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A Case for Differential Inheritance Taxation

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ABSTRACT. – This paper incorporates the case of a variable number of children in a simple model of the distribution of inherited wealth. In particular, the possibility of childless couples, and hence of bequests from relatives other than parents (*i. e.,* "collateral bequests") is considered. Within such a setting, two questions are raised. First, do collateral bequests increase the distributional inequality of wealth? Second, should one adopt inheritance tax rates that depend on the inheritor's blood relationship to the donor? In other words, does there exist an economic justification for differential inheritance taxation such as it is usual in many countries (e. g. France and Germany).

Doit-on taxer différemment les héritages?

RÉSUMÉ. – Cet article étudie les effets d'un nombre variable d'enfants dans un modèle de transmission et de répartition de la richesse. Il considère en particulier l'existence de couples sans enfants et donc d'héritages en ligne indirecte (héritages collatéraux). Dans ce cadre on aborde deux questions : (i) les héritages collatéraux augmentent-ils l'inégalité des fortunes? (ii) doit-on choisir des taux d'imposition de l'héritage dépendant du degré de parenté entre le légataire et le bénéficiaire? En d'autres termes, une fiscalité différentielle de l'héritage, telle qu'elle est pratiquée dans de nombreux pays (la France et l'Allemagne par exemple), est-elle économiquement fondée?.

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1 Introduction

This paper is one more contribution to the tradition of exploring the implications of different sets of assumptions on the distribution of wealth and its intergenerational transmission. The assumption considered here is that the number of heirs is variable, including the possibility of childless couples, and hence of bequests from relatives other than parents (*i.e.* "collateral bequests").

Two questions will be raised within such a setting. First, do collateral bequests increase the distributional inequality of wealth? Second, should one adopt inheritance tax rates that depend on the inheritor's blood relationship to the donor?

The paper is organized as follows. Some general notions are provided in the next section. In particular a possible distinction between planned and accidental bequests is presented. It is then argued that the relative importance of accidental versus planned bequests is not the same for direct (lineal) and for indirect (collateral) transmission and that this may have interesting implications as to the taxation of inheritance. Section 2 introduces the basic model in which each individual leaves an estate that is a simple fraction of his lifetime income, with a larger fraction to direct than to indirect lines. The steady-state distribution of inherited wealth is derived. Then allowing for inheritance tax rates varying according to consanguinity, the optimal tax rate structure is dealt with.

Two extensions of this basic model are then provided. In section 3, the share of lifetime wealth that is bequeathed is multiplied by a random factor reflecting the uncertainty in capital and labour markets. In section 4, there is also some randomness, but here arising from the uncertainty of the age of death; henceforth, one has both accidental and planned bequests. These rather simple bequest functions are shown, in section 5, to be derived from life-cycle utility maximization.

Anticipating the results derived below, we show that the mere existence of collateral bequeathing may increase the inequality of inherited wealth and that, on distributive grounds alone, the case for differential inheritance taxation according to consanguinity is quite strong.

2 Accidental and Planned Bequests. Lineal and Collateral Transmission

Bequests have always intrigued economists. They are indeed central to the understanding of several dynamic issues in economics. They clearly play a sizeable role in aggregate saving even though the exact magnitude of their influence on capital accumulation is vividly controversial. In the U.S. the share of inherited wealth in household wealth ranges as widely as from 80% in KOTLIKOFF and SUMMERS [1981] to 15% in MODIGLIANI [1988]. Wealth transmission is also central to BARRO's [1974] theory of the neutrality of public debt or any other attempt to socially redistribute resources between generations. Finally, uneven bequests have been shown to largely explain the observed inequality of wealth and of income (BRITTAIN [1978]).

In this paper, we focus on two ways of dividing bequests. The first division is between accidental and planned bequests; the second is between direct and indirect transmission of wealth. In the absence of perfect annuity and life insurance markets for reasons of, say, adverse selection, and in the absence of a bequest motive, individual lifetime uncertainty implies that consumers who live shorter than expected leave some wealth which is passed on to their heirs in the form of accidental bequests. On the other hand, if there exist such annuity markets and there is an explicit bequest motive, all bequests are planned. In general, however, it is not possible to sort out in actual bequests the planned from the unplanned part.

Assessing the relative part of accidental bequests in all bequests would be useful for many reasons. Were this share important, it would imply that the life-cycle model yields an adequate description of saving behaviour. It could also be used as a measure of the strength of the bequest motive, that is, the strength of intergenerational altruism. More importantly, the possibility of locating accidental bequests would have important consequences for fiscal policy as their taxation is free of any efficiency cost.

Most transmission models assume a fixed number of children, focusing on factors such as mating, estate sharing, human capital, parents' wealth, ... to account for the induced distribution of inherited wealth and income of subsequent generations. In this paper, we explicitly introduce the possibility of a variable number of children. It has been shown that this variability may be by itself a source of inequality (JENKINS [1985]); it can also act on the distribution of bequests through its interaction with social security, public debt or any intergenerational transfer of the kind (CREMER ET AL. [1987]). Another implication of a variable number of children is that it allows for the presence of childless couples or individuals. These may hold some wealth when death occurs either for a precautionary motive due to life cycle uncertainty or for a bequest motive out of altruism towards indirect relatives.

For childless couples the precautionary motive is as strong as in the case of direct transmission; one can, however, expect the explicit bequest motive, if any, to be weaker. Again, this conjecture cannot be checked with precision, though estates duty data in several countries show that indirect transmission occurs relatively more often in case of premature death. In 1977 France, out of 661,400 reported bequests, 58% concern children, the rest going to spouses, nieces or nephews. Yet, this figure ranges from 11 when the deceased is below 30 to 67 when the deceased is over 90. (Fouquet et Méron [1982]). This could be interpreted as showing that indirect bequests are essentially accidental. Were this conjecture correct, one would have a good case for taxing away all indirect bequests on efficiency grounds as least.

In that respect, it is noteworthy that in some countries such as the U.S. or the U.K., the death tax is imposed on the estate, thus regardless of the number of heirs and their relationship to the deceased, whereas in many other countries, France and Germany, for example, the tax rate varies according to the degree of consanguinity. The rationale for the latter practice is not always clear. Some authors have argued that what can be legitimately taxed is the "windfall" part of inheritance; they are thus implicitly mixing two concerns, one of efficiency alluded above and one of ethical legitimacy (see, e. g., Shoup [1969]). One does not find in this literature any justification for tax rates varying with the relationship of the heir to the deceased which would be based on equity concerns.

The rest of this paper is precisely addressed to this question. A simple model of wealth transmission is set up characterized by a variable fertility rate with the childless case, an uncertain life cycle implying accidental bequests, a bequest motive increasing with consanguinity, and a random productivity factor. It then shows to what extent and under which conditions differential tax rates can affect the degree of inequality of inherited wealth.

Throughout this paper, a linear bequest function is assumed such that bequests are proportional to lifetime income. This implies that testators derive utility from the mere amounts they bequeath and not from the expected level of their heir's utility. We thus posit paternalistic altruism as opposed to non-paternalistic altruism such as assumed by Barro [1974]. With the latter, testators would adjust their bequests to variations in taxation; they could even neutralize them completely. Such a specification would clearly induce a different model and different results.

3 Basic Model: Pure Bequest Motive and no Random Return

An individual of generation t works in period t and is characterized by life resources consisting of: inherited wealth, K_t, that varies between individuals and lifetime earnings, w, that are the same for all and constant over time. All bequests are planned (bequest motive) and are deterministically proportional to life resources. This latter assumption is consistent with the traditional life cycle theory, though highly questionable. It implies that our model is not very relevant to the upper strata of the wealth distribution in which the planning of bequests appears to be concentrated (see MODIGLIANI [1986]).

Turning to demographic hypotheses, sex differences are here assumed away so that the type of reproduction is asexual. We use the term individual and the masculine gender to designate the unit responsible for reproduction and bequeathing. The number of children this individual may have is a discrete random variable independent of his life resources. Without loss of generality, this variable takes two equally likely values: 0 or 2. Thus the size of each generation is kept constant and its structure with respect to the previous generation is constant as well: a generation consists of couples of brothers, half of which have childless uncles and half of which have uncles with children of their own. Childless individuals leave a fraction b of their wealth to their nephews if they have any; otherwise, they do not leave any inheritance. Individuals with children leave them a fraction a $(1 > a > b)$ of their life resources. In all cases, there is equal division of estates. [1] Finally, the population is sufficiently large to use the law of large numbers.

Let L be the constant size of any generation, and Ω_t, the set of members of generation t, $\omega_t \in \Omega_t$. Each generation can be divided in two equal parts: Ω_t^1 and Ω_t^2. An individual $\omega_t^1 \in \Omega_t^1$ has an uncle who has two children whereas an individual $\omega_t^2 \in \Omega_t^2$, has a childless uncle. In table 1, the structure of an generation t is depicted in relation with that of generation $t-1$.

TABLE 1

Representative members of generation t and t−1

His brother having	Member of generation $t-1$ having: no child	Member of generation $t-1$ having: two children
no child		Ω_t^2 lineal and collateral bequests size: L/4
two children	lineal and collateral bequests size: L/4 Ω_t^2	Ω_t^1 lineal bequests size: L/2

1. Other types of sharing could be dealt with, but it would complicate the analysis without affecting the nature of the conclusions.

We write $K_t(\omega_t)$ to denote the inherited wealth of ω_t and $n_t(\omega_t)$ ($=0$ or 2), the number of children of ω_t. Given all these assumptions and notation, the transmission of wealth is given by:

$$(1)\quad K_{t+1}(\omega_{t+1}) \begin{cases} =\dfrac{a}{2}\theta_1[K_t(\omega_t)+w] \\ \text{where } \omega_{t+1}\in\Omega^1_{t+1} \text{ and is the son of } \omega_t, \\ =\dfrac{a}{2}\theta_1[K_t(\omega_t)+w]+\dfrac{b}{2}\theta_2[K_t(\omega'_t)+w_t] \\ \text{where } \omega_{t+1}\in\Omega^2_{t+1} \text{ and is the son of } \omega_t \text{ and the nephew of } \omega'_t, \end{cases}$$

in which θ_1 and θ_2 represent one minus the tax rate of lineal and collateral bequests, respectively. All along, we assume that the proceeds of inheritance taxation are redistributed proportionally to inherited wealth so that it does not modify its degree of inequality. Another type of sharing could be considered. Bequests are unaffected by taxation and conversely inherited wealth is cut down by the amount of taxation. As a consequence, taxation affects the welfare of heirs but not that of the testators. In section 5, this is shown to be consistent with the maximization of a lifetime utility of Cobb-Douglas type. Other assumptions as to the functional form of the utility would surely imply gross bequests sensitive to taxation.

To measure the inequality of inherited wealth, we use the square of the coefficient of variation; this indicator is convenient for the problem at hand and for the degree of generality of this paper, it is surely as satisfactory as any other measure of inequality.

The mean value of inherited wealth can now be derived keeping in mind that the two subsets of heirs have the same size L/2 and the number of children (0 or 2) is independent of the wealth level. To denote the mean or the expected value of inherited wealth (equivalent because of the large population), we use $E(K_t)$. The mean within subgroup i is denoted by $E^i(K_t)$.

$$\begin{aligned} E(K_t) &= \Big[\sum_{\Omega^1_t} K_t(\omega^1_t)+\sum_{\Omega^2_t} K_t(\omega^2_t)\Big]\frac{1}{L} \\ (2)\qquad &= \frac{1}{2}E^1(K_t)+\frac{1}{2}E^2(K_t) \\ &= \frac{a}{4}\theta_1[E(K_{t-1})+w]+\frac{a\theta_1+b\theta_2}{4}[E(K_{t-1})+w] \\ &= A(\theta_1,\theta_2)[E(K_{t-1})+w], \end{aligned}$$

where

$$A(\theta_1,\theta_2)=\frac{2a\theta_1+b\theta_2}{4}$$

is the constant ratio of average inherited wealth to average lifetime resources in the previous generation. Equation (2) is a first-order difference equation describing the dynamics of average inherited wealth. One can similarly proceed for the variance of inherited wealth, denoted by $V(K_t)$. More precisely, we write:

$$V(K_t)=\frac{1}{L}\sum_{\Omega_t}[K_t(\omega_t)-E(K_t)]^2$$

and

$$V^i(K_t)=\frac{2}{L}\sum_{\Omega_t^i}[K_t(\omega_t^i)-E^i(K_t)]^2,$$

where $V^i(K_t)$ is the variance of wealth in subgroup i.

It can be shown that:

$$\sum_{\Omega_t}[K_t(\omega_t)-E(K_t)]^2=\sum_{i=1}^{2}\sum_{\Omega_t^i}[K_t(\omega_t^i)-E^i(K_t)]^2+\frac{L}{2}\sum_{i=1}^{2}[E^i(K_t)-E(K_t)]^2.$$

Hence, one obtains:

$$V(K_t)=\frac{1}{2}\sum_{i=1}^{2}V^i(K_t)+\frac{1}{2}\sum_{i=1}^{2}[E^i(K_t)-E(K_t)]^2$$

(3)
$$=V\left[\frac{a\theta_1}{2}(K_{t-1}+w)\right]+V\left[\frac{a\theta_1+b\theta_2}{2}(K_{t-1}+w)\right]$$
$$+\frac{(b\theta_2)^2}{16}[E(K_{t-1})+w)]^2$$
$$=[A^2(\theta_1,\theta_2)+B(\theta_2)]\,V(K_{t-1})+B(\theta_2)\,[E(K_{t-1}+w)]^2,$$

where

$$B(\theta_2)\equiv\left(\frac{b\theta_2}{4}\right)^2.$$

From (2) and (3), one derives the steady state values—supposed to exist and to be stable—for average inherited wealth E^* and its variance V^*:

(4)
$$E^*=\frac{A(\theta_1,\theta_2)}{1-A(\theta_1,\theta_2)}w,$$

(5)
$$V^*=\frac{B(\theta_2)}{[1-A(\theta_1,\theta_2)]^2[1-A^2(\theta_1,\theta_2)-B(\theta_2)]}w^2,$$

where both denominators are positive under plausible assumptions.

This yields the following steady-state square of the coefficient of variation:

(6)
$$(CV^*)^2=\frac{B(\theta_2)}{[1-A^2(\theta_1,\theta_2)-B(\theta_2)]\,A^2(\theta_1,\theta_2)}.$$

One sees right away that the steady state coefficient of variation vanishes if $B(\theta_2)=0$; that is, if there are no collateral bequests or if these are entirely taxed away. More generally, taxing collateral inheritance is desirable on distributive grounds.

Let us now turn to the taxation problem out of the steady state. We assume that the government collects a fixed amount R through inheritance taxation. For notational convenience, the tax rates are not time-indexed even though they are in fact adjusted in each period to meet the budget constraint. From (2), one has:

$$\frac{R}{L}=\frac{2a(1-\theta_1)+b(1-\theta_2)}{4}E(K_t+w)$$

or

$$\frac{R}{L}=[A(1,\ 1)-A(\theta_1,\ \theta_2)]\,E(K_t+w)$$

where $A(1,\ 1)$ is defined as above for $\theta_1=\theta_2=1$. We now introduce $\bar{A}$ such that:

$$\frac{R}{L}[A(1,\ 1)-\bar{A}]\,E(K_t+w)$$

This definition implies that for a given $E(K_t)$, the government's revenue constraint is equivalent to the equality $A(\theta_1,\ \theta_2)=\bar{A}$. Consequently, the average inherited wealth of generation $t+1$ is given by the average inherited wealth of generation t and the term A, irrespectively of the combination of tax rates actually chosen.

The taxation problem can now be solved. Our aim is to minimize the coefficient of variation of inherited wealth in $t+1$. Given the revenue constraint, $E(K_{t+1})$ is determined. We then just have to minimize the variance, that is,

$$\text{(7)}\qquad \min_{\theta_1,\,\theta_2} V(K_{t+1})=[A^2(\theta_1,\ \theta_2)+B(\theta_2)]\,V(K_t)+B(\theta_2)\,[E(K_t+w)]^2$$

subject to:

$$A(\theta_1,\ \theta_2)=\bar{A}.$$

One sees at once that this variance is a positive function of $B(\theta_2)$. Thus, the optimal solution is given by $\theta_2=0$ even though this may involve a value of θ_1 higher than unity to meet the revenue constraint. The optimal redistributive taxation problem is here rather trivial. It is introduced to present the path of reasoning to be used in the following sections. Indeed, when additional randomness is introduced, arising from either uncertain returns of variable lifetime, a possibility of crossinsurance between first relatives emerges, thus making 100% taxation of collateral bequests not necessarily optimal.

4 Random Return

In the previous section, the estate to be bequeathed was assumed to be a fixed proportion of lifetime resources, the porportion factor being the propensity to bequeath. As there is a generation gap between receiving inheritance K_t and earnings w on the one hand, and leaving bequests K_{t+1} to one's heirs on the other hand, one may expect some returns from these lifetime resources $(K_t + w)$. Let us now introduce explicitly such a factor, r, denoting the gross rate of return over a generation length. It is assumed again to be time invariant. To reflect various uncertainties on financial markets, r is assumed to be a random variable with expected value $\bar{r}$ and variance σ^2. Though they obey the same density function, the rates of return on lineal bequests and on collateral bequests are respectively denoted by r_a and r_b. To account for possible similarities in skill, luck, . . . between brothers, ρ is defined as the coefficient of correlation between rates of return on their lifetime resources. Yet, we assume the rates of return to be independent of the level of lifetime resources.

With this setting, one writes:

$$(8) \quad K_{t+1}(\omega_{t+1}) \begin{cases} = \dfrac{a\theta_1}{2} r_a [K_t(\omega_t) + w] \\ \text{where } \omega_{t+1} \in \Omega^1_{t+1} \text{ and is the son of } \omega_t, \\ = \dfrac{a\theta_1}{2} r_a [K_t(\omega_t) + w] + \dfrac{b\theta_2}{2} r_b [K_t(\omega'_t) + w] \\ \text{where } \omega_{t+1} \in \Omega^2_{t+1} \text{ and is the son} \\ \text{of } \omega_t \text{ and the nephew of } \omega'_t. \end{cases}$$

Using the same notation as above, the average value of inherited wealth and its variance are:

$$E(K_{t+1}) = A(\theta_1, \theta_2)\, E(K_t + w),$$

where

$$A(\theta_1, \theta_2) \equiv \frac{2a\theta_1 + b\theta_2}{4} \bar{r}$$

and

$$(9) \quad V(K_{t+1}) = [A^2(\theta_1, \theta_2) + B(\theta_1, \theta_2)]\, V(K_t) + B(\theta_1, \theta_2)\, [E(K_t + w)]^2$$

where

$$(10) \quad B(\theta_1, \theta_2) \equiv \frac{\bar{r}^2}{16} \left\{ (b\theta_2)^2 + \left(\frac{\sigma}{\bar{r}}\right)^2 [4(a\theta_1)^2 + 2(b\theta_2)^2 + 4a\theta_1 b\theta_2 \rho] \right\}.$$

One can then obtain the square of the coefficient of variation:

$$\text{(11)} \quad CV^2(K_{t+1}) = \frac{A^2(\theta_1, \theta_2) + B(\theta_1, \theta_2)}{A^2(\theta_1, \theta_2)} \frac{V(K_t)}{[E(K_t + w)]^2} + \frac{B(\theta_1, \theta_2)}{A^2(\theta_1, \theta_2)},$$

which in the steady state takes the following value:

$$\text{(12)} \qquad (CV^*)^2 = \frac{B(\theta_1, \theta_2)}{[1 - A^2(\theta_1, \theta_2) - B(\theta_1, \theta_2)]A^2(\theta_1, \theta_2)}.$$

Though the structure of these formulas is similar to that of the previous section, one notes that the definition (10) of $B(\theta_1, \theta_2)$ now includes the parameters of lineal bequests as well as the variance and covariance of random returns. In other words, even though $B(\theta_1, \theta_2) = 0$ is still sufficient to have perfect equality of inherited wealth in the long-run, this is not achieved by a mere 100% taxation of collateral bequests. We thus turn to the problem of the optimal redistributive taxation.

The taxation problem is formally identical to (7) and can be reduced to minimizing $B(\theta_1, \theta_2)$ such as defined in (10), subject to $A(\theta_1, \theta_2) = \bar{A}$. Assuming an interior solution, one obtains after a few manipulations:

$$\text{(13)} \qquad \theta_1^* = \frac{2[1 + \sigma^2(2 - \rho)]}{a[1 + \sigma^2(3 - 2\rho)]} \bar{A},$$

$$\text{(14)} \qquad \theta_2^* = \frac{4\sigma^2(1 - \rho)}{b[1 + \sigma^2(3 - 2\rho)]} \bar{A}.$$

From these formulas, one realizes that constraining the tax rates to be non negative ($\theta_1, \theta_2 \leq 1$) may imply corner solutions. In that respect, $\bar{A}$ plays a important role. A low value of $\bar{A}$ ($< b/4$), namely a high revenue constraint R, grants an interior solution. One sees right away that for $\rho = 1$, $\theta_2^* = 0$ which is the result of the previous section. For $\rho < 1$ one, however, has $\theta_2 > 0$. In other words, as soon as there is the slightest imperfection in the correlation between brothers, it pays from a redistributive viewpoint to leave some collateral bequests untaxed. This is a standard result in the theory of portofolio selection.

More generally, one easily checks from (13) and (14) that:

$$\frac{\theta_1^*}{\theta_2^*} = \frac{1 + (\sigma/\bar{r})^2(2 - \rho)}{(\sigma/\bar{r})^2(1 - \rho)} \frac{b}{2a}.$$

From this expression, one can derive the combinations of values for ρ and a/b, for a given $\sigma/\bar{r}$, which yield inheritance tax rates which are optimally either equal or inequal.

$$\text{(15)} \qquad \theta_1^*/\theta_2^* \gtreqless 1 \quad \text{iff} \quad \rho \gtreqless 1 - \frac{1 + (\bar{r}/\sigma)^2}{2a/b - 1}.$$

On Figure 1, we draw the relation between ρ and a/b, keeping in mind inequalities $-1 \leqq \rho \leqq 1$ and $a/b \geqq 1$. Observe that the relation so drawn moves rightward as the coefficient of variation of r decreases. This implies that the area $\theta_1 > \theta_2$ (higher rates on collateral bequests) increases as the uncertainty on the gross rates of return decreases. Further, the higher the coefficient of correlation, the likelier is the case for collateral inheritance being more heavily taxed than lineal inheritance. Both theses results are quite intuitive.

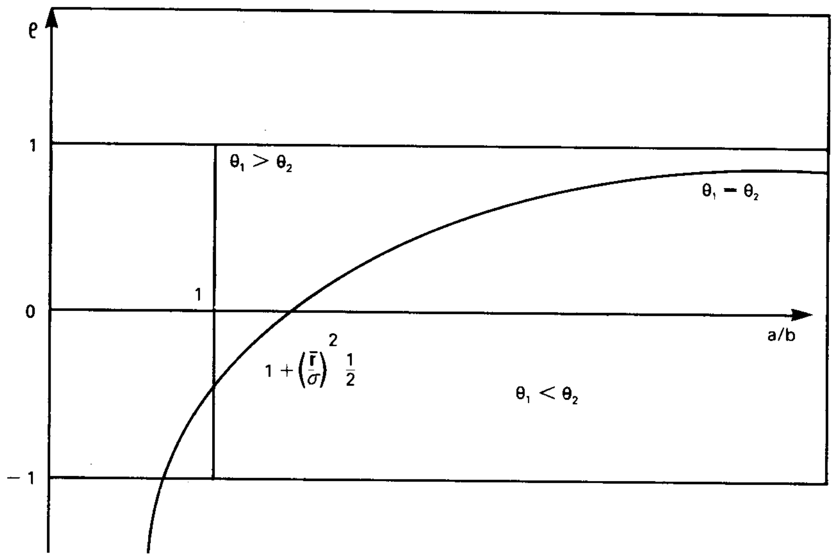

FIGURE 1

Inheritance tax rates for different parameters of the bequest functions and coefficients of correlation between rates of return

5 Uncertain Lifetime

This section reverts to the assumption that the rate of return has a constant value $r = 1$; there is still some randomness in the process of wealth transmission arising from the uncertainty in the time of death. Again, the model is very simple. All individuals face the same lifetime distribution. Yet we allow for the possibility of some correlation between brother's life duration. Life expectancy is assumed to be independent of the wealth level.

Lineal inheritance in period t, can be divided into two parts: the certain bequests, $a(K_t+w)$, and the random bequests $(1-a)\tilde{\varepsilon}(K_t+w)$ where $\tilde{\varepsilon}$ is a random variable reflecting life uncertainty ranging from 0 (no accidental bequests) to $0 < \varepsilon_{max} < 1$ (premature death). The expected value of $\tilde{\varepsilon}$ is $\bar{\varepsilon}$ $(0 \leqq \bar{\varepsilon} < 1)$, its variance is σ_ε and the coefficient of correlation between brothers' variable $\tilde{\varepsilon}$ is ρ_ε. Collateral inheritance is expressed in the same way: $b(K_t+w)$ for the certain part and $(1-b)\tilde{\varepsilon}(K_t+w)$ for the random part. Notice that this distinction between certain and random bequests is close to that between planned and accidental bequests. In section 6, both distinctions are shown to be equivalent under some particular assumptions.

In each generation, L/4 individuals die without heirs (either in direct or collateral line). They cannot avoid leaving some wealth in case of early death. We assume that this type of estate is taxed away and transferred back proportionnally to inherited wealth so that it is distributively neutral.

The structure of the model is close to that of the previous section; that is, we write the average and variance of inherited wealth for generation $t+1$ in terms of generation t's average and variance of inherited wealth:

$$E(K_{t+1}) = A(\theta_1, \theta_2)\, E(K_t+w)$$

where

$$A(\theta_1, \theta_2) = \frac{2(a+(1-a)\bar{\varepsilon})\theta_1 + (b+(1-b)\bar{\varepsilon})\theta_2}{4} \tag{16}$$

and

$$V(K_{t+1}) = [A^2(\theta_1, \theta_2) + B(\theta_1, \theta_2)]\, V(K_t) + B(\theta_1, \theta_2)\, [E(K_t+w)]^2,$$

where

$$B(\theta_1, \theta_2) = \frac{1}{16}\{\theta_2^2(b^2+(1-b)\bar{\varepsilon}) + \sigma_\varepsilon^2(4\theta_1^2(1-a)^2 + 2\theta_2^2(1-b) + 4\theta_1\theta_2\rho_\varepsilon(1-a)(1-b)\}. \tag{17}$$

The coefficients of variation out of and in the steady state are formally identical to expressions (11) and (12). A and B being of course given by (16) and (17). We now turn to the taxation problem. Following our last section approach, we write the revenue constraint of the government in terms of the equality:

$$\bar{A} = \frac{2(a+(1-a)\bar{\varepsilon})\theta_1 + (b+(1-b)\bar{\varepsilon})\theta_2}{4}. \tag{18}$$

Differentiating totally equation (18) with respect to tax rates yields:

$$\frac{d\theta_1}{d\theta_2} = -\frac{b+(1-b)\bar{\varepsilon}}{a+(1-a)\bar{\varepsilon}}. \tag{19}$$

Once again minimizing the coefficient of variation of K_{t+1} is equivalent to minimizing $B(\theta_1, \theta_2)$ with respect to θ_2, using (19). This yields:

$$(20)\quad \frac{dB}{d\theta_2} = 2\theta_2(b+(1-b)\bar{\varepsilon})^2 + \sigma_\varepsilon^2 \Big\{ 4(1-b)^2\theta_2 + 4\theta_1\rho_\varepsilon(1-a)(1-b) - 4\theta_1(1-a)^2\frac{b+(1-b)\bar{\varepsilon}}{a+(1-a)\bar{\varepsilon}} - 2\theta_2\rho_\varepsilon(1-a)(1-b)\frac{b+(1-b)\bar{\varepsilon}}{a+(1-a)\bar{\varepsilon}} \Big\},$$

with $b < a$ being a sufficient condition for the second order condition to be satisfied. To interpret (20) which is quite intricate, we take this expression at the point where $\theta_2 = 0$. Then,

$$(21)\quad \left.\frac{dB}{d\theta_2}\right|_{\theta_2=0} = 4\sigma_\varepsilon^2\theta_1(1-a)\left[\rho_\varepsilon(1-b) - (1-a)\frac{b+(1-b)\bar{\varepsilon}}{a+(1-a)\bar{\varepsilon}}\right] < 0$$

iff

$$\rho_\varepsilon < \frac{(b/(1-b))+\bar{\varepsilon}}{(a/(1-a))+\bar{\varepsilon}} < 1.$$

This says that a 100% taxation of collateral bequests is not desirable if the coefficient of correlation is not too high or if the propensity to bequeath in collateral line is not much lower than that in direct line. To put it another way, if the correlation between brothers' life expectancy were high and the parameter b much lower than a, then clearly collateral bequests would be a source of inequality of inherited wealth and their mere confiscation would be highly desirable.

More generally, from (20), one obtains the optimal ratio θ_1^*/θ_2^*:

$$(22)\quad \frac{\theta_1^*}{\theta_2^*} = \frac{1+\sigma_\varepsilon^2(1-b)[2(1-b)(b+(1-b)\bar{\varepsilon})^{-1} - \rho_\varepsilon(1-a)(a+(1-a)\bar{\varepsilon})^{-1}]}{2\sigma_\varepsilon^2(1-a)[(1-a)(a+(1-a)\bar{\varepsilon})^{-1} - \rho_\varepsilon(1-b)(b+(1-b)\bar{\varepsilon})^{-1}]}.$$

Clearly, the interpretation of equation (22) is not obvious. If condition (21) is met, the denominator of (22) is positive and $\theta_2^* > 0$. But this does not say much on the relative value of the two tax rates. Only, in the polar case of zero correlation, one can assert that $\theta_1^* > \theta_2^*$; in other words, lineal bequests should not be taxed as highly as collateral bequests.

6 Life Cycle Utility Maximization

In the previous sections, the bequest decision has been presented in a quite simple way. We want to show that it can be the result of a life cycle utility maximization under some particular assumptions. The model goes as follows. An individual ω_t, member of generation t, works in period t and retires in period $t+1$. Let $K(\omega_t)$ be the level of bequests received by ω_t. Let w denotes his first period earnings, $c_0(\omega_t)$ and $c_1(\omega_t)$ his first and second period consumption flows, r the gross rate of return and $H(\omega_t)$ the level of bequests he wants to leave to his heirs if any. The budget constraint of ω_t is:

$$c_0(\omega_t) = w + K(\omega_t) - \frac{c_1(\omega_t) + H(\omega_t)}{r}. \tag{23}$$

We adopt a loglinear utility function:

$$u(\omega_t) = \alpha \log c_0(\omega_t) + \beta \log c_1(\omega_t) + \gamma \log H(\omega_t), \tag{24}$$

where $\alpha + \beta + \gamma = 1$ and $\gamma = a$ for individuals with children, $\gamma = b$ for childless individuals with nephews, $\gamma = 0$ for heirless individuals. Maximizing (24) subject to (23) yields the bequest functions (1).

Allowing for a random rate of return, is quite straightforward with a utility function such as (24). One just has to assume that $\tilde{r}$ is a random variable with $\bar{r}$ and σ^2 for expected value and variance which denotes the gross rate of return of an optimally chosen portofolio (see PESTIEAU and POSSEN [1979]).

The case of random duration of life is not as easily manageable. We assume that individuals live one or two periods with a probability $(1-\Pi)$ and Π respectively. One can then think of two different behavioural assumptions that lead to bequest functions as those used in section I. A straightforward, but not very satisfactory way is to assume that individuals are "optimistic" in the sense that they simply ignore the possibility of premature death. In other words, they anticipate with probability one to live until the end of period 2 and hence maximize (24) subject to (23). Observe that in this case the above partition into certain and random bequests is actually equivalent to that into planned and accidental bequests.

An alternative way for obtaining the bequest functions is to assume a more sophisticated behaviour on behalf of the individuals. Using a loglinear utility function, we make a distinction between the treatment of consumption and that of bequests. We suppose that the objective is to maximize the consumption expected utility plus the utility of the expected bequests. In other words, individuals have a sort of risk neutrality towards bequests. Formally, we write,

$$u(\omega_t) = \alpha \log c_0(\omega_t) + \Pi\, \beta \log c_1(\omega_t) + \gamma \log E[H(\omega_t)], \tag{25}$$

where

$$E[H(\omega_t)] = [\omega + K(\omega_t) - c_0(\omega_t)]\, r - \Pi\, c_1(\omega_t). \tag{26}$$

Such a specification is clearly at odds with standard expected utility models. It can only be defended on the grounds that it implies a simple bequest function and that it reflects a differential treatment of risk towards one's own consumption and that of one's heirs.

Maximizing (25) subject to (26), assuming that $\alpha+\Pi\beta+\gamma=1$, yields the following two estate values:

$$H(\omega_t)=r[w+K(\omega_t)](1-\alpha) \quad \text{with probability } 1-\Pi,$$
$$H(\omega_t)=r[w+K(\omega_t)](1-\alpha-\beta) \quad \text{with probability } \Pi.$$

Defining $a\equiv(1-\alpha-\beta)$, $\tilde{\varepsilon}\equiv0$ with probability Π and $\beta/(\alpha+\beta)$ with probability $(1-\Pi)$, one obtains the bequest function used in section 5 for lineal inheritance. For collateral inheritance, (24) must be modified accordingly with parameters α', β', γ' $(\alpha'+\Pi\beta'+\gamma'=1, \gamma>\gamma')$ thus giving $b=1-\alpha'-\beta'$. Notice that this formulation implies that it is, even at a conceptual level, difficult to asses the relative part of planned and accidental bequests. In this case, the above division into certain and random bequests can thus no be seen as a distinction between planned and unplanned bequests.

The production side could also be developped in the same direction thus making both the rate of return r and the wage rate w endogenous and variable out of the steady state [2].

7 Conclusion

Are there any sound arguments for taxing differently lineal and collateral bequests? Or should they both be taxed identically as it is practice with the U.S. estate taxation? This is clearly an important and difficult question. It is important because inheritance taxation is often the center of hot debates by social reformers and revenue seeking governments. It is difficult because it involves conflictual issues of equity and efficiency within a dynamic framework.

In view of this complexity, this paper has dealt with a particular issue. Assuming away efficiency concerns, it has concentrated on the optimal redistributive structure of inheritance taxation. Furthermore, some restrictive assumptions were imposed to make the problem tractable. In that respect, the rather simple demographic structure, as well as the specification of the bequest motive should be mentioned. Within these limits, we have shown that under plausible assumptions, tax rates should be higher on collateral than on lineal bequests. But because of these limits, the results are very much model specific.

Paradoxically, with the sole exception of RICHTER [1987], this question has hardly been discussed in Public Finance. One can wish that this state of affairs be modified in the future. In particular, it might be interesting

2. Making w uncertain would make the model more difficult.

to examine the incidence of differential inheritance taxation under alternative sets of assumptions, particularly towards the functional form of utility and the nature of altruism.

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