Learning and Rationality: an Empirical Study of Investment Managers' Stock Market Predictions

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ABSTRACT. – The rationality of investment managers' expectations with respect to changes in the London FTA All Share stock market index is examined using survey data. Respondents to the survey include chief investment managers from over fifty leading investment houses in the City of London. By modelling the actual current and expected future changes as a jointly covariance stationary process, the time series properties of the data are exploited to yield an efficient test of the rational expectations hypothesis as a set of non-linear restrictions on the vector autoregressive representation. Rationality for the full sample period August 1980 to July 1985 is rejected. However, tests carried out for a smaller sample period starting in July 1981, which arguably allows agents enough time to have adapted to the new monetary measures instigated by the first Thatcher government, are unable to reject rationality.

Apprentissage et rationalité : une étude empirique des prévisions des gérants de portefeuille sur l'évolution du marché boursier

RÉSUMÉ. – A partir d'une enquête d'opinion, on étudie la rationalité des anticipations des gérants de portefeuille sur le mouvement de l'indice général FTA des cours de la Bourse de Londres. L'enquête est effectuée, entre autres, auprès des responsables de la gestion de portefeuille des cinquante plus importants organismes de placement de la Cité de Londres. En représentant les évolutions courantes et anticipées par un processus stationnaire du second ordre, on dérive un test efficace de l'hypothèse d'anticipations rationnelles sous la forme d'un ensemble de restrictions non linéaires sur la représentation autorégressive du processus. On rejette la rationalité sur l'ensemble de la période d'observation Août 1980-Juillet 1985. En revanche, si l'on considère la sous-période commençant en Juillet 1981, ce qui donne aux agents le temps de s'adapter aux nouvelles mesures monétaires du premier gouvernement Thatcher, l'hypothèse de rationalité n'est plus rejetée.

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HOWARD: *Dandy, the Vikings had the momentum throughout the first half. Let's see if the momentum stays with the Vikes into the second half. Otherwise, it's going to be a long night for the Bears.*

DANDY DON: *Yes, Howard. It'll also be interesting to see if the Bears continue to be confused by the new formation that Bud Grant has installed for this game. We haven't seen the Vikings throw play-action passes as much as they have this evening, and this has surprised the Bears. If the Bears can figure out the Vikes' new strategy and adjust to it, it will be a new ball game.*

MONDAY NIGHT FOOTBALL [1].

1 Introduction

The rational expectations hypothesis (REH) has had a major impact on macroeconomics during the last decade or so. In its strongest form, the REH maintains that not only do agents not make systematic errors when forecasting the future, but that they also efficiently utilise all available information when doing so (MUTH [1961], SHILLER [1978], BEGG [1982]). [2] While it may be argued that in general such information assumptions are rather extreme (FRIEDMAN [1979]), one area in which the application of the REH is apparently less controversial is in the study of asset markets. In fact, the application of something akin to the REH to asset markets, namely the efficient markets hypothesis (EMH), actually predates the rational expectations revolution of the 1970s (SAMUELSON [1965], MANDELBROT [1966], FAMA [1970]). The EMH maintains that an asset market is "efficient" if prices reflect all available information and no profit opportunities are left unexploited. The macroeconomic importance of such markets derives from the fact that agents transacting in an efficient market will ensure an optimal allocation of resources FAMA [1970], [1976]). Since typical participants in well developed asset markets are highly motivated professionals with access to potentially vast information sets literally at the touch of a button, many economists who would demur at the REH in general, would accept it as a good approximation to behaviour in such markets. This line of thought underlies, for example, the "partly rational" (i. e. rational asset markets, non-rational goods markets) models of e. g. DORNBUSCH [1976], or BLANCHARD [1981].

The purpose of this paper is to test the rationality of investment managers' expectations of the stock market index over a 5 year period in the City of London, one of the world's leading financial centres, using survey data. Since the respondents to the survey in question are typically managers of very large investment portfolios, it seems reasonable to model the evolution of expected changes together with actual changes in the market index, as a jointly determined process which, according to certain statistical theorems, one can then model as a finite-order vector autoregression. The REH can then be efficiently tested as a set of non-linear cross-equation restrictions on the vector autoregressive representation.

The data used, which we discuss in detail in section 3, was available from the end of 1979. However, in June 1979 a Conservative government, headed by Mrs Thatcher, came to power in the UK, committed to broadly monetarist methods of running the economy. Some of the administration's new measures were announced almost immediately, such as the relaxation of exchange controls and of the Special Depository Regulations (the "corset") on the banking system, while the main thrust of the "Medium Term Financial Strategy" was announced in the March 1980 Budget Statement. It might therefore be argued that the early part of our data sample represents a major regime shift during which agents were learning about the new economic environment (see e. g. SARGENT'S [1986 *b*] discussion of British economic policy during this period). Thus, a test of investment managers' rationality using the whole sample period would appear as rather too stringent a test of the REH. On the other hand, testing for rationality over a smaller period which allows for investment managers to have learnt the new "rules of the game" would be a better indicator of the validity of the REH, so that any differences in results between the longer and shorter periods may be indicative of learning.

Using survey data to test rationality has the advantage that the econometrician is not forced to admit into the maintained hypothesis an assumed model of equilibrium behaviour in order to work out what the rational expectation should be. Thus, the excess volatility tests of LE ROY and PORTER [1981] and SHILLER [1981], for example, assume that stock prices are equal to the discounted present value of expected future dividends. Similarly, as noted by HANSEN and HODRICK [1980], the mounting evidence on the failure of the forward foreign exchange rate to act as an optimal predictor of the future spot rate may be symptomatic of the presence of a time-varying risk premium rather than a failure of the REH (see e. g. TAYLOR [1987]). Combining the use of survey data with the unrestrictive vector autoregressive methodology (see e. g. SIMS [1980]) means that the maintained hypothesis in our tests is highly unrestrictive.

Thus the key innovations in this paper are two-fold. The first is the use in this context of the vector autoregressive methodology to test the REH, which we outline in Section 4. The second is the use of survey data on market participants' expectations. Studies, for example, by LAKONISHIK [1980], BROWN and MAITAL [1981], and PEARCE [1984], have tested the rationality of stock price predictions using the Livingston survey data on economists' expectations. Whilst one would expect (or at least hope) that economists' expectations are highly influential, one could always argue that economists are not given quite the same motivation to forecast stock price movements as actual market participants. Also, our analysis to some extent cuts through the caveat that individual irrationality does not imply market irrationality since it only takes a few well-informed individuals to police the rational expectations equilibrium: the respondents to the survey we analyse are the very agents whom one would expect to be policing the equilibrium.

1. Quoted in SARGENT [1986 *a*].
2. Thus, MUTH [1961] writes: "I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory."

The remainder of the paper is set out as follows. The next section briefly discusses the REH in order to provide some motivation for the tests of rationality described later. Section 3 discusses the data and section 4 describes the econometric methodology. In section 5 we present our empirical results and a final section summarises and concludes.

2 Testing the rational expectations hypothesis

Let x be an economic or financial variable which agents wish to forecast, and let ${}_tx_{t+n}$ be their subjective expectation for x at time $t+n$, formed at time t using information up to time t. Then if expectations are rational in the sense of MUTH [1961] we have

$$(1) \qquad {}_tx_{t+n} = \mathrm{E}(x_{t+n} \mid \mathrm{I}_t)$$

where I_t is the information set at time t, and $\mathrm{E}(. \mid \mathrm{I}_t)$ is the mathematical expectations operator conditional on I_t. Of course, the realisation of x_{t+n} will differ from this expectation even under the REH because of "news" arriving between times t and $t+n$. Thus, x_{t+n} can be written as the sum of its expected value plus a rational expectations forecasting error, μ_{t+n} say:

$$(2) \qquad x_{t+n} = {}_tx_{t+n} + \mu_{t+n}.$$

Taking conditional expectations of (2) conditional on I_t and rearranging, we have:

$$(3) \qquad \mathrm{E}(\mu_{t+n} \mid \mathrm{I}_t) = 0.$$

Expression (3) is a fundamental property of rational expectations, and is sometimes referred to as the "orthogonality condition" – forecasting errors should be orthogonal to information available at the time the forecast was made (see e. g. SHILLER [1978]). The tests outlined and applied below are essentially tests of this orthogonality condition.

The data used in this paper, which we describe in the next section, is designed as an aggregate or average measure of expectations across a cross section of investment managers. Given that there is a dispersion of expectations across this cross section, it is quite feasible that one could fail to reject "average rationality" while certain survey respondents have persistently biased expectations. That is, the mean of the expectations distribution may be an optimal predictor, whilst some survey respondents have mean expectations which are more often located in *one* of the tails of this distribution (and which are therefore biased).

3 The data

On the first working day of each month since November 1979, a firm of British management consultants, Godwins, has asked the chief investment manager from each of just over fifty leading investment houses in the City of London, to predict the direction of change of the London Financial Times Actuaries All Share (FTA) index for 1 year ahead. Since the survey results are published in three-category response form (percentage of respondents expecting "up", "down" or "same"), we used the well-known subjective probability method of CARLSON and PARKIN [1975] and KNOBL [1974] to derive a series corresponding to aggregate point expectations. This involves assuming that at each observation point, each respondent has a subjective probability distribution concerning the outcome of the variable in question; and secondly that the means of the individual probability distributions are themselves normally distributed [3] – this is sometimes termed the "expectations distribution". It is the series of means of the expectations distribution that is taken as the aggregate expectations series. It is usually estimated as:

$$_t x_{t+n} = \rho \frac{F^{-1}(EF_t) + F^{-1}(1-ER_t)}{F^{-1}(EF_t) - F^{-1}(1-ER_t)}$$

where EF_t and ER_t are the proportions of respondents at time t expecting the variable in question to fall or rise over the given period respectively; F(.) is the standard normal quantile [so that e. g. $F^{-1}(EF_t)$ is the abscissa of the quantile corresponding to the proportion EF_t]; and ρ is a scaling factor, normally chosen so that the mean expected change over the whole sample period is equal to the mean actual change. Further discussion of the formula can be found for example in HOLDEN, PEEL and THOMPSON [1985], chapt. 1, and BATCHELOR [1986]. The scaling factor ρ is in fact the "just noticeable difference" – i. e. agents will report "no expected change" if in fact they expect the variable in question to alter by $\pm\rho$ per cent over the relevant period. Our choice of ρ follows CARLSON and PARKIN [1975] and has been attacked, for example by FOSTER and GREGORY [1977], for biasing rationality tests towards non-rejection. KNOBL [1974] sets $\rho = 2$ whilst BATCHELOR [1982] chooses ρ to minimise the sum of squared expectations errors. However, there is no reason to suppose that there is a superior method for choosing ρ. In the present context, long-term unbiasedness would seem to be a fairly minimal requirement, so that choosing ρ in the fashion indicated will tend to strengthen the implications of a rejection of the REH.

3. The idea that the distribution of means is normal can be given some motivation by appeal to the Central Limit Theorem, and there appears to be some evidence that expert forecasts are distributed in this way (CARLSON [1975]). At a practical level, assuming different expectations distributions may make little difference – as evidenced by AGENOR [1982] who compares assumptions of normal, log-normal and Cauchy distributions.

Since conversion of our raw response data into quantitative expectations series in the above fashion requires realisations of the actual stock market index 1 year later, we were able to construct a 1-year-ahead expectations series for the proportionate annual change in the FT index for the period November 1979 through July 1985 (69 data points).

4 Econometric methods

If the actual yearly proportionate change in the FTA index at month t, f_t, and the 1-year-ahead expected change, ${}_tf_{t+12}$, together form a linearly indeterministic, jointly covariance stationary process, then the multivariate form of a statistical theorem, known as Wold's decomposition (HANNAN [1970]) implies that the process has a unique, infinite-order moving average representation. For a suitably chosen value of n, this can be approximated in finite samples by an n-th order bivariate vector autoregression. If f_t and ${}_tf_{t+12}$ are in mean deviation form, this can be written:

$$(4) \qquad \begin{pmatrix} 1-\alpha(L) & -\beta(L) \\ -\gamma(L) & 1-\delta(L) \end{pmatrix} \times \begin{pmatrix} f_t \\ {}_tf_{t+12} \end{pmatrix} = \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix}$$

where $\alpha(L)$, $\beta(L)$, $\gamma(L)$, and $\delta(L)$ are n-th order scalar polynomials in the lag operator, L:

$$\theta(L) = \sum_{i=1}^{n} \theta_i L^i, \qquad \text{for} \quad \theta = \alpha, \beta, \gamma, \delta$$

and $\omega_t = (\varepsilon_t \, \eta_t)'$ is a vector white noise process:

$$E(\omega_t \omega'_{t-i}) = \begin{cases} \Omega, & i=0 \\ 0, & i \neq 0 \end{cases}$$

The requirement that the variables in question together form a jointly covariance stationary process is required in order to infer the existence of a moving average (and hence an autoregressive) representation from Wold's decomposition. In terms of the autoregressive parameters, stationarity requires at least the necessary condition that all the roots of the determinental equation

$$\begin{vmatrix} 1-\alpha(z) & -\beta(z) \\ -\gamma(z) & 1-\delta(z) \end{vmatrix} = 0$$

lie outside the unit circle (where z is a real variable).

If we further assume that ω_t has a bivariate normal distribution, then asymptotically efficient estimates of the parameters can be obtained by individual application of ordinary least squares. If λ denote the stacked coefficient vector:

$$\lambda=(\alpha_1 \ldots \alpha_n \beta_1 \ldots \beta_n \gamma_1 \ldots \gamma_n \delta_1 \ldots \delta_n)'$$

and $\hat{\lambda}$ denote the corresponding ordinary least squares estimator based on T observations, then the asymptotic distribution of $\hat{\lambda}$ is given by:

$$T^{1/2}(\hat{\lambda}-\lambda) \tilde{\alpha} N(0, \Omega \otimes M^{-1})$$

where

$$M = \underset{T \to \infty}{\text{plim}} \frac{(X'X)}{T}$$

and X is the matrix of observations on the regressors, with t-th row

$$(f_{t-1}, \ldots, f_{t-n}, {}_{t-1}f_{t+11}, \ldots, {}_{t-n}f_{t+12-n}).$$

The system (4) can be expressed in first-order form as

$$\begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-n+1} \\ {}_tf_{t+12} \\ {}_{t-1}f_{t+11} \\ \vdots \\ {}_{t-n+1}f_{t+12-n+1} \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_n & \beta_1 & \beta_2 & \ldots & \beta_n \\ 1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ . & . & \ldots & . & . & . & \ldots & . \\ \gamma_1 & \gamma_2 & \ldots & \gamma_n & \delta_1 & \delta_2 & \ldots & \delta_n \\ 0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 \\ . & . & \ldots & . & . & 1 & \ldots & . \\ 0 & 0 & \ldots & . & . & . & 1 & 0 \end{bmatrix} \times \begin{bmatrix} f_{t-1} \\ f_{t-2} \\ \vdots \\ f_{t-n} \\ {}_{t-1}f_{t+11} \\ {}_{t-2}f_{t+10} \\ \vdots \\ {}_{t-n}f_{t+12-n} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \\ \eta_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or, more compactly, in an obvious notation:

(5) $$Z_t = \Phi Z_{t-1} + v_t.$$

We can now use this framework to derive a test for the rationality of investment managers' expectations. If investment managers' expectations are rational then

$$f_{t+12} = {}_tf_{t+12} + \mu_{t+12}$$

where μ_{t+12} is the rational expectations forecasting error, so that

(6) $$E(\mu_{t+12} \mid I_t) = 0$$

But, using the first-order formulation (5):

(7) $$ {}_{t}f_{t+12} = g'\Phi Z_{t-1} + \eta_t $$

where g' is a $(1 \times 2n)$ selection vector with unity in the $(n+1)$th element and zeros elsewhere. It is also easy to show, by recursive substitution –

$$ Z_{t+12} = \Phi^{13} Z_{t-1} + \sum_{i=n}^{12} \Phi^i v_{t+12-i} $$

so that

(8) $$ f_{t+12} = e'\Phi^{13} Z_{t-1} + \zeta_t $$

where e' is a $(1 \times 2n)$ selection vector with unity in the first element and zeros elsewhere, and

$$ \zeta_t = e' \sum_{i=0}^{12} \Phi^i v_{t+12-i} $$

Now define the restricted information set Λ_t:

$$ \Lambda_t = \{f_{t-1}, f_{t-2}, \ldots, {}_{t}f_{t+12}, {}_{t-1}f_{t+11}, \ldots\} $$
$$ \Lambda_t \subset I_t $$

– i. e. Λ_t contains only information on lagged expected and actual changes in the index.

Subtracting (7) from (8) and taking conditional expectations with respect to Λ_t:

(9) $$ E(f_{t+12} \mid \Lambda_t) - E({}_{t}f_{t+12}) \mid \Lambda_t) = (e'\Phi^{13} - g'\Phi) Z_{t-1}. $$

But, under the rational expectations hypothesis (9) should be identically equal to zero (take expectations of (6) conditional on I_t and apply the law of iterated expectations). Hence, the non-linear rational expectations restrictions are:

(10) $$ e'\Phi^{13} - g'\Phi = 0. $$

A convenient way of testing these restrictions is to estimate the unrestricted vector autoregression (5) by ordinary least squares, and then use the resulting estimates to calculate a Wald test statistic. If we write the parameter constraints as a $(2n \times 1)$ vector, $r(\lambda)$:

$$ r(\lambda)' = e'\Phi^{13} - g'\Phi $$

then the Wald test statistic is:

(11) $$ W = r(\hat{\lambda})' [\widehat{D(\lambda)}' \{\hat{\Omega} \otimes (X'X)^{-1}\} \widehat{D(\lambda)}]^{-1} r(\hat{\lambda}) $$

where a circumflex denotes that the quantity has been evaluated at the unrestricted ordinary least squares estimate, and where $D(\lambda)$ is the matrix of first derivatives of $r(\lambda)$. Using a matrix differentiation result due to

SCHMIDT [1974], it can be shown:

$$\text{(12)} \qquad D(\lambda) = \begin{bmatrix} \sum_{i=0}^{12} (e'\Phi'^{i} e)\Phi^{12-i} \\ \dots\dots\dots\dots\dots\dots \\ \sum_{i=0}^{12} (g'\Phi'^{i} e)\Phi^{12-i} - I \end{bmatrix}$$

where I is an identity matrix of order $2n$. Note that $e' \Phi'^{i} e$ and $g'\Phi'^{i} e$ select, respectively, the (1,1)th and $(n+1, 1)$th elements of Φ'^{i}, so that $D(\lambda)$ is in practice relatively straighforward to compute.

An alternative approach to testing for orthogonality of the forecasting errors to the information set at $t-1$ is to regress them onto known elements of the information set and test for a zero coefficient vector. Thus, in the regression

$$\text{(13)} \qquad f_{t+12} - {}_{t}f_{t+12} = \kappa' x_{t-1} + \mu_{t+12}$$

where x_{t-1} is a vector of variables such that $x_{t-1} \subseteqq I_{t-1}$, then the rational expectations restrictions are simply $\kappa = 0$. Problems with this approach include determining appropriate elements of x_{t-1}, and also that the rational expectations forecasting error, μ_{t+12}, will have a moving average representation which cannot be dealt with by standard generalised least squares estimators (FLOOD and GARBER [1980], HANSEN and HODRICK [1980]). Moreover, comparing (13) with (9), it can be seen that the methodology outlined above is tantamount to testing $\kappa = 0$ in (10) when $x_{t-1} = Z_{t-1}$. Since Z_{t-1} contains only lagged values of actual and expected changes in the stock market index, orthogonality with respect to Z_{t-1} would seem to be a fairly basic requirement for the rationality of investment managers' expectations. Further, since the above methodology exploits the time series properties of the data, it provides a more efficient test of the rational expectations restrictions.

Applying a result concerning the linearised minimum chi-square estimator (ROTHENBERG [1973]), it is also straightforward to calculate the restricted parameter estimates, $\tilde{\lambda}$, say, under the restrictions (10), as a function of the restricted parameter estimates, $\hat{\lambda}$:

$$\text{(14)} \qquad \tilde{\lambda} = \hat{\lambda} - \hat{V}D(\hat{\lambda})[D(\hat{\lambda})'\hat{V}D(\hat{\lambda})]^{-1} r(\hat{\lambda}).$$

Where $\hat{V} = \hat{\Omega} \otimes (X'X)^{-1}$.

This estimator has an asymptotic covariance matrix $\tilde{V}$ given by:

$$\text{(15)} \qquad \tilde{V} = \hat{V} - \hat{V}D(\hat{\lambda})[D(\hat{\lambda})'\hat{V}D(\hat{\lambda})]^{-1} D(\hat{\lambda})'\hat{V}$$

which is equivalent to the asymptotic covariance matrix of the constrained maximum likelihood estimator. Expressions (14) and (15) are clearly the nonlinear analogues of expressions for the linearly restricted multivariate least squares estimator (see e.g. JUDGE *et al.* [1982], p. 326). Application of this result allows the restricted estimates to be obtained without the need to employ computationally burdensome non-linear optimisation routines (see e.g. SARGENT [1979], HAKKIO [1981]). As noted by SIMS [1980], however, the estimated coefficients in vector autoregressions, being projection coefficients, do not have a direct economic interpretation. Our chief interest in the restricted estimates therefore lies in being able to construct likelihood ratio and Lagrange multiplier statistics to supplement the Wald test statistic for the rational expectations restrictions. Since these tests are asymptotic, it would seem desirable to produce a cross-check in finite samples. The likelihood ratio statistic is:

$$LR = T(\ln|\tilde{\Omega}| - \ln|\hat{\Omega}|) \tag{16}$$

where an upper tilde denotes that the quantity has been evaluated at the restricted vector, $\tilde{\lambda}$. For the case in hand, the Lagrange multiplier statistic is

$$LM = \tilde{u}'[\tilde{\Omega}^{-1} \otimes X(X'X)^{-1}X']\tilde{u} \tag{17}$$

where $\tilde{u}$ denotes the $(2T \times 1)$ stacked vector of residuals from (4), $(\varepsilon_1, \ldots, \varepsilon_T, \eta_1, \ldots, \eta_T)'$, evaluated at $\tilde{\lambda}$.

The likelihood ratio and Lagrange multiplier statistics have the same asymptotic distribution as the Wald statistic under the null hypothesis, i.e. central chi-square with $2n$ degrees of freedom.[4, 5]

Note that none of these tests require homogeneity of disturbance variances across the equations of the vector autoregression. One might, for example, expect the variance of ε_t in (4) to be larger than that of η_t, since the former contains both systematic and random components (MULLINEAUX [1978]). This in no way affects the test procedures applied in this paper since no restrictions are placed on Ω.

5 Empirical results

After allowing for lags, the largest data set available was from August 1980 through July 1985. However, as noted in the introductory section, it would seem desirable to run the tests not only over the whole period but also over a shorter period which allows agents enough time to have adapted to the new regime. We therefore also considered the sample period from July 1981 through July 1985.

The initial empirical task is to determine the order of the vector autoregression, i.e. n. This is a rather delicate operation since underparameterisation will tend to bias the tests, whilst undue overparameterisation will tend to diminish the test power. Maximisation of standard goodness of fit measures – such as the coefficient of determination (R^2) will naturally tend to lead to over-parameterisation. For this reason, it might be better to make an allowance for parsimony (i.e. legitimate contraction of the parameter space) as well as goodness of fit by choosing n to minimise the Akaike Information Criterion (AIC) (AKAIKE [1973]):

$$\mathrm{AIC} = -2\ln \mathrm{L}(\hat{\lambda}) + 2p$$

where $\mathrm{L}(\hat{\lambda})$ denotes the likelihood function evaluated at $\hat{\lambda}$ and p is the number of estimated parameters in the system (equal to $4n$ in this case). Clearly, one should also test for misspecification by examining the whiteness of residuals using standard diagnostics such as the Ljung-Box (i. e. modified Box-Pierce) portmanteau statistic.[6] Broadly speaking, our strategy in choosing n was to seek the smallest parameterisation which yielded serially uncorrelated residuals and which could not be rejected, by a likelihood ratio test, against the next highest parameterisation (i. e. $n+1$). This in fact turned out to be more or less consistent with minimisation of the AIC.

Our empirical results are reported in Table 1. For both of the data periods n was chosen at 8. For the full data period, the AIC could have been marginally reduced by reducing n by one, but this led to unsatisfactory values of the portmanteau statistics.

The interesting feature of Table 1 is however the reported values of the Wald, likelihood ratio and Lagrange multiplier test statistics for the rational expectations restrictions. For the full sample period, the restrictions are rejected by all three tests at nominal significance levels of less than 5%. For the smaller sample period, however, only the Wald statistic is large enough to reject the restrictions at the 5% level, and then only marginally so. Thus, rationality for the shorter period cannot be decisively rejected, suggesting a "learning period" following the change of administration in 1979 up to about mid 1981. Note however, that this inference is at most only suggested by our empirical results, since we have only tested for orthogonality of agents' expectations with respect to a restricted information set (Λ_t in section 4). It is quite possible that rationality would also be rejected for the shorter period if we were to test for othogonality with respect to a larger information set.

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4. Although the restricted estimator (14) is not the restricted maximum likelihood estimator (RMLE), since its asymptotic properties are identical to those of the RMLE, the LR and LM statistics, (16) and (17), are valid test statistics. A discussion of the Wald, likelihood ratio and Lagrange multiplier tests can be found in ENGLE [1984].

5. The vector autoregressive methodology outlined above seems to have been originally suggested by SARGENT [1979]. It was subsequently used to test the hypothesis that the forward exchange rate is an optimal predictor of the future spot rate by HAKKIO [1981], BAILLIE, LIPPENS and MCMAHON [1983], and LEVY and NOBAY [1986]. BAILLIE et al. and LEVY and NOBAY estimate the unrestricted model and report Wald statistics, whilst Hakkio estimates the restricted model using a nonlinear Gauss-Seidel optimisation routine and reports likelihood ratio statistics. See also BAILLIE and MCMAHON [1987].

6. The Ljung-Box statistic is

$$Q = T(T+2)\sum_{t-1}^{P} (T-t)^{-1} r(t)^2$$

where T is the number of observations and $r(t)$ denotes the autocorrelation statistic at lag t. Under the null of white noise residuals, Q will have an asymptotically central chi-square distribution with $(P-n)$ degrees of freedom (LJUNG and BOX [1978]).

TABLE 1

Wald, Likelihood Ratio and Lagrange Multiplier Tests for The Rational Expectations Restrictions [a]

Period	n	n'	R_1^2	R_2^2	Q_1	Q_2	$L(n-1)$	$L(n+1)$	W	LR	LM
1980: 8					18.08	11.33	2.75	3.27	39.54	32.94	27.87
1985: 7	8	7	0.71	0.56	(0.15)	(0.58)	(0.60)	(0.51)	(0.00)	(0.01)	(0.03)
1981: 7					14.73	20.30	7.99	1.83	27.01	23.35	20.38
1985: 7	8	8	0.77	0.64	(0.32)	(0.09)	(0.09)	(0.77)	(0.04)	(0.10)	(0.20)

[a] R_1^2 and R_2^2 denote the coefficients of determination for the n-th order actual and expected change regressions respectively; Q_1 and Q_2 are the corresponding Ljung-Box statistics, evaluated at 21 autocorrelation, and are asymptotically central chi-square variates under the null of white noise residuals, with $(21-n)$ (i.e. 13) degrees of freedom; $L(n-1)$ in a likelihood ratio statistic for a vector autoregression of order $(n-1)$ (VAR$(n-1)$) against the alternative VAR(n), whilst $L(n+1)$ tests VAR(n) against VAR$(n+1)$: each is an asymptotically central chi-square variate with four degrees of freedom, and was constructed with a finite sample correction for degrees of freedom as suggested in SIMS [1980]; the Wald, likelihood ratio and Lagrange multiplier statistics for the rational expectations restrictions are each asymptotically central chi-square under the null with $2n$ (i.e. sixteen) degrees of freedom; figures in parentheses denote marginal significance levels in all cases; n' denotes the value of n that minimised the AIC.

6 Conclusion

The results reported in the previous section suggest that investment managers' expectations of changes in the London stock market index were rational over the period July 1981 through July 1985, although the same cannot be said for the full sample period August 1980 through July 1985. The differing results for these two periods is therefore consistent with a learning period during which agents adjusted to the new Thatcher regime. One might perhaps object that 2 years would seem to be a long time for agents to learn the new rules of the game; but given the very dramatic nature of the regime shift, a 2 year learning period is perhaps not incredible. As argued for example by SARGENT [1986 *b*], the relaxation of exchange controls and the removal of the corset alone almost certainly shifted the demand configuration for a broad range of assets in historically uncertain and largely unpredictable ways.

As pointed out by LUCAS and SARGENT [1979], the omission of specific allowance for learning in many rational expectations models, as noted for example by FRIEDMAN [1979], is not in itself sufficient to overturn the REH. Moreover, models embodying learning have been advanced by e. g. FELDMAN [1982] and BRAY and SAVIN [1986]. Although it is hard to model empirically parameter-adaptive learning within the fixed coefficient, vector autoregressive framework used in this paper, modelling learning behaviour using the Kalman filter would seem to be an obvious avenue for future research (see e. g. CUTHBERTSON and TAYLOR [1986].

• References

AGENOR, P. R. (1982). – "Le Traitement des Anticipations en Économétrie", unpublished doctoral Thesis, *Université de Paris-I, Pantheon-Sorbonne.*

AKAIKE, H. (1973). – "Information Theory and an Extension of the Maximum Likelihood Principle", *in* PETROV B. N., and CSAKI, F. (Eds.), *Second International Symposium on Information Theory*, Akademiai Kiado, Budapest.

BAILLIE, R. T., LIPPENS, R. E. and MC MAHON, P. C. (1983). – "Testing Rational Expectations and Efficiency in the Foreign Exchange Market", *Econometrica*, 51, pp. 553-563.

BAILLIE, R. T. and MC MAHON, P. C. (1987). – "Rational Forecasts in Models of the Term Structure of Interest Rates" *in* GOODHART, C. A. E., CURRIE, D. and LLEWELLYN, D. T. (Eds.), *The Operation and Regulation of Financial Markets*, MacMillan, London.

BATCHELOR, R.A. (1982). – "Expectations, Inflation and Growth: The European Experience", *European Economic Review*, 17, pp. 1-25.

BEGG, D. K. H. (1982). – *The Rational Expectations Revolution in Macroeconomics*, Philip Allan, Oxford.

BLANCHARD, O. J. (1981). – "Output, the Stock Market, and Interest Rates", *American Economic Review*, 71, pp. 132-143.

Bray, M. M. and Savin, N. E. (1986). – "Rational Expectations Equilibria, Learning, and Model Specification", *Econometrica*, 54, pp. 1129-1160.

Brown, B. W. and Maital, S. (1981). – "What do Economists Know? An Empirical Study of Experts' Expectations", *Econometrica*, 49, pp. 491-504.

Carlson, J. A. (1975). – "Are Price Expectations Normally Distributed?", *Journal of the American Statistical Association*, 70, pp. 749-754.

Carlson, J. A. and Parkin, M. J. (1975). – "Inflation Expectations", *Economica*, 42, pp. 123-138.

Cuthbertson, K. and Taylor, M. P. (1986). – "Monetary Anticipations and the Demand for Money: Results for the UK using the Kalman Filter", *National Institute of Economic and Social Research*, Discussion Paper 90.

Dornbusch, R. (1976). – "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, 84, pp. 1161-1176.

Engle, R. F. (1984). – "Wald, Likelihood Ratio, and Lagrange Multiplier Tests in Econometrics", *in* Griliches, Z. and Intrilligator, M. D. (Eds.), *Handbook of Econometrics*, 2, pp. 775-826, North-Holland, Amsterdam.

Fama, E. F. (1970). – "Efficient Capital Markets: A Review of Theory and Empirical Work", *Journal of Finance*, 25, pp. 383-417.

Fama, E. F. (1976). – "Forward Rates as Predictors of Future Spot Rates", *Journal of Financial Economics*, 3, pp. 361-377.

Feldman, M. (1982). – "Learning and Convergence to Rational Expectations", *mimeo*, University of California, Santa Barbara.

Flood, R. P. and Garber, P. M. (1980). – "A Pitfall in the Estimation of Models with Rational Expectations", *Journal of Monetary Economics*, 6, pp. 433-435.

Foster, J. and Gregory, M. (1977). – "Inflation Expectations: The Use of Qualitative Survey Data", *Applied Economics*, 9, pp. 319-329.

Friedman, B. M. (1979). – "Optimal Expectations and the Extreme Information Assumptions of Rational Expectations Macromodels", *Journal of Monetary Economics*, 5, pp. 23-41.

Hakkio, C. S. (1981). – "Expectations and the Forward Exchange Rate", *International Economic Review*, 22, pp. 663-678.

Hannan, E. J. (1970). – *Multiple Time Series*, John Wiley, New York.

Hansen, L. P. and Hodrick, R. J. (1980). – "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: an Econometric Analysis", *Journal of Political Economy*, 88, pp. 829-853.

Holden, K., Peel, D. A. and Thompson, J. L. (1985). – *Expectations: Theory and Evidence*, MacMillan, London.

Judge, G. J., Carter Hill, R., Griffiths, W. E., Lutkepohl, H. and Lee, T. C. (1982). – *Introduction to the Theory and Practice of Econometrics*, John Wiley, New York.

Knöbl, A. (1974). – "Price Expectations and Actual Price Behaviour in Germany", *International Monetary Fund Staff Papers*, 21, pp. 83-100.

Lakonishik, J. (1980). – "Stock Market Return Expectations: Some General Properties", *Journal of Finance*, 35, pp. 921-931.

Le Roy, S. F. and Porter, R. D. (1981). – "The Present Value Relation: Tests Based on Implied Variance Bounds", *Econometrica*, 49, pp. 555-574.

Levy, E. and Nobay, A. R. (1986). – "The Speculative Efficiency Hypothesis: a Bivariate Analysis", *Economica Journal*, 96 (Conference Supplement), pp. 109-121.

Ljung, G. M. and Box, G. E. P. (1978). – "On a Measure of Lack of Fit in Time Series Models", *Biometrika*, 66, pp. 67-72.

LUCAS, R. E. Jr. and SARGENT, T. J. (1979). – "After Keynesian Macroeconomics", *Federal Reserve Bank of Minneapolis Quarterly Review*, 3.

MANDELBROT, B. (1966). – "Forecasts of Future Prices, Unbiased Markets and Martingale Models", *Journal of Business*, 39, pp. 242-255.

MULLINEAUX, D. J. (1978). – "On Testing for Rationality: Another Look at the Livingston Price Expectations Data", *Journal of Political Economy*, 86, pp. 329-336.

MUTH, J. F. (1961). – "Rational Expectations and the Theory of Price Movements", *Econometrica*, 29, pp. 315-335.

PEARCE, D. K. (1984). – "An Empirical Analysis of Expected Stock Price Movements", *Journal of Money Credit and Banking*, 16, pp. 317-327.

ROTHENBERG, T. J. (1983). – *Efficient Estimation with A Priori Information*, Yale University Press.

SAMUELSON, P. A. (1965). – "Proof that Properly Anticipated Prices Fluctuate Randomly", *Industrial Management Review*, 6, pp. 41-49.

SARGENT, T. J. (1979). – "A Note on Maximum Likelihood Estimation of the Rational Expectations Model of the Term Structure", *Journal of Monetary Economics*, 5, pp. 133-143.

SARGENT, T. J. (1986*a*). – "Reagonomics and Credibility", *in* SARGENT, T. J., *Rational Expectations and Inflation*, Harper and Row, New York.

SARGENT, T. J. (1986*b*). – "Stopping Moderate Inflations: the Methods of Poincaré and Thatcher", *in* SARGENT, T. J., *Rational Expectations and Inflation*, Harper and Row, New York.

SCHMIDT, P. (1974). – "The Asymptotic Distribution of Forecasts in the Dynamic Simulation of an Econometric Model", *Econometrica*, 42, pp. 303-309.

SHILLER, R. J. (1978). – "Rational Expectations and the Dynamic Structure of Macroeconomic Models", *Journal of Monetary Economics*, 4, pp. 1-44.

SHILLER, R. J. (1981). – "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?". *American Economic Review*, 71, pp. 421-436.

SIMS, C. A. (1980). – "Macroeconomics and Reality", *Econometrica*, 48, pp. 1-47.

TAYLOR, M. P. (1988). – "A DYMIMIC Model of Forward Foreign Exchange Risk, with Estimates for Three Major Exchange Rates", *The Manchester School* (forthcoming).