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Uncertainty and Stability in a Macro-Econometric Model

Carlo BIANCHI, Jean-Louis BRILLET, Lorenzo PANATTONI *

ABSTRACT. – One of the main techniques for determining the long term stability properties of a macro-econometric model has been for some time now to compute the eigenvalues of the linearized reduced form of the model. But these eigenvalues are affected by uncertainty, coming mostly from the error on the estimated coefficients. In this paper we study, using the French macro-economic model Mini-DMS, how taking into account the uncertainty can affect the conclusions. We shall give particular attention to the following points: are the conclusions concerning convergence certain, do the eigenvalues change significantly with the period at which the linearization is made, does the use of elasticities instead of multipliers produce significantly more stable eigenvalues.

Incertitude et stabilité dans un modèle macroéconomique

RÉSUMÉ. – Une des techniques principales pour déterminer les propriétés de stabilité de long terme d'un modèle macroéconométrique a été de calculer les valeurs propres de la forme réduite linéarisée. Mais les valeurs propres sont entachées d'une incertitude provenant essentiellement de l'erreur sur les coefficients estimés. Dans cet article on étudie, en utilisant le modèle Mini-DMS, comment la prise en compte de l'incertitude modifie les conclusions. On s'intéresse particulièrement aux points suivants : les conclusions sur la convergence sont-elles certaines, les valeurs propres changent-elles avec la période où la linéarisation est faite, l'usage des élasticités au lieu des multiplicateurs produit-il des valeurs propres plus stables.

* C. BIANCHI, L. PANATTONI :Centro Scientifico IBM, via S Mavia 67, Pisa, Italy. J. L. BRILLET : INSEE, 18, boulevard A. Pinard, 75675 Paris, France.

1 Introduction

This paper deals with the interpretation of the eigenvalues of the reduced form of a macro-econometric model in terms of long term stability, adding to this already well studied subject the dimension of the uncertainty on the eigenvalues. In fact as the estimated structural coefficients of the model are subject to sampling errors, so are the characteristic roots.

The principles of this type of study, (for more details, see Deleau and Malgrange [1979]), will be briefly discussed in this section.

Let us represent a structural nonlinear econometric model by

$$y_t = f(y_t, y_{t-1}, x_t, a) + u_t, \qquad t = 1, 2, \ldots, \mathrm{T}$$

where:

f is a column vector of functional operators (f_i, $i = 1, 2, \ldots, m$), continuous and differentiable with respect to the elements of y_t, y_{t-1}, x_t and a, with continuous derivatives up to the second order;

y_t, y_{t-1} and x_t are the column vectors of endogenous, lagged endogenous and exogenous variables at time $t = 1, 2, \ldots, \mathrm{T}$ (y_{it}, $i = 1, 2, \ldots, m$; x_{jt}, $j = 1, 2, \ldots, n$);

a is the column vector (a_k, $k = 1, 2, \ldots, s$) of structural coefficients to be estimated (the other known coefficients of the model being included in the functional operators);

u is the column vector of structural stochastic disturbances at time t (u_{it}, $i = 1, 2, \ldots, m$).

The most natural way of studying the dynamics of the model is to measure how a perturbation of the conditions at time t affects the solution of the following periods, and in particular if in the long run the solution gets back to its unperturbed path and what is the shape of this return : monotonic or cyclical, with which period.

This is done by derivating the model around a particular solution, producing the following linearized form:

$$\Delta y_t = \partial f / \partial y_t \, \Delta y_t + \partial f / \partial y_{t-1} \, \Delta y_{t-1} + \partial f / \partial x \, \Delta x_t$$

or, in reduced form:

$$\Delta y_t = \mathrm{A}_t \Delta y_{t-1} + \mathrm{B}_t \Delta \mathrm{x}_t$$

where

$$\mathrm{A}_t = (\mathrm{I} - \partial f / \partial y_t)^{-1} \, \partial f / \partial y_{t-1}$$

and

$$\mathrm{B}_t = (\mathrm{I} - \partial f / \partial y_t)^{-1} \, \partial f / \partial x_t.$$

If we suppose that the matrix A_t is constant over time (which is true if the model is linear in the variables, with constant coefficients), the computation of its eigenvalues gives the characteristics of the dynamics.

In this case, writing $A = V^{-1} L V$ (with L diagonal containing the eigenvalues and V matrix of eigenvectors) gives:

$$V \Delta y_t = L V \Delta y_{t-1}, + V B \Delta x_t$$

so if a variation of x at time t has on the endogenous variables an effect Δy_t, at time $t+k$ the effect will be:

$$V \Delta y_{t+k} = L^k (V \Delta y_t).$$

In other terms, each component of the variation Δy in the basis of the eigenvectors of matrix A will be multiplied at each period by a factor equal to the associated eigenvalue. In particular, the comparison of the modulus of the eigenvalue to one will show if the dynamics is convergent, and the type (real or complex) will determine if the evolution is cyclical or monotonic.

But it is clear that one condition is crucial for the use of this technique: the matrix A must be stable, not only in its eigenvalues, but also in its eigenvectors (see MALGRANGE [1981]); the stability of the basis for the decomposition of the variations of y is necessary to establish the last above formula. [1]

As said before the stability of the matrix A can be easily stated if the model is linear in the variables and constant in the coefficients. If this is not true, we could try to enforce the condition through a formal change in the variables. For instance if the model is purely multiplicative (or log-linear), by simply expressing the equations in terms of the logarithms of the variables we would get a linear model; in this case the variations would be interpreted in relative terms instead of absolute.

It is questionable whether in the case of operational models (the ones used for forecasting) the linear or log-linear approach gives the best results as to the stability of the matrix A: all of these models include formulations of the two types, and it does not seem possible to get a quantitative measurement of their relative influence.

These mathematical considerations are not the only aspect in the choice between absolute and relative variations. We can also consider the problem from an economic point of view, in which case the users of the model must choose between two definitions of stability:

– absolute stability, meaning that a one-period perturbation produces in the long run a return to the absolute value of the unperturbed trajectory,

– relative stability, if the relative difference between the two trajectories decreases to zero.

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1. Although it can be seen that under a weaker condition:

$$V_t^{-1} V_{t+1} = L1 \text{ constant.}$$

The same type of reasoning can be made, using this time the product of the two matrices. In particular, if L1 is diagonal, the case can be associated with the use of elasticities, which we shall present shortly.

For instance, the long-term effect on GDP of a given instantaneous perturbation could be constant or growing in absolute terms, but if this growth is slower that the one of GDP itself, we can wonder if the agents will keep taking into account a variation which becomes infinitesimal compared to the actual value. Anyway, this very long term reasoning seems somewhat artificial: if we consider the practical application of this kind of study to the operational long term (a twenty year horizon for instance), the choice is no longer so clear.

The above considerations show that the interest of this kind of study, as well as the choice between the possible techniques, are dependent on the actual model to which they are applied. So it appeared that the most efficient way to treat this problem was to take an operational model, as representative as possible of the models used for medium-long term forecasting, and to subject it to a set of experiments. We elected the French Mini-DMS model (Brillet [1986]) and computed for two different years of simulation the eigenvalues and eigenvectors using the two methods, observing how they changed with respect to the period (the actual association of eigenvalues of different periods was made using the usual method of interpreting the effect of the suppression of a particular column of the matrix A_t on each set of eigenvalues). But we did not not only compute the deterministic values: we added the consideration of the uncertainty of the dynamic process, seen through the analysis of the uncertainty on the eigenvalues.

Of course this uncertainty can be studied for itself, and indeed we shall first look into the stability properties associated to the eigenvalues in each case for a particular year, examining in particular how the conclusions on stability changed if we used, instead of deterministic values a measure of the probability of being stable. [2] Furthermore the evolution of the uncertainty with the period at which the derivatives are computed, and the method applied, can be used also as an element in the choice between absolute and relative measurements, in the following way:

– we can compare for one particular year the size of the uncertainty using alternative methods, and see if one gives more precise conclusions than the other;

– we can try to determine if the variation of the eigenvalues with time, using one of the two methods, is small enough to be attributed to this uncertainty, or must come from a more structural source such as a significant time evolution.

For this purpose we have computed, on the Mini-DMS model, the eigenvalues and associated standard errors for two different years using the two methods.

The results show that the size, uncertainty, probability of divergence are strongly reduced in the case of elasticities, (although some exceptions appear), and that the stability of the results over time is also strongly enhanced.

2 Computing Eigenvalues for Nonlinear Models

Given the nonlinear econometric model formalized in the previous section let us assume the existence of a vector $\hat{a}$ of consistent estimates of the vector of the all structural coefficients a, the asymptotic normality of $T^{1/2}(\hat{a}-a) \sim N(0, \Psi)$ and the availability of a consistent estimate of Ψ, $\hat{\Psi}$.

The dynamic behavior (and stability) of a local linearization of this model at time t is determined by the characteristic roots of the already defined matrix A_t. A convenient way to get an estimate $\hat{A}_t$ of this matrix is the computation of the partial derivatives, in the neighborhood of the solution point at time t, of y_t with respect to y_{t-1} [or $\log(y_t)$ with respect to $\log(y_{t-1})$ in the case of elasticities].

Let λ_t be a real characteristic root of A_t and $\hat{\lambda}_t$ the corresponding characteristic root of $\hat{A}_t$. Under the assumptions of continuity and differentiability of the functions involved in the structural form, since $T^{1/2}(\hat{a}-a)$ is asymptotically normally distributed as $N(0, \Psi)$, then $T^{1/2}(\hat{\lambda}_t-\lambda_t)$ is asymptotically normally distributed as $N(0, j_t' \Psi j_t)$, where j_t is the vector of partial derivatives of λ_t with respect to the elements of the vector a (Rao, [1965, p. 321]). If the computation is performed for λ_t, through the matrix $\hat{A}_t$, we get a consistent estimate $\hat{j}_t$ of j_t and the square root of $(\hat{j}_t' \hat{\Psi} \hat{j}_t)/T$ is the estimated asymptotic standard error of the given root.

If λ_t is complex, the above results hold both for the modulus and for the argument (or for the corresponding period); in this case we have to compute two vectors $\hat{p}_t$ and $\hat{q}_t$, partial derivatives of the modulus and of the argument, respectively.

The $\hat{j}_t$ (or $\hat{p}_t$ and $\hat{q}_t$) vectors for all the characteristic roots can be computed using a simulation method, which makes no difference between linear and nonlinear models; the method is based on numerical differentiation of the characteristic roots of the matrix $\hat{A}_t$, with respect to the structural estimated coefficients, where the matrix $\hat{A}_t$ is repeatedly constructed using numerical differentiation of the endogenous variables with respect to the lagged endogenous. The procedure can be summarized as follows.

1) A scan is preliminarily performed on the model to determine which are the lagged endogenous variables actually present and the maximum lag of each of them.

2) The model is numerically solved at time t.

2. In linear econometric models the estimation of the asymptotic standard errors of the characteristic roots has been dealt with by Theil and Boot [1962], Neudecker and Van de Panne [1966], Oberhofer and Kmenta [1973] and Schmidt [1974]. By means of these asymptotic standard errors, it is possible to test the model for stability. However, the power of the test is open to question, because the null hypothesis must be always stability, rather than instability, as well pointed out in Oberhofer and Kmenta [1973, fn.5] and Gustafson [1978].

3) An increment is given to the value of a lagged endogenous variable, the model is again solved at time t and the partial derivatives of y_t with respect to the given lagged endogenous are numerically computed. This step is repeated for all the lagged endogenous variables and the computed derivatives are stored into a matrix ($\hat{A}_t$).

4) The characteristic roots of the matrix are computed and stored; if they are complex, we compute and store moduli and arguments (or periods).

5) An increment is given to one structural estimated coefficient.

6) The process is repeated from step 2 to step 5 as many times as the number of structural estimated coefficients.

7) The partial derivatives of each root, with respect to the structural coefficients, are then computed, stored into the vector $\hat{f}_t$ (or the two vectors $\hat{p}_t$ and $\hat{q}_t$, if the root is complex) and the asymptotic standard error of the root is finally obtained.

Some care must be taken in the choice of the increments to be given to the lagged endogenous variables and to the coefficients to compute the derivatives. In all the performed experiments, relative increments in the range 0.001-0.000001 have always led to results equal at least in the first 2-3 significant digits (quite enough for a standard error). Of course, to appreciate these small increments, the tolerance at convergence in the iterative Gauss-Seidel method had to be much smaller than the one usually adopted by econometricians for the solution of simultaneous systems; 10^{-9} has been used for these experiments.

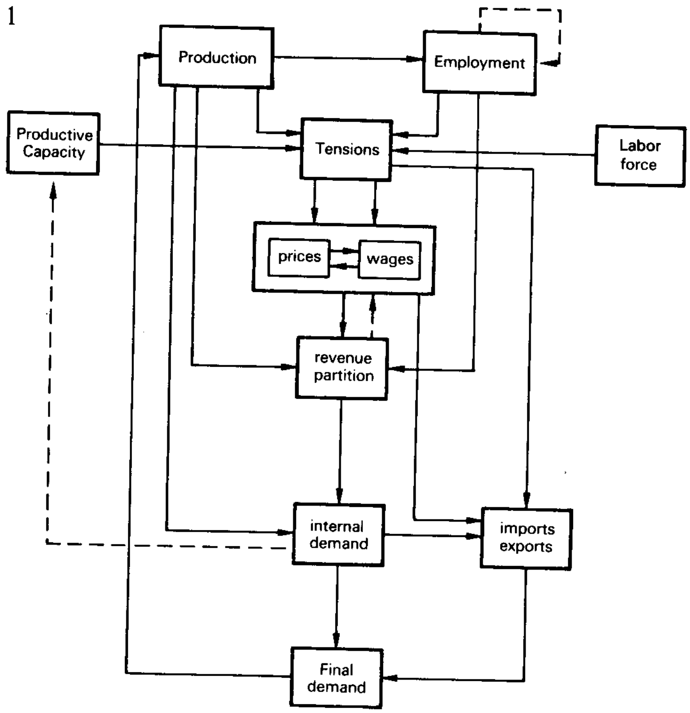
3 The Practical Study

3.1. The Model Used in the Experiments: Mini-DMS

The Mini-DMS model (Brillet [1986]) constitutes a smaller version of the Dynamic Multi Sectorial model of the French economy (Fouquet *et al.*, [1978]) built in 1974-1976 at INSEE (National Institute for Statistics and Economic Studies) to be used as a medium term forecasting tool, in particular for national planning studies conducted through the Commissariat General au Plan (General Planning Agency). Largely reduced in size (the present version contains 235 equations, 71 of which are behavioral, as compared to more than 2400 for the larger version) Mini-DMS nevertheless preserves the same economic structure as well as most of the theoretical mechanisms of the original model. The economic equilibrium is reached through two simultaneous iterative processes: a Keynesian process on demand (a given value of demand induces a level of production from which a new value is determined as the given sum of its individual elements) and the price/wage rate loop.

Figure 1 gives a very schematic view of the process: from final demand the model deduces production and desired employment level, to which the effective level adjusts only partially; comparison between potentials (predetermined production capacity, labor force, job supply) and the quantities actually used produces disequilibrium or tension variables, which determine the level reached by the iterative loop between wage rate and price index; the subsequent partition of the revenue between business firms and households gives their respective demand elements: investment (through an accelerator-profit formulation) and consumption, thus global domestic demand which, corrected of the external trade elements (influenced, besides demand itself, by available productive capacity and competitiveness), produces a new value for final demand, allowing a reinitialization of a process which hopefully leads to an equilibrium value after some iterations.

FIGURE 1



In its present state, the Mini-DMS model can be considered as being half way between an operational-forecasting tool: its acceptable forecasting qualities, as well as its rather detailed set of decisional variables, can lead to its use for simple enough macro-economic studies, and a research tool, used for carrying out mathematical economic experiments, some of which have already been made in the near past, concerning in particular multiplier analysis, optimal control problems or dynamic properties of alternate formulations.

Estimates of the structural coefficients the model have been obtained by means of a straightforward extension of BRUNDY and JORGENSON'S [1971] instrumental variables method (limited information) to the case of nonlinear models. The method has been applied iteratively, till convergence has been reached, so that the final estimates of coefficients are not affected by the choice of the initial coefficients values. In each iteration, the instrumental variables are computed as deterministic solution values of the system (which is the simplest choice, although not the best in the class of nonlinear estimators as well explained in AMEMIYA, [1983]). Since the number of stochastic equations in the model is considerably larger than the sample period length, the estimate of the covariance matrix of the disturbance process would be singular, and the standard system estimation methods could not be applied.

3.2. The Experiments Conducted on the Model

Computing the eigenvalues for the Mini-DMS model shows a rather high level of dynamicity for the medium-term annual case: 105 lagged influences can be numbered. The eigenvalues of the reduced form have been computed, and although the dimension of the associated matrix was 105, due to null or negligible influences and collinearities, only 41 eigenvalues have appeared to be significantly different from zero, even if, for the sake of brevity, only the first 20 are displayed in the tables.

The process has been repeated for two different years, 1974 and 1981, using either the multiplier form or the elasticities; we then have four sets of results, in which the actual average eigenvalue is accompanied by its dispersion statistics, on the modulus as well as the occasional period.

First of all we shall consider each set individually.

3.2.1. *Elasticities*

3.2.1.1. *Individual study for 1974*

Let us look at table 1: considering average values (first column) we see that there is only one value with a modulus larger than one, thus showing divergence properties. Four eigenvalues are exactly one, but many others can be considered rather large, and show dynamic influences which will stay significant in the medium term: 17 eigenvalues are larger than 0.80, and 27 larger than 0.60. So we would conclude that the model is very slowly diverging, through a single dynamic process. Let us see now how taking uncertainty into account affects this conclusion.

For this we will us the actual value of the standard error (column 2), as well as graph number 2, using the same order for the eigenvalues, and in which to the average value (lower zone) is added twice the size of its standard error: the top of the bar should then represent a higher limit for the modulus.

Considering the information given by the standard error, we first see that the unitary eigenvalues are certain. As this value was clearly independant from chance, we can feel that this was to be expected. But we can also have the feeling that this type of dynamic has some artificial character, and could perhaps be eliminated by a simple transformation in the variables.

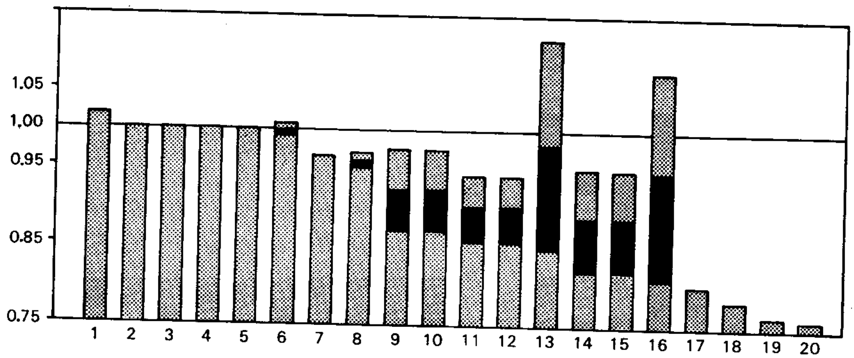
To put some light on this subject, we will identify the most natural source for these unitary eigenvalues: the formulation of some equations as growth rates.

TABLE 1

Eigenvalues for 1974 (elasticities)

Value	Modulus	Std. Error
1.	1.0107	.0032
2.	1.	0.
3.	1.	0.
4.	1.	0.
5.	1.	0.
6.	.9905	.0070
7.	.9648	0.
8.	.9469	.0108
9.	.8698	.0512
10.	.8698	.0512
11.	.8580	.0422
12.	.8580	.0422
13.	.8511	.1312
14.	.8274	.0639
15.	.8274	.0639
16.	.8240	.1279
17.	.8079	.0021
18.	.7928	.0001
19.	.7758	0.
20.	.7692	0.

FIGURE 2



Let us consider the following two-equations model:

$$\dot{y} = a\dot{x} + b\,g\,(z) + c$$
$$x = d\,f\,(y) + e$$

with y and x endogenous, a, b, c, d and e parameters (estimated or not).

In this model, the only dynamics come from the growth rates equation, which, in addition to growth rates, only contains exogenous elements.

The linearization of the model gives:

$$\Delta y/y - \Delta y_{-1}/y_{-1} = a(\Delta x/x - \Delta x_{-1}/x_{-1}) + b(y_{-1}/y)\,\partial g/\partial z\,\Delta z$$

and

$$\Delta x = d\,\partial f/\partial y\,\Delta y.$$

The first linearized equation can be written as:

$$\Delta y/y - a\,\Delta x/x = (\Delta y_{-1}/y_{-1} - a\,\Delta x/x_{-1}) + b(y_{-1}/y)\,\partial g/\partial z\,\Delta z$$

It is clear that, in this case, a unitary value appears: this is true even if we take into account the simultaneity of the system through the second equation, as long as no additional function of x or y appears in the first one. This unitary value is not affected by uncertainty, even if the value of parameters are. More generally, we can see that the existence of a linear combination between growth rates, containing possibly also exogenous elements of any form, will produce unitary eigenvalues with zero uncertainty.

As an example, let us take a wage rate equation: if the growth of the wage rate is exogenous in real terms, the effect of a one-time perturbation of this exogenous value will be maintained on the real wage rate indefinitely; only if the real growth is endogenous can the dynamics of the model change the variation over time.

The above example shows also why the dynamics cannot be eliminated through a change in the variables: we could indeed replace this equation by another, linking the absolute wage rate with the level of the price index and the cumulated exogenous increases. In this case the equation would give the same solution, without dynamics this time. But we have changed also the meaning of a perturbation of the exogenous variable: applying the same perturbation as before would clearly bring back the same dynamics; so the change in the formulation did not really modify the dynamics, but instead the definition of a perturbation on the exogenous variable.

Let us now consider the other eigenvalues of table 1. We can see that although the standard errors are in general very low with respect to the size of the eigenvalues, four values now show some probability of being strictly[3] larger than one: eigenvalues 1 (modulus 1.010, standard error 0.003), 6 (modulus 0.990, standard error 0.007), 13 (modulus 0.851, standard error 0.131) and 16 (modulus 0.824, standard error 0.128). For none of these values is the divergence certain; thus the diagnosis on the stability properties of the model linearized in 1974, based on the average values, is not confirmed by the consideration of the uncertainty, and must be replaced by a range of possibilities going from a moderately slow divergence (something like 10% each year) to a slow convergence.

Moreover, this new element shows that possible divergence comes not only from the largest eigenvalues, but also from much smaller ones with particularly large standard errors (here, for number 13 and 16, 40 times larger than for number 1): thus for individual dynamics, the notion of "having good convergence properties" gives very different results whether we consider average convergence or the probability of showing divergence. In particular if we are interested in knowing what dynamics can induce instability in the model (which is generally the goal of this type of study), it seems more interesting to point out elements which have a significant probability of divergence, rather than certainly converging ones, even if this convergence is very slow.

The consideration of the periods and their standard errors is less interesting: in fact it is harder to interpret the uncertainty on the period, which has in general a non-zero probability of being infinite, and which in any case shows a strongly asymmetric distribution. We can observe nonetheless that the standard error on the period is rather large compared to the average value.

3.2.1.2 *Time evolution for 1981*

The average values in table 2 produce the same conclusions as table 1: four unitary eigenvalues, 17 larger than 0.80, 27 larger than 0.60. The only difference is that now there are two eigenvalues larger than one.

Concerning uncertainty, we get similar conclusions: the general size of the standard errors does not change, we still get eight eigenvalues with zero standard error, and two (13 and 16) with a large one. Instead of four, we get five processes with a strong probability of divergence: values 1, 2, 7, 13 and 16. However there is an important exception: one of the larger than one values has a zero standard error, introducing a certainly diverging process.

3.2.1.3. *Interpretation in terms of influences*

If we want to interpret the new results to show how individual eigenvalues change with time, it is of course necessary first of all to establish a one to one correspondance between the elements of the two tables. The statistics presented in tables 1 and 2 are clearly not sufficient, although we can suppose that the size of the standard error of a particular influence does not change too much over time: for instance having a zero value should be a constant property. To produce this association we shall us the same technique as DELEAU and MALGRANGE [1979] for the economic interpretation of the eigenvalues: we shall replace in turn each of the columns of the matrix A_t by zero values (thus eliminating the dynamic influence of the corresponding variable), compute the eigenvalues of the restricted matrix for each of the two years, and try to associate the eigenvalues which are influenced in the same way by a particular suppression.

This technique works very well in this case, and leads to the classification showed in table 3, about which we can make the following remarks:

– this technique proved very useful, as it prevented from making such errors as associating values 1/1974 (1.0107) and 2/1981 (1.0082), or 13 and

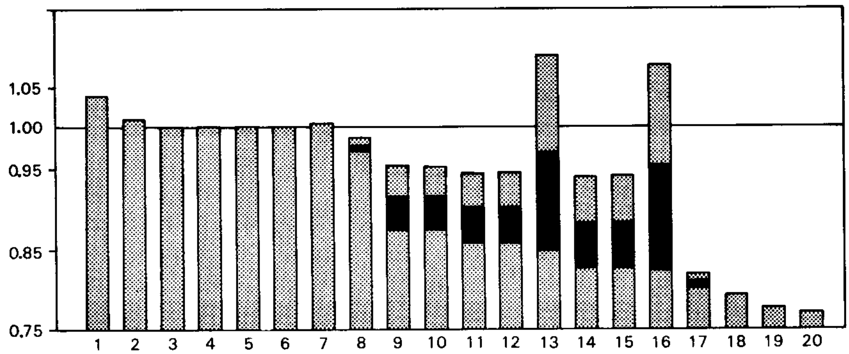
3. The uncertainty equal to zero of the unitary eigenvalues, means that they are associated in fact with dynamic identities (thus the measurement of uncertainty can also be a tool for the economic interpretation of the values). This property is also observed for eigenvalues 7, 18, 19 and 20.

TABLE 2

Eigenvalues for 1981 (elasticities)

Value	Modulus	Std. Error
1.	1.0391	0.
2.	1.0082	.0028
3.	1.	0.
4.	1.	0.
5.	1.	0.
6.	1.	.0001
7.	.9957	.0044
8.	.9727	.0078
9.	.8732	.0393
10.	.8732	.0393
11.	.8586	.0413
12.	.8586	.0413
13.	.8479	.1207
14.	.8263	.0564
15.	.8263	.0564
16.	.8233	.1273
17.	.8045	.0083
18.	.7928	.0001
19.	.7762	.0004
20.	.7692	.0002

FIGURE 3



16/1974 (0.8511 and 0.8240) and 13 and 16/1981 (0.8479 and 0.8233) instead of the inverse.

– no eigenvalue has a modulus larger than one in the two sets, showing that it is not improbable that the divergence is only a temporary phenomenon, and that the average steady state solution of the model is stable;

– the eigenvalue inducing certain divergence in 1981 (1.0391) has a modulus of 0.9648 in 1974; the zero standard error helps us to interpret this value as being associated with the fact that a linear combination between dynamic variables shows a constant relative variation from one period to another:

$$a\,\Delta y_t/y_t = \lambda(a\,\Delta y_{t-1}/y_{t-1}) + b\,\Delta x_t$$

where λ is not affected by uncertainty; in this particular case, the proximity to unity makes us suspect that we have:

$$a\Delta y_t = a\Delta y_{t-1} + b\Delta x_t$$

and that λ is the ratio between two consecutive values of y; this would happen for instance in an equation where the only endogenous variables are presented as first differences.

In this case the use of elasticities has a perverse effect on the stability of the eigenvalue, exactly similar to the one we get using plain multipliers for elasticity equations, as pointed out in MALGRANGE [1981]. But the problem is not so important here, as a diverging eigenvalue is associated with a decreasing variable.

TABLE 3

Comparison between 1974 and 1981 eigenvalues

Ranking and Values 74		81		Standard Errors 74	81
1.	1.0107	7.	.9957	.0032	.0045
2.	1.	3.	1.	0.	0.
3.	1.	4.	1.	0.	0.
4.	1.	5.	1.	0.	0.
5.	1.	6.	1.	0.	0.
6.	.9905	2.	1.0082	.0070	.0028
7.	.9648	1.	1.0391	0.	0.
8.	.9469	8.	.9727	.0108	.0078
9.	.8698	9.	.8733	.0512	.0393
10.	.8698	10.	.8733	.0512	.0393
11.	.8580	11.	.8586	.0422	.0413
12.	.8580	12.	.8586	.0422	.0413
13.	.8511	16.	.8233	.1312	.1273
14.	.8274	14.	.8263	.0639	.0564
15.	.8274	15.	.8263	.0639	.0564
16.	.8240	13.	.8479	.1279	.1207
17.	.8079	17.	.8045	.0021	.0083
18.	.7928	18.	.7928	.0001	.0001
19.	.7748	19.	.7762	.0001	.0004
20.	.7692	20.	.7692	.0001	.0002

– for two other large eigenvalues we have a very high precision, but the average values are so close to unity that they can change their average convergence property, without getting out of the confidence interval associated with the other period. This confirms the uncertainty on the nature of the process they define, between very low divergence and convergence (we do not know either if this nature is stable with time).

– this remark holds for all the other values (except maybe for N° 19): no significant change can be found from one period to the other. Generally the difference is very small compared to the standard error;

– as to the standard errors themselves, they change more or less in size: from 0.0512 to 0.393 for eigenvalues 9 and 10, but from 0.0422 to 0.0413 for eigenvalues 11 and 12. However the classification into large ones (eigenvalues 13 and 16), moderate (from 9 to 12, 14 and 15), low (1, 6, 8, 17) and null (from 2 to 5, 7, from 18 to 20) is not modified.

On the whole, we can conclude that there is a strong probability that an individual year defines a very slow divergence (but even in that case it is not certainly persisting), and that the stationarity of the eigenvalues is very good, especially when compared to the uncertainty on their yearly values.

3.2.2. *Multipliers*

3.2.2.1. *Individual study for 1974*

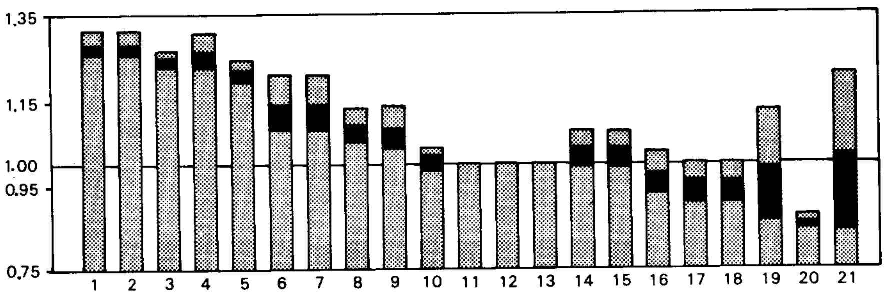
Comparing the average results in table 4 to the equivalent statistics for elasticities (table 1), we get a quite different situation: now 10 eigenvalues have a modulus larger than one and 3 exactly one. The increase is stable at first: 18 values are larger than 0.90; but if we take more eigenvalues into account, the new values get nearer to the previous experiments on elasticities: just as before, the 0.60 level is associated with eigenvalue N° 27.

TABLE 4

Eigenvalues for 1974 (multipliers)

Value	Modulus	Std. Error
1.	1.2589	.0307
2.	1.2589	.0307
3.	1.2318	.0202
4.	1.2276	.0437
5.	1.1952	.0272
6.	1.0786	.0679
7.	1.0786	.0679
8.	1.0495	.0391
9.	1.0340	.0484
10.	1.0031	.0152
11.	1.	0.
12.	1.	0.
13.	1.	0.
14.	.9930	.0464
15.	.9930	.0464
16.	.9309	.0509
17.	.9163	.0492
18.	.9163	.0492
19.	.8704	.1323
20.	.8575	.0141
21.	.8457	.1909

FIGURE 4



Considering the uncertainty, we see now that the size of the standard errors has not really increased, on the whole: this was not evident since the values of the variables entering in the computation of the elasticities are affected by uncertainty. But, with the change on the average values, now the first five eigenvalues are associated with a significantly diverging process, and the following five, as well as values 14, 15, 19 and 21, (plus maybe from 16 to 18) with an uncertain one. Of course, this result was expected: numerous equations in the Mini-DMS model are formulated as naturally positive growth rates:

$$y_t = y_{t-1}(1 + f_t(\ldots))$$

thus producing, if f_t and y_t are not too strongly correlated, an eigenvalue of approximately $1 + f_t$ (see appendix 1 for a detailed explanation), in which most of the uncertainty comes from f_t and not the multiplier effect due to the simultaneity of the model. This is the case for instance for prices and wages, which in the year 1974 presented a very high growth.

So in this case the largest eigenvalues are associated with the strongest divergence probability, although some smaller ones can produce divergence too. But the interpretation of this divergence, measured in absolute rather than relative difference to naturally increasing variables, can of course be questioned. Using the same technique as before, we shall now try to associate eigenvalues obtained using multipliers and elasticities. This proves much harder than before and we were only able to get the results shown in the uncompleted table 5.

TABLE 5

Comparison between elasticity and multiplier eigenvalues (1974)

Elasticities		Multipliers		Std. Errors	
Order	Value	Order	Value	Elast	Mult.
1.	1.0107	9.	1.0040	.0032	.0484
2.	1.	5.	1.1952	0.	.0272
3.	1.	4.	1.2276	0.	.0437
4.	1.	3.	1.2318	0.	.0202
5.	1.	8.	1.0495	0.	.0391
6.	.9905	- - - - - - -	- - - - - - - -	.0070	- - - - - - - -
7.	.9648	12.	1.	0.	0.
8.	.9469	11.	1.	.0108	.0078
9.	.8698	- - - - - - -	- - - - - - - -	.0512	- - - - - - - -
10.	.8698	- - - - - - -	- - - - - - - -	.0512	- - - - - - - -
11.	.8580	- - - - - - -	- - - - - - - -	.0422	- - - - - - - -
12.	.8580	- - - - - - -	- - - - - - - -	.0422	- - - - - - - -
13.	.8511	19.	.8703	.1312	.1322
14.	.8274	- - - - - - -	- - - - - - - -	.0639	- - - - - - - -
16.	.8274	- - - - - - -	- - - - - - - -	.0639	- - - - - - - -
16.	.8240	- - - - - - -	- - - - - - - -	.1279	- - - - - - - -
17.	.8079	- - - - - - -	- - - - - - - -	.0021	- - - - - - - -
18.	.7928	- - - - - - -	- - - - - - - -	.0001	- - - - - - - -
19.	.7748	17.	.9162	.0001	.0492
20.	.7692	18.	.9162	.0001	.0492

3.2.2.2. *Time evolution in 1981*

Compared to the 1974 average eigenvalues, now we get only 9 values larger than one (one less than before), still three exactly one, and the following decrease is quite similar.

But concerning the size of the eigenvalues, we get quite important differences, especially concerning the highest ones: for instance the fifth one for 1974 is greater than the first of 1981.

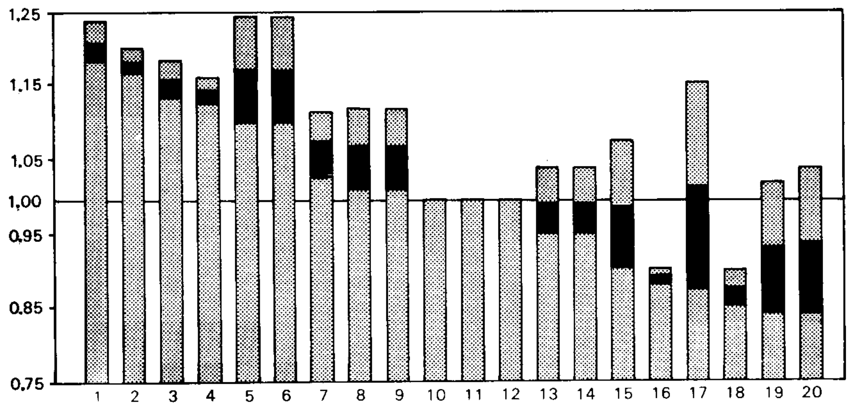
The global size of the standard errors does not seem to have much changed either; but as they apply to different average values, the conclusion as to the divergence of the process can be different from before. Now only the first four values are significantly greater than one, and there is a strong

TABLE 6

Eigenvalues for 1981 (multipliers)

Value	Modulus	Std. err.
1.	1.1791	.0278
2.	1.1582	.0176
3.	1.1303	.0240
4.	1.1238	.0184
5.	1.0980	.0714
6.	1.0980	.0714
7.	1.0273	.0460
8.	1.0094	.0559
9.	1.0094	.0559
10.	1.	0.
11.	1.	0.
12.	1.	0.
13.	.9454	.0435
14.	.9454	.0435
15.	.8973	.0879
16.	.8821	.0102
17.	.8717	.1388
18.	.8543	.0229
19.	.8425	.0982
20.	.8425	.0982

FIGURE 5



probability for 6 others: from 5 to 9 and N° 17 (plus maybe 13, 14, 15, 19 and 20). But this result does not differ too much from 1974, where we had five significant, and nine insignificant divergences.

Using again the same method, we can establish a correspondence between eigenvalues of different periods: this will show if eigenvalues associated with a particular dynamic influence change significantly with time, contrary to eigenvalues using elasticities. This proves easy, at least in the cases for which we were able to produce an association for 1974 between multiplier and elasticity eigenvalues.

Table 7 shows in the first four columns the values and standard errors of the moduli of eigenvalues measured in 1974 and 1981 (in decreasing order). The fifth column gives for the 1981 value of each row the number of the 1974 associated value. The last four columns give, with the order of eigenvalues used for 1974, the associated values for the two years (when the interpretation has proved possible).

As we can see, establishing a correspondence across time for multiplier eigenvalues proves easy for those which have been already associated with elasticity eigenvalues; but for the others the interpretation is much more difficult, and sometimes it is apparently impossible to evidence a clear link. [4]

In particular the two largest eigenvalues (modulus 1.2589) in the 1974 case cannot be associated with a 1981 value, at least ones with a modulus larger than unity. Moreover, the uncertainty of the associated eigenvalues can have a very different size, which does not help in the interpretation.

But despite those difficulties, we see clearly that even if we suppose that the effects of uncertainty are independent across time [5], at least some of the eigenvalues are significantly different: for instance numbers 3+14 (1974)/1+4 (1981) (a couple of values is associated in this case with a couple of columns), and 10 (1974)/16 (1981). The convergence properties can also change over time, for the deterministic value as well as for the uncertainty: for values 10/16, value 16 (0.8821) of 1981 is clearly converging, while the associated 1.0031 of 1974 has a quite uncertain property. If we make experiments for additional years there is high chance of finding that for the same dynamic influence one year produces significant divergence and the other significant convergence.

So even though the difficulties in the interpretation of the multiplier case are important, the partial elements we get show clearly that the conclusions as to the convergence of the dynamic processes are strongly linked with the year under investigation, even if we try to relativize our conclusions by introducing uncertainty. Indeed, we see that the differences between conclusions made for different years are larger than could be expected from the uncertainty on the eigenvalues.

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4. This is the reason why we use a different presentation as we cannot present the 1981 reordering and the total list of the eigenvalues in the same table.

5. In fact they must be positively correlated, as a particular error on coefficients must influence economic equilibrium in a similar way for different periods. So the significant level of difference should probably be lowered.

TABLE 7

Comparison between 1974 and 1981 eigenvalues (multipliers)

Ranking	Value 74	Std. Err. 74	Value 81	Std. Err. 81	Equivalence 81/74	Value 74	Value 81	Std. Err. 74	Std. Err. 81
1	1.2589	.0307	1.1791	.0278	3+14				
2	1.2589	.0307	1.1582	.0176	---				
3	1.2318	.0201	1.1303	.0240	8	1.2318	1.1791	.0201	.0278
4	1.2276	.0437	1.1238	.0184	3+14	1.2276	1.0980	.0437	.0714
5	1.1952	.0272	1.0980	.0714	4	1.1952	1.0980	.0272	.0714
6	1.0786	.0679	1.0980	.0714	5				
7	1.0786	.0679	1.0273	.0460	9				
8	1.0495	.0391	1.0094	.0559	6 (?)	1.0495	1.1303	.0391	.0240
9	1.0340	.0484	1.0094	.0559	7 (?)	1.0340	1.0273	.0484	.0460
10	1.0031	.0152	1.	0.	11	1.0031	.8821	.0152	.0102
11	1.	.0002	1.	0.	12	1.	1.	.0002	0.
12	1.	0.	1.	0.	13	1.	1.	0.	
13	1.	.0001	.9454	.0435	---	1.	1.	.0001	0.
14	.9930	.0464	.9454	.0435	---	.9930	1.1238	.0464	.0278
15	.9930	.0464	.8973	.0879	---				
16	.9309	.0509	.8821	.0102	10				
17	.9163	.0492	.8717	.1388	19	.9163	.8425	.0492	.0892
18	.9163	.0492	.8543	.0229	18	.9163	.8425	.0492	.0892
19	.8704	.1323	.8425	.0892	17	.8704	.8717	.1323	.1388
20	.8575	.0141	.8425	.0892	---				

4 Conclusions

In this paper we have shown, on a pratical example based on a model similar to those producing operational forecasts, that the study of the dynamics of the model using elasticities produces results rather stable with time, both on average and standard errors of eigenvalues. On the other hand it came out that, in the case of an eigenvalue near to unity, the conclusion as to the convergence of the process should be relativized.

On the contrary, using multipliers we have much more difficulty in interpreting the dynamics, and although the uncertainty does not really increase, we get significant differences between years, and significantly different conclusions as to the convergence of the dynamic processes.

Perhaps, to get more definitive conclusions, this study should be pursued using a smaller model, which would allow to track year by year these statistics over a large period (compared possibly with the dynamics of a steady-state growth path), and to point out clearly the economic mechanisms involved, without losing too much of the representativity of the experiment.

This would also allow us to study the eigenvectors, the stability of which proves quite difficult to measure in a large scale model, but nevertheless must be necessarily stated if we want to use the same matrix of derivatives over time.

The uncertainty of a Growth Rate Equation

Let us consider a model with only one lagged endogenous variable, appearing through a growth rate:

$$\dot{z}=f(y,\ x)$$
$$y=g\ (y,\ z,\ x)$$

z : endogenous variable which is lagged;

y : vector of other endogenous variables.

x : vector of exogenous variables.

In the multiplier case the derivation gives :

$$\Delta z=(1+f)\,\Delta z_{-1}+z_{-1}\,\partial f/\partial y\,\Delta y$$
$$\Delta y=\partial g/\partial y\,\Delta y+\partial g/\partial z\,\Delta z$$

and

$$\Delta z=[\mathrm{I}-z_{-1}\,\partial f/\partial y(\mathrm{I}-\partial g/\partial y)^{-1}(\partial g/\partial z)]^{-1}(1+f)\,\Delta z_{-1}$$

while for elasticities we get:

$$\Delta z/z=[\mathrm{I}-z_{-1}\,\partial f/\partial y(\mathrm{I}-\partial g/\partial y)^{-1}(\partial g/\partial z)]^{-1}\,\Delta z_{-1}/z_{-1}$$

In other terms, in the first case the eigenvalue can be interpreted as the correction of an autonomous growth term by a simultaneity effect, while in the second only the simultaneity effect appears. Of course, if z does not appear in g, or y in f, this effect disappears, and we get eigenvalues of $1+f$ and 1, respectively.

These formulas, which can be extended to the case of several simultaneous growth rates, do not allow absolute considerations on the relative size and uncertainty of the two methods; but they show that, while in the elasticity case the uncertainty comes only from the simultaneity (the lagged value of z being predetermined), in the multiplier case it comes also from the actual value of f; in particular, most grow rates being positive with a strong enough probability, we can expect in this case eigenvalues significantly greater than one, if the simultaneity effect is not too large.

These remarks also point out that the crucial element is not the linearity of the whole model, but of the formulas including lagged influences.

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