

Looking for the News in the Noise. Additional Stochastic Implications of Optimal Consumption Choice

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ABSTRACT. – In neoclassical models of consumption choice under earnings uncertainty changes in consumption programs from one period to the next are determined by new information received about future earnings over the period. This proposition suggests that actual consumption choices imbed extractable information about the extent and time resolution of earnings uncertainty. The primary goal of this paper is to demonstrate how one can infer the extent of earnings uncertainty from information on consumption choices. We obtain a theoretical relationship between the revision in the present expected value of consumption (noise) and the revision in the expectation of lifetime earnings (news) that can be used to measure subjective earnings uncertainty.

A la recherche de l'information nouvelle (news) dans le bruit (noise). Implications stochastiques de choix de consommation optimaux

RÉSUMÉ. – Dans les modèles néoclassiques de consommation où les revenus sont incertains, les révisions du programme de consommation d'une période à l'autre sont déterminées par l'information nouvelle sur les revenus futurs acquise au cours de la période présente. Cette proposition implique que les choix de consommation observés incorporent une information relative au degré d'incertitude des revenus et à sa résolution au cours du temps. L'objet principal de cet article est de montrer comment on peut effectivement connaître le degré d'incertitude du revenu à partir d'informations sur les choix de consommation. On obtient une relation théorique entre la révision de la valeur anticipée de la consommation (noise) et la révision des anticipations des ressources vitales (news). Cette relation peut être utilisée pour évaluer le degré subjectif d'incertitude des revenus.

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1 Introduction

In neoclassical models of consumption under uncertainty optimizing agents utilize only current information about present and future prices and endowments in making current consumption decisions. This proposition has two implications. First, what was learned in the past is relevant to current behavior only in so far as past experience is incorporated in current knowledge of distributions of present and future prices and endowments. Second, given current information, new information about distributions of prices and endowments completely govern changes in the consumption program over time.

This paper, unlike much of the related literature, ¹ focuses on the second implication of the life cycle theory. Limiting ourselves to the special case of earnings uncertainty, we provide conditions under which the revision in the expected discounted value of lifetime earnings (the news) equals the revision in the expected discounted value of consumption (the noise). There are two implications of this result. First, since new information represents the resolution of past subjective uncertainty, the result can be used to measure the extent and the time resolution of earnings uncertainty. Second, since the revision in the expected discounted value of consumption need not equal the revision in the expected discounted value of lifetime earnings, one can measure the extent to which the revisions in consumption programmes are, indeed, determined by revisions in earnings expectations; *i. e.*, one can test the news equals noise proposition. We present one such test, using time series data, which unambiguously rejects the news equals noise proposition.

In the course of deriving the noise equals news results we clarify the functional relationship between the disturbance in the Euler equation

1. Recently, considerable attention has been given to testing for optimal intertemporal consumption choice in stochastic environments. In principle both implications noted above provide a basis for testing the theory. A test of the first implication is that theoretically irrelevant information is in fact irrelevant to current consumption choices. A test of the second implication is that, given current information, new information about prices and endowments fully determines the precise time path of consumption. While the two implications are closely related, one can easily construct examples of non optimizing consumption choice that satisfy tests of one implication but not of the other. Nevertheless, most past tests have focused on the first implication, defined a revision in the consumption programme (more precisely in the programme for the marginal utility of consumption), and then tested whether this revision is correlated with information available before it was made (see HALL [1978], HANSEN and SINGLETON [1983], MANKIW, ROTENBERG and SUMMERS [1982]). One difficulty in evaluating these studies as a group is that they involve repeated use of much the same data for consumption and leisure choices while using a variety of different time series that incorporate past information. In a finite sample revisions in marginal utility will be significantly correlated with a multitude of variables in the "past" information set, and one will surely find many such variables in repeated searches.

governing the evolution of the marginal utility of consumption and the underlying innovation in earnings. We also indicate conditions under which the expectation of future consumption, conditional on current information, depends only on current consumption, and the additional restrictions needed for this conditional expectation to be linear in current consumption.

The next section derives our main theoretical proposition. Section 3 discusses the implications of the proposition for measuring earnings uncertainty and testing the life cycle model. We then illustrate the test of the noise equals news proposition by applying it to aggregate time series data (Section 4). The results indicate that, at least at the level of aggregate data, the assumptions underlying a linear relationship between expected future consumption and current consumption are false, and that unexpected changes in aggregate labor earnings explain only a small portion of the variance in aggregate consumption innovations. Apparently, in aggregate consumption data the noise greatly exceeds the news.

2 Relating Consumption Innovations to New Information on Lifetime Earnings

In the life cycle model with earnings uncertainty considered here current consumption and plans about future consumption depend on preferences, the level of current assets, and probability distributions governing the stream of lifetime labor earnings. Revisions in consumption plans between two different periods are determined by revisions in the probability distribution of lifetime earnings associated with new information gathered between the periods. Using this fact, and assuming that distributions of revisions in the expected present value of lifetime earnings do not depend on past information, the life cycle model implies the existence of two functions, one depending only on c_{t+1} , and the other only on c_t , where c_t denotes period t consumption. The difference between these two functions is exactly equal to the revision in the expected discounted value of lifetime earnings between periods t and $t+1$.

This proposition and some of its corollaries are presented in this section. The first two corollaries consider implications of the proposition for the stochastic process generating consumption. In contrast to other results in the literature, these corollaries deal with consumption per se, which is observable, rather than the marginal utility of consumption, which is not. The third corollary concerns the relationship between the stochastic process generating consumption and that generating earnings. This corollary states that the covariance of realized lifetime earnings and the revision

in the present expected value of consumption (defined in the proposition) equals the variance of the revision in consumption, which in turn, equals the variance in the unobservable revision in the expected discounted value of lifetime earnings. It is this corollary which provides a basis for both an empirical analysis of the extent of household uncertainty about earnings, and for testing the extent to which revisions in consumption programmes are indeed determined by revisions in expected earnings.

Assumption 1 (A1) provides the model of consumption behavior that underlies our results.

ASSUMPTION 1 (A1): The consumer chooses a consumption program to $\max E_t \left\{ \sum_{\tau=0}^{T-t} \beta^\tau U(c_{t+\tau}) \right\}$ subject either to $A_t = \sum_{\tau=0}^{T-t} R^{\tau+1} (c_{t+\tau} - w_{t+\tau})$, with probability one, or

$$A_t = E_t \sum_{\tau=0}^{T-t} R^{\tau+1} (c_{t+\tau} - w_{t+\tau}),$$

where $U(\cdot)$ is a monotonically increasing strictly concave utility function possessing a continuous second derivative, β is a subjective discount factor, T is the known length of economic life, t is the age of the agent, c_t is consumption in period t , $w_{t+\tau}$ is earnings in period $t+\tau$, A_t is nonhuman wealth at age t , $R = \frac{1}{1+r}$, where r is the known real interest rate, and E_t

denotes the expectation operator conditional on the information set at time t (where required we explicitly denote this information set by I_t). Moreover, the consumption programme is assumed to be bounded away from zero with probability one.²

Proposition 1 (P1) underlies the results presented in HALL [1978].

PROPOSITION 1 (P1): (See HALL [1978]). Given A1,

$$U'(c_{t+1}) = \lambda U'(c_t) + \xi_{t+1}$$

where $E_t \xi_{t+1} = 0$ and $\lambda = \beta/R$.

Proposition 1 is the first order condition arising from expected utility maximization. It states that the expected marginal utility of consumption in period $t+1$ conditional on the information set in period t is a function of only consumption in period t ; that is, it does not depend on any other variable in I_t . Note, however, that P1 has implications only for

2. More precisely we require that there exists an $\varepsilon > 0$, such that $C' \geq \varepsilon$ with probability one, where C' denotes the consumption programme formulated in period t . This assumption underlies the proof of the result in HELLMIG [1978] (recorded here as proposition 1). It can be insured by appropriately restricting the utility function and the statistic process generating earnings (*i.e.* the primitives of the model; see for example, HELLMIG [1977]). Finally, note that the propositions and corollaries presented here will be true under either the "probability" or the "expectational" specification of the budget constraint (it is possible for a property of the consumption programme to be true for one specification, but not the other).

the stochastic process generating the marginal utility of consumption. In particular, the expectation of future consumption could depend on any or all variables in the current information set without violating P1. Each of the following two assumptions is sufficient to restrict the elements of the current information set which determine the expectation of future consumption. A1 is an assumption about preferences, while A2 is an assumption about the stochastic earnings process. As indicated in corollary 2, under either assumption expected future consumption depends only on current consumption.

ASSUMPTION 2 (A2): The utility function is quadratic, *i. e.*, $U(c_t) = a_0 + a_1 c_t + a_2 c_t^2$.

ASSUMPTION 3 (A3): Let $\eta_{t+\tau} = (E_{t+\tau} - E_{t+\tau-1}) \sum_{j=\tau}^{T-t} R^{j-\tau} w_{t+j}$, and $F_{t+\tau}(\eta_{t+\tau} | I_{t+\tau-1})$ be the distribution of $\eta_{t+\tau}$ conditional on the information set in period $t+\tau-1$. Then

$$\{dF_{t+\tau}(\eta_{t+\tau} | I_{t+\tau-1}) = dF_{t+\tau}(\eta_{t+\tau})\}_{\tau=0}^{T-t}$$

In A3 $\eta_{t+\tau}$ is the revision in the expected discounted value of lifetime earnings arising from information that accumulates between $t+\tau-1$ and $t+\tau$. Clearly, since revisions in expectations cannot be predicted, $E[\eta_{t+\tau} | I_{t+\tau-1}] = 0$. A3 states that not only the expectation, but also the entire distribution of $\eta_{t+\tau}$ is independent of $I_{t+\tau-1}$.

Proposition 2 is central to the remainder of this paper.

PROPOSITION 2 (P2): If A1 and either A2 or A3 are satisfied, then there exist monotonically increasing continuously differentiable functions,

$$\delta^{(t+1)}(c_{t+1}) \text{ and } \delta^{(t)}(c_t),$$

such that

$$\delta^{(t+1)}(c_{t+1}) = R^{-1} \delta^{(t)}(c_t) + \eta_{t+1},$$

where η_{t+1} is defined in A3 and, hence, $E_t \eta_{t+1} = 0$.

The proof of P2, presented in the appendix, shows that $\delta^{(t+1)}(c_{t+1})$ is equal to the expected discounted sum of consumption expenditures between $t+1$ and the end of the planning horizon conditional on the information set in period $t+1$; $\delta^{(t)}(c_t)/R$ is the expectation of this same variable conditional on the information set in period t .³ The proposition states that these expectations can be written as functions of only c_{t+1} and c_t , respectively. It follows from the budget constraint that the revision in the expectation of the discounted value of consumption expenditures must be equal to the revision in the expectation of the discounted value of lifetime earnings (η_{t+1}).

3. It should be noted that the proof is constructive in that it provides a method of calculating these functions from the utility function and probability distributions of revisions in the expected discounted values of lifetime earnings.

It is worth comparing P1 and P2. Both propositions establish the existence of two functions, one dependent only on c_{t+1} and one only on c_t , such that the difference between them is "unexpected"; that is, both differences have an expectation, conditional on the information set in period t , of zero. In P2, however, this difference is precisely the revision in the expected discounted value of lifetime earnings. P1, in itself, does not provide information on the source of $\xi_{t+1} = U'(c_{t+1}) - \lambda U'(c_t)$, nor does it indicate anything about the properties of ξ_{t+1} except that $E_t \xi_{t+1} = 0$. It should be clear, however, that ξ_{t+1} is determined by η_{t+1} . In fact, given A3, there is a one to one relationship between the realizations of the two random variables (see the appendix).

We first use proposition 2 to clarify two properties of the stochastic process generating consumption, and then discuss how it can be used to investigate the stochastic relationship between consumption and earnings. Corollary 3 is an immediate consequence of P2 and provides sufficient conditions for HALL'S [1978] statement that "no variable apart from current consumption should be of any value in predicting future consumption."

COROLLARY 3: *If A1, and either A2 or A3 are satisfied, then there exists a (monotonically increasing and continuously differentiable) function, $g_t(c_t)$, such that*⁴

$$c_{t+1} = g_t(c_t) + u_{t+1}$$

where

$$E_t u_{t+1} = 0.$$

The assumptions underlying P2 and Corollary 3 are quite general, requiring no explicit specification of the utility function or stochastic process generating earnings. As a consequence, the function $g_t(\cdot)$ could be quite complicated. Corollary 4 notes, however, that if the utility function either displays constant absolute risk aversion, as specified in A4, or is quadratic, then $g_t(\cdot)$ is linear.

ASSUMPTION 4 (A4): The utility function exhibits constant absolute risk aversion, i. e., $U'(c_t) = B e^{-\gamma c_t}$ ($B, \gamma > 0$).

COROLLARY 4 (proved in the Appendix): Provided A1 and either A2, or A3 and A4, are satisfied, then $c_{t+1} = \alpha_{0t} + \alpha_1 c_t + \alpha_{2t} \eta_{t+1}$.

Many of the tests of proposition 1 presented in the literature assume $g_t(c_t)$ is linear in c_t (e. g., HALL [1978], HALL and MISHKIN [1982], and FLAVIN [1981]). Corollary 4 indicates that those results are somewhat more general than noted by HALL [1978], who justified linearity by quadratic utility, since $g_t(c_t)$ will also be linear if A3 and A4 are satisfied. Note that

4. P2 and the implicit function theorem imply the existence of a monotonically increasing continuously differentiable function $Q^{(t+1)}(\cdot)$ such that $c_{t+1} = Q^{(t+1)}\{R^{-1} \delta^{(t)}(c_t) + \eta_{t+1}\}$. The function $g_t(c_t)$ is constructed by integrating $Q^{(t+1)}(\cdot)$ over the probability measure, $dF_{t+1}(\eta_{t+1})$.

in the case of constant absolute risk aversion, $\alpha_1 = 1$. Note also that if the assumptions underlying this corollary are valid, the revision in the expected discounted value of lifetime earnings is simply proportional to the difference between c_{t+1} and a linear function of c_t . Hence, under the assumptions of the corollary, the η_{t+1} revisions defined in P2, can be identified in a straightforward manner, and this identification does not require any additional information on the sequence of distribution functions $\{F_{t+1}(\eta_{t+1})\}$. Our final corollary concerns the relationship between η_{t+1} and the revisions in the expected discounted value of the consumption program in a more general setting. This corollary requires only the assumptions underlying proposition 2.

COROLLARY 5: Let $L_t = \sum_{\tau=0}^{T-t} R^\tau w_{t+\tau}$ and $r_{t+1} = \delta^{(t+1)}(c_{t+1}) - R^{-1} \delta^{(t)}(c_t)$;

and assume A1 and either A2 or A3. Then

$$E_{(j)}[r_{t+1} L_{t+1}] = E_{(j)}[r_{t+1}^2] = E_{(j)}[\eta_{t+1}^2]$$

for $j \leq t$.

L_{t+1} is the realized discounted value of labor earnings between t and the end of the planning horizon. It can be partitioned into the revision, between $t+1$ and the end of the planning horizon, in the expected discounted value of lifetime earnings, $(L_{t+1} - E_{t+1} L_{t+1})$, the period $t+1$ revision in that expectation that occurred because of information accumulated between t and $t+1$, η_{t+1} (recall that $E_{t+1} L_{t+1} - E_t L_{t+1} = \eta_{t+1}$, and the period t expected discounted value of lifetime earnings $(E_t L_{t+1})$; that is

$$L_{t+1} = (L_{t+1} - E_{t+1} L_{t+1}) + \eta_{t+1} + E_t L_{t+1}.$$

Provided the assumptions underlying proposition 2 are correct, the revision in the expected discounted value of the consumption program, *i. e.*, $r_{t+1} = \delta^{(t+1)}(c_{t+1}) - R^{-1} \delta^{(t)}(c_t)$, just equals η_{t+1} . Corollary 5 follows from noting that $L_{t+1} - E_{t+1} L_{t+1}$ cannot be correlated with any variable in I_{t+1} including η_{t+1} , while η_{t+1} cannot be correlated with any variable in I_t including $E_t L_{t+1}$.

EDEN and PAKES [1981] appear to be the first to utilize the fact that the revision in consumption expenditures should contain information on changes through time in the expected discounted value of lifetime earnings. They note that the total variance in the individual's expected discounted value of lifetime earnings at time t is just $\sum_{j=1}^{T-t} R^{2(j-1)} E \eta_{t+j}^2$, and that the sequence $\{E \eta_{t+j}^2\}$ provides a measure of the age profile of the realizations of the variance in lifetime earnings. The article by EDEN and PAKES [1981] assumes a quadratic utility function, and uses only information on consumption expenditures to estimate $E \eta_t^2$. Corollary 5 provides the analogue of the Eden and Pakes result for an arbitrary concave utility function, and indicates that there are, in principal, two unbiased estimates of $E \eta_{t+1}^2$. This latter fact has two implications. First it provides a basis for testing the model,

since, in principal, we can compare the variance in the consumption innovation to the covariance between it and lifetime earnings. Section 3.2 illustrates this point. Second, but perhaps more important, the fact that there are two unbiased $E \Sigma \eta_{t+1}^2$ allows us to relax different assumptions and still extract information on the parameters of interest. In particular it permits us to allow for heterogeneity and errors in measurement when using micro panel data to analyze the uncertainty in earnings streams. Section 3.1 elaborates this point.

3 Some Implications

3.1 Panel Data

In this section we consider in more detail the special case of assumptions A 1 and A 2 (quadratic utility) or A 3 plus A 4 (the case where the one-period utility function exhibits constant absolute risk aversion, and the distributions of the revisions in expected lifetime earnings are independent of past information). Our purpose is to illustrate how the results of the last section enable us to account for heterogeneity and errors in measurement when constructing measures of earnings uncertainty from micro panel data sets. Partly for notational convenience we begin by explicitly incorporating (quite arbitrary) heterogeneity in the preference parameters, β and γ , and in the distributions of the revisions in expected lifetime earnings, $dF_{t+\tau}(\eta_{t+\tau})$; we assume, initially, correctly measured data. Under these assumptions, the results in the last section and those in the appendix imply that the change in consumption and the lifetime earnings of consumer i at age t can be written, respectively, as

$$(1) \quad \Delta C_{t+1, i} = C_{t+1, i} - C_{t, i} = \alpha_{0, i, t} + \alpha_{2, t+1} \eta_{t+1, i}$$

and

$$L_{t+1, i} = (L_{t+1, i} - E[L_{t+1, i} | I_{t+1, i}]) + \eta_{t+1, i} + E[L_{t+1, i} | I_{t+1, i}]$$

where

$$\alpha_{0, i, t} = \gamma_i^{-1} \ln \left\{ \int \exp(-\gamma_i \eta_{t+1, i} / \alpha_{2, t+1}) dF_{t+1, i}(\eta_{t+1, i}) - \ln(\beta_i / R) \right\},$$

$$\text{and } \alpha_{2, t+1}^{-1} = \sum_{\tau=0}^{T-(t+1)} R^\tau.$$

Suppose that we have N observations on the couple $(\Delta C_{t+1, i}, L_{t+1, i})$ in each of J different population groups, and that we were interested in

estimating and comparing the means of

$$E[\eta_{t+1, i}^2 | I_{i, t}] \equiv \int \eta_{t+1, i}^2 dF_{t+1, i}(\eta_{t+1, i}),$$

say comparing the means of $E_{(i)} E[\eta_{t+1, i}^2 | I_{i, t}] = \sigma_{t+1}^2$ in the different groups. (The first expectation is understood to be taken with respect to the measure of consumers in the group of interest; when we wish to distinguish more explicitly between groups, we will use a superscript j). We consider using the group sample covariance of $\Delta C_{t+1, i}$ and $L_{t+1, i}$, say $S_{t+1}^{2(j)}$, as an estimate of $\sigma_{t+1}^{2(j)}$, and we seek conditions under which

$$(2) \quad S_{t+1}^{2(j)} = 1/N \sum_{i \in j} (\Delta C_{t+1, i} - \bar{\Delta C}_{t+1}^{(j)}) (L_{t+1, i} - \bar{L}_{t+1}^{(j)}) \xrightarrow{P} \sigma_{t+1}^{2(j)} \alpha_{2, t+1}$$

where $\bar{\Delta C}_{t+1}^{(j)} = N^{-1} \sum_{i \in j} \Delta C_{t+1, i}$, $\bar{L}_{t+1}^{(j)} = N^{-1} \sum_{i \in j} L_{t+1, i}$, and \xrightarrow{P} reads converges in probability to. Clearly, provided the sample realizations of $(\Delta C_{t+1, i}, L_{t+1, i})$ are mutually independent, and

$$E \Delta C_{t+1, i} (L_{t+1, i} - \bar{L}_{t+1}^{(j)}) = \sigma_{t+1}^{2(j)} \alpha_{2, t+1}$$

for all $i \in j$ (and some finite $\sigma_{t+1}^{2(j)}$), the strong law of large numbers will insure that property (2) is indeed satisfied for every j (see, for example, RAO [1973] chapter 2).

It is worth considering situations in which the first two conditions may be questionable. The independence assumption is likely to be problematic if all observations are drawn in the same year, since then there may be market factors whose realizations affect all the $\eta_{t+1, i}$ in a given group. In this case, what we can (and cannot) learn from the data depends upon the structure of the dependence induced by the market factors. In the case where $\eta_{t+1, i} = \zeta_{t+1, i} + \theta_{t+1}$ for all i , and the $\{\zeta_{t+1, i}\}$ are mutually independent, we could still obtain consistent estimates of the difference between the $\sigma_{t+1}^{2(j)}$ among different groups, though not of the level of any of them. In considering the second condition, recall that the model implies that

$$\alpha_{2, t+1} E[\eta_{t+1, i} L_{t+1, i} | I_{i, t}] = \alpha_{2, t+1} E[\eta_{t+1, i}^2 | I_{i, t}],$$

and this insures that

$$\alpha_{2, t+1} E[\eta_{t+1, i} E[\eta_{t+1, i} L_{t+1, i}]] = \alpha_{2, t+1} E_{(i)} E[\eta_{t+1, i} L_{t+1, i} | I_{i, t}] = \alpha_{2, t+1} \sigma_{t+1}^2.$$

It follows that

$$E \Delta C_{t+1, i} (L_{t+1, i} - \bar{L}_{t+1}^{(j)}) = \alpha_{2, t+1} \sigma_{t+1}^{2(j)} \quad (\text{for } i \in j),$$

provided $E \alpha (L_{t+1, i} - \bar{L}_{t+1}^{(j)}) = 0$, that is, provided the covariance between $\alpha_{0, i, t}$ and lifetime earnings vanishes. Since the model assumes that the individual knows his preference parameters and the distributions of the revisions in lifetime earnings, and since these determine $\alpha_{0, i, t}$, there should be no correlation between $\alpha_{0, i, t}$ and the revisions in the expectation of lifetime income. However, there may be a cross-sectional correlation

between $\alpha_{0, i, t}$ and the initial (period zero) expected discounted value of lifetime earnings, and this could cause some inconsistency.

In order to determine the effect of errors in measurement on our results, we must specify how these errors are generated. Under the classical assumption of additive measurement errors which are uncorrelated with the true deviates and with each other—that is, under the assumption that we observe couples $(\Delta C_{it}^m, L_{it}^m)$ governed by

$$(3) \quad \begin{cases} \Delta C_{i,t}^m = \Delta C_{i,t} + v_{i,t}^C \\ L_{i,t}^m = L_{i,t} + v_{i,t}^L \end{cases}$$

where $E[v_{i,t}^q, L_{i,t}] = E[v_{i,t}^q, C_{i,t}] = E[v_{i,t}^L, v_{i,t}^C] = 0$ (for $q = L, C$), and $(\Delta C_{i,t}, L_{i,t})$ abide by the model in equation (1)—it is clear that the presence of measurement error does not affect the consistency property described in equation (2). In this case, then, both $S_{L, \Delta C}^{(j)}$ and $S_{L^m, \Delta C^m}^{(j)}$ (within group covariance of the observed deviates) are consistent estimators for $\alpha_{t+1} \sigma_{L, \Delta C}^{2(j)}$. On the other hand, $S_{L^m \Delta C^m}$ will be a less precise estimator for $\alpha_{t+1} \sigma_{L, \Delta C}^2$ and presumably one could derive more precise estimates for $\alpha_{t+1} \sigma_{L, \Delta C}^2$ than $S_{L^m \Delta C^m}^{(j)}$.⁵

3.2 A Time Series Example

Having considered applying Corollary 5 to measuring earnings uncertainty, we now turn to an aggregate time series example that uses Corollary 5 to test the new equals noise proposition. Here we ignore issues of aggregation over individuals, let T (the planning horizon) approach infinity, and assume the stochastic process generating earnings is (strictly) stationary and normal.⁶ These assumptions simplify the testing procedure considerably.

The assumption of stationarity allows us to write the earnings process as an infinite autoregression with an independent and identically distributed disturbance. This disturbance is proportional to η_{t+1} , the revision in

5. This is because earnings and consumption levels, as well as other household characteristics, are likely to contain extractable information on the extent of measurement error.

6. Strictly speaking, the assumption of stationarity is not necessary in our framework. Under mild regularity conditions on the boundedness of the variance of the earnings process, the

fact that $R < 1$, implies that, as T grows, the difference, $\sum_{\tau=0}^T R^\tau w_{j+\tau} - \sum_{\tau=0}^{\infty} R^\tau w_{j+\tau}$, converges,

in mean square, to zero. That is, if we formed $L_j^{(T)} = \sum_{\tau=0}^T R^\tau w_{j+\tau}$, then, by choosing T

large enough, we can insure that the difference $r_j L_j^{(T)} - r_j L_j^{(\infty)}$ is smaller than any positive ϵ with probability very close to one. On the other hand the larger is T , the less data is

available to form sample averages $\left(J^{-1} \sum_{j=0}^j r_j L_j^{(T)} \text{ and } J^{-1} \sum_{j=0}^j r_j L_j^{(\infty)} \right)$ and the larger will be the standard error of these estimates of $EJ^{-1} \sum r_j L_j^j$ and $EJ^{-1} \sum r_j^j$.

We actually tried to form these means sample averages empirically for the special case of quadratic utility functions (see Corollary 4), but it became clear that sufficiently large T resulted in the loss of too many degrees of freedom.

expected lifetime earnings between t and $t + 1$ (see ANDERSON [1971] and the definition of η_{t+1} in A 3). That is

$$(4) \quad w_{t+1} = \sum_{\tau=0}^{\infty} \gamma_{\tau} w_{t-\tau} + \varepsilon_{t+1},$$

and

$$\eta_{t+1} = \theta \varepsilon_{t+1},$$

where $\{\varepsilon_{\tau+1}\}$ is a sequence of independently and identically distributed random variables ⁷.

Given stationarity of the earnings process, it is assumed that $\delta^{t+1}(\cdot) \rightarrow \delta^*(\cdot)$ as $T \rightarrow \infty$, where the function $\delta^*(\cdot)$ can be expressed as the n -th order polynomial

$$(5) \quad \delta^*(c_t) = \sum_{i=0}^n m_i c_t^i.$$

Note that if $m_i = 0$, for $i \geq 2$, the assumption of a quadratic or constant absolute risk aversion utility function (Corollary 4), and the corresponding linear predictor function for c_{t+1} used in previous analyses (e. g., FLAVIN [1981], HALL and MISHKIN [1982]), is valid.

To provide a more informative summary of the data than can be obtained from the value of a test statistic, we allow a disturbance term, say v_{t+1} , to affect the revision in the consumption programme [*i. e.*, $r_{t+1} = \delta^{t+1}(c_{t+1}) - R^{-1} \delta^t(c_t)$]. We then ask what percentage of the variance in r_{t+1} (the noise) is accounted for by the disturbance, and what percentage is accounted for by the revision in the expected discounted value of lifetime earnings, *i. e.*, by the news, η_{t+1} .

Letting $\{v_t\}$ be a sequence of independent random variables whose joint distribution is assumed to be independent of the joint distribution of earnings (and whose realizations cannot, therefore, be accounted for by the lifecycle model with earnings uncertainty), we write

$$(6a) \quad r_{t+1} = \eta_{t+1} + v_{t+1}$$

7. If the process generating earnings has a convergent autoregressive representation, then θ can be expressed as a function of the autoregressive coefficients (the γ_t), and one could impose, or test, this constraint. In the empirical work the value of θ varied with the order of the autoregression we assumed, though the estimated variance of the disturbance from the wage equation, and its covariance with the residual in the consumption equation (see below) did not vary significantly. This is another example of the familiar observation that the residuals formed after estimating a stationary process do not vary much with the precise form of the process estimated, though other properties of the estimated process may vary substantially. Since our theoretical results are independent of the precise form of the earnings process, we thought it best to leave θ unconstrained. We also used NIA observations on compensation of employees as the earnings variable. None of the empirical results were materially affected by using employee compensation rather than wages and salaries.

and consider estimates of ρ^2 , where

$$(6b) \quad \rho^2 = \frac{E \eta_{t+1}^2}{E \eta_{t+1}^2 + E v_{t+1}^2}$$

To obtain the system of equations to be estimated, we use the relation $\delta'(c_t) = \delta^t(c_t) - c_t$ (see the appendix). Equation (7a) is derived from this fact, (4), (5), (6) and the definition of r_{t+1} presented in Corollary 5, while equation (7b) comes directly from (4). This produces the system

$$(7a) \quad c_{t+1} = k_0 + R^{-1} \sum_{i=1}^n k_i c_t^i - \sum_{i=2}^n k_i c_{t+1}^i + \theta m_1^{-1} \varepsilon_{t+1} + m_1^{-1} v_{t+1}$$

$$(7b) \quad w_{t+1} = \sum_{\tau=0}^{\infty} \gamma_{\tau} w_{t-\tau} + \varepsilon_{t+1}$$

where $k_0 = m_0(R^{-1} - 1)/m_1$, $k_1 = (m_1 - 1)/m_1$, and $k_i = m_i/m_1$ ($i=2, \dots, n$). Note that, if the model is correct, the coefficient of c_t^i in equation (7a) should be opposite in sign, and a bit larger (in absolute value) than the coefficient of c_{t+1}^i ($i=2, \dots, n$) with the difference determined by R . Thus, for $i \geq 2$ we can obtain an estimate of R , and for $i > 2$ we can test the model's implications by testing if the coefficient of c_t^i equals R^{-1} times the coefficient of c_{t+1}^i .

Since both ε_{t+1} and v_{t+1} are determinants of c_{t+1} they will, in general, be correlated with powers of that variable. Therefore, consistent estimates of the coefficients in equation (7a) require the use of instruments for c_{t+1}^i ($i=2, \dots, n$). Clearly, the assumptions of the model imply that

$$E(c_t^i v_{t+1}) = E(c_t^i \varepsilon_{t+1}) = E(w_{t-\tau} v_{t+1}) = E(w_{t-\tau} \varepsilon_{t+1}) = 0 \quad \text{for } i, \tau \geq 0.$$

In the example in section 3 equation (7a) is estimated by two-stage least squares using current and lagged earnings and powers of current consumption as instruments. Equation (7b) is estimated by ordinary least squares. Let e_{t+1}^c and e_{t+1}^w be the estimated residuals from the consumption and earnings equations, respectively, that is

$$(8) \quad e_{t+1}^c = c_{t+1} - \hat{k}_0 - R^{-1} \sum_{i=1}^n \hat{k}_i c_t^i - \sum_{i=2}^n R^{-1} \hat{k}_i c_{t+1}^i,$$

and

$$e_{t+1}^w = w_{t+1} - \sum_{\tau=0}^{\infty} w_{t-\tau} \hat{\gamma}_{\tau}$$

where a circumflex over a variable indicates its estimated value. Then, letting $S(x, y)$ represent the sample covariance of x and y ,

$$(9) \quad r_{e^c e^w}^2 = \frac{S(e^w, e^c)^2}{S(e^w, e^w)^2 S(e^c, e^c)^2} \xrightarrow{P} \frac{\theta^2 E \varepsilon^2}{\theta^2 E \varepsilon^2 + E v^2} = \rho^2$$

where \xrightarrow{P} reads converges in probability, and the last equality follows from the fact that $\eta = \theta \varepsilon$ [equation (4)] and the definition of ρ^2 [equation

(6b)]. That is, the r^2 from the residuals of the two equation system in (7) provides us with a consistent estimate of ρ^2 , the fraction of the variance in the revision in the expected discounted value of consumption expenditures that is accounted for by the lifecycle model with earnings uncertainty.

4 Results From An Illustrative Test of the Noise Equals News Proposition

The data used to carry out the illustrative time series test described above are National Income Accounts (NIA) quarterly observations of consumption of nondurables and services and quarterly NIA observations of wages and salaries⁸. There are 147 observations corresponding to the first quarter of 1947 through the third quarter of 1983. All observations were expressed in percapita terms and converted to 1972 dollars using a weighted average of the NIA nondurables deflator and the NIA services deflator, with the fixed weight determined by the average share of nondurables consumption in total consumption of nondurables plus services. Since our empirical approach assumes stationarity in earnings, we detrended wages and salaries with the trend path estimated by regressing the logarithm of wages and salaries against a constant and time.

Empirical Results

Table 1 presents the coefficients from estimating equation (7a) assuming first through fourth order polynomial functions for $\delta^*(.)$ [equation (5)]. Estimation of the linear model is by OLS, while the second, third, and fourth order models are estimated by two stage least squares⁹.

The higher order terms in each of the regressions are highly significant suggesting that the linear model posited by FLAVIN [1981] and HALL and MISHKIN [1982] is inappropriate. The appropriateness of a higher order model is also suggested by a test of the linearity of the function $g_t(.)$ of Corollary 3. Specifically, we regressed c_{t+1} on successive higher order polynomials of c_t . In the regression of c_{t+1} on c_t and c_t^2 the coefficient of c_t^2 has a t ratio of -2.30 which is significant at the 5% level. The F

8. We also used NIA observations on compensation of employees as the earnings variable. None of the empirical results were materially affected by using employee compensation rather than wages and salaries.

9. At this stage the analysis is indistinguishable from estimating the first order condition (M) with a polynomial approximation of the utility function.

statistic for the inclusion of third order terms in the approximation for $g_t(\cdot)$ was (marginally) insignificant.

TABLE 1

Regression Results: First Order Through Fourth Order Consumption Models*

Variable	First Order Model	Second Order Model	Third Order Model	Fourth Order Model
Constant	-3.417 (5.973)	-2.914 (8.818)	14.986 (12.825)	-22.224 (10.845)
C_t	1.006 (.002)	1.006 (.007)	.988 (.014)	1.040 (.017)
C_t^2		-.184E-3 (.805E-5)	-.405E-3 (-.158E-4)	-.583E-3 (.115E-4)
C_{t+1}^2183E-3 (.843E-5)	.407E-3 (.180E-4)	.557E-3 (.908E-5)
C_t^3			-.521E-9 (.380E-10)	.140E-8 (.459E-10)
C_{t+1}^3521E-9 (.406E-10)	-.133E-12 (.475E-10)
C_t^4				-.123E-12 (.631E-14)
C_{t+1}^4116E-12 (.677E-14)
Ratio of Standard Error of Regression to Mean Value of Consumption00542	.00116	.00024	.00003

* Two stage least squares estimates of equation (5a). w_{t-1} and c_t^i ($i=0, 1, \dots, 7$ and $i=1, \dots, N$) are used as instruments. There are 129 observations. Numbers in parentheses below coefficient estimates are estimated asymptotic standard errors. E1 is 10.

Recall that if the model is appropriate the coefficient of c_t^i equals minus R^{-1} times the coefficient of c_{t+1}^i for $i \geq 2$ [see equation (7a)]. Looking at the unconstrained parameters estimates in Table 1 it is clear that they are close to satisfying these constraints. However, a formal test of these constraints clearly rejects them; the observed value of the F(2, 131) test statistic is 21.36. This occurs because the fourth order model has a near perfect fit, making even those alternatives that are close to the null hypothesis very powerful. The estimate of R^{-1} , that is of one plus the annual real interest rate, obtained from the constrained 4th order model has the reasonable value of 1.032 with a standard error of .018.

Table 2 provides the estimated fractions of the variance in consumption innovations (noise) explained by earnings information (news) [see equation (6b)]. As indicated by equation 9, this ratio is equal to the squared correlation coefficient between the residuals in the consumption and the earnings equations (7a) and (7b). The earnings equation was estimated using eight lagged values of quarterly earnings. We also conducted the analysis using four rather than eight lags of earnings and obtained results essentially identical to those reported in Table 2.

TABLE 2

Estimated Ratios of News to Noise and Estimated Asymptotic Standard Errors

Model	Ratio of News to Noise	Standard Error of Ratio
First Order181	0.77
Second Order511E-3	.084
Third Order743E-2	.84
Fourth Order020	.050

All of the ratios reported in Table 2 are quite small. In the first order, linear model the innovation in earnings explains less than a fifth of the innovation in consumption. For the higher order models "news" is 2% or less of "noise". Only in the first order model is the estimated ratio of news to noise statistically significantly different from zero.

5 Summary and Conclusion

This paper provides conditions under which the life cycle model with earnings uncertainty implies a simple functional relationship between revisions in the expected discounted value of consumption programmes and revisions in the expected discounted value of lifetime earnings. This relationship can be used to infer the extent and time resolution of uncertainty from panel data on consumption choices. The paper also indicates conditions under which the expected discounted value of future consumption depends only on current consumption. Finally, the paper presents a new test of optimal consumption choice under earnings uncertainty.

Applying the results of this paper to micro panel data should be particularly fruitful since they permit comparisons across demographic and occupational groups of the magnitude and time resolution of earnings uncertainty. Much of the uncertainty in earnings in the cross section is, of course, averaged out in macro data. Indeed, in illustrating our theoretical results on aggregate time series data, we find that new information about earnings has little bearing on aggregate consumption innovations.

APPENDIX

Proof of P2: Let $\{c_{j+\tau}^{(j)}\}_{\tau=0}^{T-j}$ be the optimal consumption program for period j . Since this program must satisfy the budget constraint in year j ,

$$E_j \sum_{\tau=0}^{T-j} R^\tau c_{j+\tau}^{(j)} = A_j R^{-1} + E_j \sum_{\tau=0}^{T-j} R^\tau w_{j+\tau}.$$

Using this condition for period t and $t+1$, and the fact that

$$A_{t+1} = A_t R^{-1} + w_t - c_t,$$

one can show that

$$(10) \quad \delta_{t+1} = R^{-1} \delta_t + \eta_{t+1},$$

where $\delta_{t+1} = E_{t+1} \sum_{\tau=0}^{T-(t+1)} R^\tau c_{t+1+\tau}^{(t+1)}$ and $\delta_t = \delta_t - c_t$. The term δ_{t+1} equals the expected discounted value of current and future consumption conditional on the information set in period $t+1$ and is, therefore, a function of I_{t+1} ; i. e., $\delta_{t+1} = \delta_*^{(t+1)}(I_{t+1})$; and $\delta_t = \delta_*^{(t)}(I_t)$. To prove the proposition, it suffices to prove the following lemma.

LEMMA: If A1 and either A2 or A3 is satisfied then for $t=1, \dots, T-1$, $\delta^{(t)}(I) = \delta^{(t)}(c_t)$; with $\delta^{(t)}(c_t)$ monotonically increasing and continuously differentiable in c_t .

Proof: If the utility function is quadratic (A2), then the lemma follows directly from Proposition 1 and the definition of δ_t , for quadratic utility implies that $E_t[c_{t+\tau}^{(t)}] = (\lambda^\tau - 1)\alpha_0 + \lambda^\tau c_t$ for $\tau \geq 0$; where $\lambda = \beta/R$, and α_0 is determined by the parameters of the utility function. If A2 is not satisfied but A3 is, the lemma is proved by induction. Thus assume $\delta_*^{(j+1)}(I_{j+1}) = \delta^{(j+1)}(c_{j+1})$, the latter function being monotonically increasing and continuously differentiable in c_{j+1} . Then equation (10) and the implicit function theorem imply the existence of a continuously differentiable monotonically increasing function $Q^{j+1}(\cdot)$ such that

$$(11) \quad c_{j+1} = Q^{(j+1)}(\delta_j R^{-1} + \eta_{j+1}).$$

Also from Proposition 1,

$$(12) \quad U'(c_{j+1}) = \lambda U'(c_j) + \xi_{j+1} \quad \text{with} \quad E_j \xi_{j+1} = 0.$$

Substituting (11) into (12) and taking expectations we have

$$(13) \quad H^{(j+1)}(\delta_j, c_j) = \int [U' \{ Q^{(j+1)}(\delta_j R^{-1} + \eta_{j+1}) \}] dF_{j+1}(\eta_{j+1}) - \lambda U'(c_j) = 0,$$

with $H_{\delta_j}^{(j+1)} = R^{-1} \int U'' \{ Q^{(j+1)'}(R^{-1} \delta_j + \eta_{j+1}) \} dF_{j+1}(\eta_{j+1})$, which is negative and continuous in δ_j by virtue of the continuity of $U''(\cdot)$ and $Q^{(j+1)'(\cdot)}$; and $H_{c_j}' = -\lambda U''(c_j)$, which is positive and continuous in c_j . The

implicit function theorem therefore implies the existence of a monotonically increasing continuously differentiable function $\delta^{(j)}(c_j)$ such that $\delta_j = \delta^{(j)}(c_j)$. Since $\delta_j = \delta_j + c_j$, it follows that $\delta_*^{(j)}(I_j) = R[\delta^{(j)}(c_j) + c_j] = \delta^{(j)}(c_j)$, is also monotonically increasing and continuously differentiable in c_j . To complete the inductive argument one need only observe that $\delta_*^{(T)}(I_T) = c_T$, and construct $\delta^{T-1}(c_{T-1})$ from equations (11), (12), and (13) substituting $T-1$ for j . \square

Two points are worthy of note here. First the proof clarifies the roles of assumptions 2 and 3 in the text in deriving proposition 2. If the utility function is quadratic (assumption 2) the both $Q^{j+1}(\cdot)$ and $U(\cdot)$ are linear. In that case equation (13) involves integrating over a linear function of η_{j+1} , so that $H^{j+1}(\cdot)$ depends on the distribution of η_{j+1} , only through $E_j \eta_{j+1}$, which is zero by construction. For quadratic utility then the proposition is true regardless of whether the conditional distribution of η_{j+1} depends on any variables in I_j . If the utility function is not quadratic, then equation (13) involves integrating over a convex function of η_{j+1} . The integral will then depend on higher order moments of η_{j+1} , and though $E_j \eta_{j+1} = 0$, the conditional variance, say, of η_{j+1} may depend on variables in I_j . Thus, without either quadratic utility or assumption 3, $H^{j+1}(\cdot)$ will be a function of more variables in I_j than c_j , and neither proposition 2 nor the statement that $E_j c_{j+1}$ is only a function of c_j are true. The second point is that the proof is constructive in the sense that given any $U(\cdot)$, and any sequence $\{dF_{j+1}(\eta_{j+1})\}$, the proof explains exactly how to construct $\{\delta^{j+1}(c_{j+1})\}$ and $\{\delta^j(c_j)\}$.

Proof of Corollary 4: If A2 is satisfied then Corollary 4 follows directly from the proof of Proposition 2. To prove the corollary when A3 and A4 are satisfied we first use an inductive argument to show that $\{\delta^{(t+1)}(c_{t+1}) = \psi_{0,t+1} + \psi_{1,t+1} c_{t+1}\}_{t=1}^{T-1}$ and then derive the implied relationship between c_{t+1} and c_t . Assuming $\delta^{(t+1)}(c_{t+1}) = \psi_{0,t+1} + \psi_{1,t+1} c_{t+1}$, equation (A2) in the proof of Proposition 2 becomes,

$$c_{t+1} = \frac{1}{\psi_{1,t+1}} (\delta_t R^{-1} + \eta_{t+1} - \psi_{0,t+1}).$$

Substituting this equation into (12) and solving (13) for δ_t yields:

$$\delta_t = R \psi_{1,t+1} c_t + \frac{R \psi_{1,t+1}}{\gamma} \log \left(\frac{k_{t+1}}{\lambda} \right) + \psi_{0,t+1} R,$$

where $k_{t+1} = \int e^{-(\gamma/\psi_{1,t+1} R) \eta_{t+1}} dF_{t+1}(\eta_{t+1})$. Noting that $\delta_t = (\delta_t + c_t) = \delta^{(t)}(c_t)$, and that $\delta^T(c_T) = c_T$, completes the inductive argument. Clearly this argument implies that the sequences $\{\psi_{0,t}\}$ and $\{\psi_{1,t}\}$ are determined by the recursions

$$\psi_{1,t} = \psi_{1,t+1} R + 1, \quad \psi_{0,t} = \frac{R \psi_{1,t+1}}{\gamma} \log \frac{k_{t+1}}{\lambda} + \psi_{0,t+1} R,$$

with initial conditions $\psi_{1,T} = 1$; and $\psi_{0,T} = 0$. This solution and equation (10) in the proof of Proposition 2, imply the corollary. Assuming

that the limit of the solution to the finite horizon problem converges to the solution of the infinite problem and assuming a stationary distribution of η_t , then as $T \rightarrow \infty$, $\psi_{1,t} \rightarrow \frac{1}{1-R}$, and

$$\psi_{0,t} \rightarrow \frac{1}{\gamma} \frac{R}{(1-R)^2} \log \frac{k^*}{\lambda}, \quad \text{where } h^* = \int e^{-\gamma((1-R)/R)\eta} df(\eta). \quad \square$$

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