount of disaggregation we use is by type of asset (machinery, building, land, inventory, and depletable assets). Further disaggregation is usually done on a piecemeal basis. We have variously disaggregated by industry, by size of firm, by province, and by year. Whatever the level of aggregation, our procedure is to compute the parameters of r_g and r_n at the same level of aggregation rather than aggregating effective tax rates that were calculated from more disaggregated parameters. (This latter procedure is followed in KING and FULLERTON [1984].)

b. *The Arbitrage Assumption*

A key distinguishing feature of alternative effective tax calculations concerns the arbitrage assumption chosen and consequently which rates of return are taken as given. The need for an arbitrage assumption arises because of the fact that tax systems impose varying burdens on different sources of finance – debt, retained earnings and new issues. This implies that differential burdens must be imposed on some agents in capital markets. The arbitrage assumption stipulates which agents in the market are able to compete away differential tax burdens. The arbitrage assumption we adopt is partly based on data considerations and partly on institutional ones. It is referred to as the *open economy assumption* and seems particularly appropriate for the Canadian setting.

The basic assumption is that, because Canada is a small open economy with a capital market which is highly integrated with world capital markets, the costs of debt and equity finance are determined by the latter. More particularly, for debt the after-tax return to foreign debt-holders is given exogenously. If starred values refer to foreign ones, the following international arbitrage condition must hold:

$$(i+\pi)(1-m^*)-(1-c^*)(\pi-\pi^*)=(i^*+\pi^*)(1-m^*). \tag{23}$$

This arbitrage equation, which determines i, assumes that exchange rate movements reflect differences in expected inflation (and are taxed as capital gains).

On the equity side, a further assumption is made for data reasons. As discussed below, the rate of return on equity is measured from stock market data in a manner which does not allow us to distinguish the rate of return on retained earnings from that on new share issues. The rate of return on equity paid by firms is therefore assumed to be the same for new issues as for retained earnings. This implies that

$$\frac{\rho}{1-c}=\frac{\sigma}{1-\theta}+\pi\left(1-\frac{1-c}{1-\theta}\right)=\rho_g. \tag{24}$$

The return ρ_g paid by firms is calculated from observed stock market data. The value of ρ_g must satisfy an international arbitrage condition analogous to (23) with ρ_g replacing i. The net return received by household savers, ρ and σ, can then be computed from (24) and used to obtain r_n. The value of r paid by firms is simply:

$$r=\beta i(1-u)+(1-\beta)\rho_g. \tag{25}$$

Thus, given observed measured of i and ρ_g, all financial rate of return variables can be computed. Also, comparative static or counterfactual computations can be done by considering changes in tax or inflation rates domestically given that the righthand side of (23) is exogenous.

The above arbitrage assumptions differ considerably of those of King and Fullerton, who adopt two alternative ones. Their fixed-p assumption is analogous to assuming r_g is the same on all projects (and $=10\%$). Notice that this implies that different firms face different interest rates and rates of return on equity. Their fixed-s case is that in which all arbitrage occurs at the household level so that, in our notation, $i(1-m)=\rho=\sigma$. This implies that firms face different costs of all three sources of finance and would be consistent with perfect substitutability at the household level of different sorts of assets. A variant of the fixed-p case was used by BRADFORD and FULLERTON [1981] who assumed that arbitrage occurred at the firm level so

$$i(1-u)=\rho/(1-c)=\sigma/(1-\theta)+\pi(1-(1-c)/(1-\theta)).$$

It should be noted that one further advantage of the open economy arbitrage assumption is that it allows us to disaggregate effective tax rate calculations into that due to the corporate tax and that due to the personal tax. In an open economy facing fixed world rates of return, corporate taxation affects mainly the investment decision while personal taxes affect savings. The magnitudes of the relevant distortions can be obtained by taking the difference between the world cost of funds r^* and either r_g or r_n as appropriate, where $r^*=\beta i+(1-\beta)\rho-\pi$.

C. *Loss Offsetting and Risk*

The above formulations were based on two implicit assumptions. The first is that negative tax liabilities are treated symmetrically with positive ones. The other is that the analysis is based on a deterministic model of household choice.

The absence of full loss offsetting can, in principle, be incorporated into the above theory. In theory, its effect can either increase or decrease marginal tax rates, though the former seems most likely to occur. In the context of depreciable assets, the absence of full loss offsetting reduces the present value of depreciation write-offs ($u\alpha/(r+\alpha)$) and reduces the value of interest write-offs ui (thereby increasing the effective cost of debt finance). Both these increase r_g to the extent that depreciation or interest write-offs are postponed. MINTZ [1986] has amended the above methodology to take these influences into account. On the other hand, to the extent that revenues are earned while the firm is in a non-taxpaying position, r_g will fall [since the grossing-up of the user cost in (10) will be at a rate greater than $(1-u)$].

The incorporation of risk is somewhat more difficult. One simple way to think of risk affecting r_g is through its effect on the rate of return to equity, ρ (or σ). One can think of the return to equity ρ as comprising a safe return i^* plus a risk premium h which can be estimated under certain circumstances. It has been established in the literature (e. g. MINTZ [1982], GORDON [1985]) that full loss offsetting is equivalent to allowing a deduction for the cost of risk-taking. To the extent that loss offsetting of risks does occur, the risk premium itself ought to be reduced by the tax, $h(1-u)$. Since our methodology does not reduce the risk premium by the tax, it will yield an overestimate of r_g to the extent that loss offsetting of risk occurs.

Whether or not loss offsetting occurs depends on the source of the risk. If the risk takes the form of capital risk as discussed in BULOW and SUMMERS [1984], loss offsetting does not occur. On the other hand, risks reflected in varying revenues will almost certainly be partly offset.

d. *The Data*

We briefly outline here the manner in which we attach numbers to the variables in r_g and r_n. The exact manner in which data are obtained depends upon the level for aggregation at which effective tax rates are being computed. Nonetheless the same general approach is followed in all cases. The following summarizes the principles followed in constructing the various types of data.

(i) *Financing Ratios* (β, a). These were constructed using the structure of liabilities from published balance sheets. Depending on the application, either differences between end-of-year values of liabilities of debt, retained earnings and new issues, or the stock values themselves were used to estimate β and a.

(ii) *Rates of Return* (i, ρ, σ). The bond rate i was calculated from a 12-month average of long-term nominal corporate bond yields. The required return on equity before personal tax was calculated from the inverse of price-earnings ratios, where book earnings were corrected to account for inflation's effects on the capital stock, inventories and debt liabilities. The arbitrage assumption requires that this also equal the before-tax return on new issues. For calculations at the industry level, a risk premium was calculated from capital-asset pricing model studies and adjusted for leverage.

(iii) *Inflation Rate* (π). The expected inflation rate was estimated using an ARIMA five-year forecast (using the sample method) based on the consumers price index.

(iv) *Real Capital Gains* ($\dot{q}/q$, $\dot{p}/p$). Expected capital gains on capital goods and resources were estimated by the same procedure as for inflation, using the appropriate capital good series or resource price index. For resources subject to royalties, it is also necessary to know the profit margin $(p-c')/p$. For the results we report here on mining, no royalty exists so it was not necessary to make this calculation. However, the methodology has been extended to oil and gas in Canada by SIMARD [1985]. She calculated profit margins by using estimates of the short run cost function. (The results reported below assume $\dot{q}/q=0$, however.)

(v) *Depreciation Rate* (δ). Depreciation rate calculations were based upon length of life data for various types of capital. Where necessary, service lives were aggregated using as weights the proportions of gross investment. Service lives L were converted to equivalent exponential depreciation rates by the formula $\delta=2/L$.

(vi) *Holding Period for Inventories* (T). These were calculated from the ratio of average monthly inventories to average monthly shipments.

(vii) *Corporate Tax Parameters* (α, u, φ). In Canada, the corporate tax rate and the investment tax credit rates depend upon the type and size of industry, the province of location and the type of investment. For each type of capital good, statutory tax rates were aggregated appropriately according to the share of income taxable at various rates. A similar procedure was used for the investment tax credit. For depreciation rates, when the tax system allowed declining balance write-offs, α was calculated as an average of the rates applicable to various types of capital using as weights the amounts of gross investment. When straight-line depreciation was allowed, the expression for r_g had to be amended as indicated earlier.

(viii) *Personal Tax Rates* (m, θ, c). The personal tax rate on interest income was calculated as an average of marginal tax rates on capital income across all income taxes. The dividend tax rate corrected this for the dividend tax credit. The capital gains tax rate, for those years in which capital gains were taxed, was somewhat more difficult to calculate since c is an accrued tax rate whereas capital gains are actually taxed on realization. The accrued tax rate c was calculated such that the present value of capital gains tax payment based on realized taxation was equal to the present value of taxes levied on accrued gains discounted by the shareholders' after-tax cost of equity finance. The average holding period of shares was taken as the ratio of shares floated to volume of shares traded. (A 10 year average was used.) The realized capital gains tax rate was taken to be one-half of the dividend tax rate.

5 An Outline of Some Results

Using the above methodology, a variety of effective tax computations have been undertaken for Canada from 1980 to the present. In a series of applications Boadway *et al.* have calculated effective tax rates according to the following aggregations.

- time series of aggregate effective tax rates by type of capital good (buildings, machinery, land and inventories);
- cross-section of effective tax rates by size of firm and same four types of capital;
- cross-section of effective tax rates by industry and same four types of capital;
- cross-section of effective tax rates for mining firms for two provinces (Ontario and Quebec) and eight types of investment.

These are the results that we briefly report here. Other applications have been made by other authors. The Department of Finance in Canada [1985] have computed a variety of effective corporate tax rates by industry for background use in their corporate tax reform process. Mintz [1986], has computed effective tax rates in situations of risk and imperfect loss offsetting. Simard [1985] has extended the resource industry methodology to oil and gas.

A representative sample of the results achieved is shown in Tables 1-4, which refer to each of the four cases mentionned above.

(i) *Effective Tax Rates Over Time.* For each year from 1963-81 effective corporate and personal tax rates were calculated by each of four types of capital goods aggregated over all sectors. For presentation purposes here, the rates are further aggregated into two sub-periods, 1963-71 and 1972-81. The break point was chosen to coincide with a major reform of the income tax system. The tax reform involved a lowering of personal and corporate tax rates, the including of capital gains in the tax base, the liberalization of deductions for capital expenditures, and some increase in the generosity of the personal tax treatment of capital income. In addition, over the two periods inflation rose, while real rates of return fell.

As Table 1 indicates, effective corporate tax rates fell over the two periods. This was accompanied by a fall in the gross rate of return on investment, with the real cost of capital changing very little. Inventories bore the highest tax and land the lowest. The results generally indicate that the liberalization of capital write-offs reduced effective tax rates, and that the rise in inflation had little effect on effective tax rates except for inventories.

Effective personal tax rates, on the other hand, showed a significant rise. This was not due to a change in rates, but to the detrimental effect of inflation in an unindexed system. It was this rise in effective personal tax rates which caused the overall effective tax rate to rise over the two periods.

(ii) *Effective Tax Rates by Firm Size.* Table 2 reports on effective tax rate calculations done for a single year by size of business. The interest in this is due partly to the fact that statutory corporate tax rates are significantly lower for small businesses (i. e., those primarily in the smallest asset-sized category in Table 2). Besides facing different statutory rates, small businesses have other structural differences which contribute to the results of Table 2. For example, they tend to hold a higher proportion of their real assets as inventory and a lower proportion as machinery, they tend to use more debt finance and less new equity issues, and they face higher costs of equity finance.

The results indicate that, despite the preferential rate, the tax system does not appear to discriminate in favour of small business. The smallest asset-size firms have slightly larger effective tax rates than the next firm size (though smaller than the largest firms). Thus, the lower tax rate is presumably offset by other factors, such as the structure of real capital. Notice that the same relative pattern of effective tax rates by type of capital good are observable here.

(iii) *Effective Tax Rates by Industry.* Table 3 shows effective corporate tax rates by industry according to two alternative values of the cost of funds. For these calculations, the financial structure and the cost of finance r was taken to be the same for all industries. Inter-industry differences can be attributed solely to differences in tax provisions and real asset structures.

The results indicate considerable variability across industries and capital goods in the effective tax rate. Again, inventories seem to be the most highly taxed, except in agriculture, forestry and fishing where they benefit

TABLE 1

The Cost of Capital, Effective Tax Rates and Net of Tax Return to Savers Averaged for the 1963-71 and 1971-81 Periods (in percentages)

	1963-71					1972-81				
	Buildings	Machinery	Land	Inventories	Average	Buildings	Machinery	Land	Inventories	Average
Rate of Return to Capital (r_g):										
Gross of All Taxes	6.4	6.4	6.0	8.5	6.9	5.5	5.4	4.1	9.1	6.3
Net of Corporate Tax	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8	4.8
Net of All Taxes	3.6	3.6	3.6	3.6	3.6	2.2	2.2	2.2	2.2	2.2
Effective Tax Rates [1]:										
Corporate and Personal	43.3	43.0	40.0	57.4	47.2	58.9	58.1	45.6	75.2	64.0
Corporate Only	24.6	24.1	19.9	43.3	29.8	12.5	11.0	−15.7	47.2	23.4
Personal Only	24.8	24.8	24.8	24.8	24.8	52.9	52.9	52.9	52.9	52.9

[1] The corporate rate is $(r_g - r^*)/r_g$; the personal rate is $(r^* - r_n)/r^*$; the total rate is $(r_g - r_n)/r_g$, where $r^* = \beta i + (1-\beta)\rho - \pi$.

Source: BOADWAY, BRUCE and MINTZ [1987].

TABLE 2

Effective Tax Rates by Size of Business [1] ***(in percentages)***

Asset Size ($ Mil)	Land	Buildings	Machinery	Inventories	Aggregate
0-1/4	51.8	55.2	56.1	71.5	64.0
1/4-1	48.3	52.0	52.5	69.4	61.9
1-5	40.3	51.4	54.4	75.0	67.2
5-10	36.4	51.3	55.6	76.7	68.8
10-25	38.9	52.6	57.8	77.0	69.4
25+	45.5	55.4	59.3	79.0	66.9

[1] Effective tax rates are calculated as $(r_g - r_n)/r_g$.

Source: BOADWAY, BRUCE and MINTZ [1987].

TABLE 3

Effective Corporate Tax Rates by Industry, $(r_g - r)/r_g$

Industry	Buildings	Machinery	Land	Inventories	Aggregate
I. *Real Cost of Funds of* 10.0% [1]:					
Agriculture, Fishing, Forestry	7.7	0.4	12.8	−14.6	2.1
Manufacturing	25.3	7.6	22.9	33.6	20.5
Construction	17.3	6.7	15.1	33.5	16.0
Utilities	32.5	31.6	25.8	42.3	32.2
Wholesale Trade	22.9	18.9	24.3	31.2	27.1
Retail Trade	16.9	25.2	20.6	26.9	24.3
Services	15.6	23.5	15.3	28.2	20.8
TOTAL	24.0	20.5	18.3	32.1	24.0
II. *Real Cost of Funds of* 5.0% [2]:					
Agriculture, Fishing, Forestry	5.0	−0.6	6.0	−29.8	−2.0
Manufacturing	23.8	5.9	10.1	45.1	25.8
Construction	16.6	8.5	6.9	47.4	21.3
Utilities	31.0	33.2	11.7	54.8	34.1
Wholesale Trade	21.6	24.8	10.9	43.2	36.2
Retail Trade	15.3	34.2	9.2	38.5	32.3
Services	15.3	31.8	7.0	40.5	26.1
TOTAL	22.7	22.5	8.2	43.8	28.6

[1] Based on the 1984 nominal interest rate of 12%, the cost of equity finance of 16.3%, and the expected inflation rate (5%).

[2] Based on the 1977-80 nominal interest rate of 10.2%, the cost of equity finance of 14.5%, and the expected inflation rate (7.5%).

Source: BOADWAY, BRUCE and MINTZ [1987].

from cash flow accounting provisions. Land is least taxed, while depreciable capital is in between. The most favourably-treated sector is agriculture, forestry and fishing, then manufacturing and services due mainly to their favourable tax provisions. Trade industries face high effective tax rates due to their relatively large holdings of inventories. The wide dispersion

Table 4

1985 Effective Tax Rates for Ontario and Quebec Mining (processing profit constraint not binding) [a]

	Federal corporate income tax		Provincial corporate income tax		Total corporate income tax		Provincial mining tax		Total tax	
	Ont	Que	Ont	Que	Ont	Que	Ont	Que	Ont	Que
Depletable asset:										
$-t$	−1.9	−1.9	−0.5	−0.3	−2.4	−2.2	0	0	−2.4	−2.2
$-t_r$	−19.4	−19.4	−4.6	−2.5	−25.9	−23.0	0	0	−25.9	−23.0
Buildings:										
$-t$	−0.1	−0.6	1.3	1.1	0.3	−0.6	1.9	1.9	2.3	0.6
$-t_r$	−1.0	−5.6	9.9	8.4	2.7	−5.2	14.4	13.8	16.4	4.6
Equipment:										
$-t$	−1.7	−2.4	0.3	−0.1	−1.1	−2.3	0.7	0.6	0.8	−1.4
$-t_r$	−16.7	−25.8	1.4	−1.1	−10.3	−24.7	5.9	5.1	6.4	−13.1
Exploration and development [b]:										
$-t$	−28.8	−28.8	0	−3.4	−33.2	−35.1	0	−7.3	−48.5	−59.4
$-t_r$	−28.8	−28.8	0	−3.4	−33.2	−35.1	0	−7.3	−48.5	−59.4
Inventories:										
$-t$	2.6	2.6	1.0	0.3	4.4	3.2	3.5	3.1	13.0	9.4
$-t_r$	18.3	18.3	7.5	2.5	27.3	−21.5	23.1	20.9	52.5	44.4
Processing assets [c] smelting:										
$-t$	−4.1	−4.7	0.3	−0.4	−3.8	−5.2	−0.8	−3.1	−6.6	−11.2
$-t_r$	−53.3	−67.5	2.9	−3.1	−46.9	−78.0	−7.2	−35.3	−125.8	−1947.5
Concentrating and smelting:										
$-t$	−4.1	−4.7	0.3	−0.4	−3.8	−5.2	−.1	−5.0	−8.9	−14.4
$-t_r$	−53.3	−67.5	2.9	−3.1	−46.9	−78.0	−21.1	−74.6	−317.0	NA
Refining concentrating and smelting:										
$-t$	−4.1	−4.7	0.3	−0.4	−3.8	−5.2	−5.0	−5.6	−14.0	−15.0
$-t_r$	−53.3	−67.5	2.9	−3.1	−46.9	−78.0	−39.4	−74.6	−2616.3	NA

[a] *The marginal tax* is $t=r_g-r_n$, where r_g is gross of tax return to capital, and r_n is the net of tax return to capital.

The *marginal tax rate* is $t_r=(r_g-r_n)/r_g$.

[b] For exploration and development the *marginal tax rate* is $t_r=(r_g-w)/w$, where w is normalized to unity.

[c] In some cases the processing allowance is so generous that r_g is negative. Hence NA is reported.

Source: Boadway, Bruce, McKenzie and Mintz [1987].

of effective tax rates typifies the results that have been found by other studies in Canada (e. g. Daly *et al.* [1985], Department of Finance [1985]) and lends credence to the view that intersectoral misallocation of capital is at least as important as distortions imposed on aggregate capital accumulation.

(iv) *Effective Tax Rates in Mining.* Table 4 shows a sample of effective corporate tax rate calculations done for mining using the methodology described above. Given the various items of preferential treatment for resource industries, it is not surprising that, for almost all decisions, effective tax rates turn out to be negative. The exceptions are inventory and buildings. Exploration and development and processing are particularly favoured owing to the generous write-offs and special allowances given on expenditures for these reasons. These include the immediate write-off of exploration and development, the depletion allowance, and a processing allowance, all in addition to interest deductibility.

One must be somewhat cautious in interpreting these results for, say, welfare purposes since they really only apply to firms in a fully tax paying position. Because of the absence of full loss offsetting, firms in non-taxpaying positions cannot obtain the full benefit of the generous deductions. For these firms, effective tax rates may well be considerably higher.

This completes our summary of effective tax rate calculations for Canada. The methodology can be seen to be different in some important ways from the King-Fullerton technique. As with the latter, it would be straight-forward to adopt it to the institutional context of other countries.

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