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Long-Run Properties of Large-Scale Macroeconometric Models

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ABSTRACT. – We consider alternative approaches to the evaluation of the long-run properties of dynamic nonlinear macroeconometric models, namely dynamic simulation over an extended database, or the construction and direct solution of the steady-state version of the model. An application to a small model of the UK economy is presented. The model is found to be unstable, but a stable form can be produced by simple alterations to the structure.

Les propriétés de long terme des grands modèles macroéconométriques

RÉSUMÉ. – Nous envisageons différentes approches à la détermination des propriétés de long terme des modèles macroéconométriques dynamiques et non linéaires : c'est-à-dire soit la simulation dynamique sur une longue base de données, soit la construction et la solution directe du régime stationnaire du modèle. Une application à un petit modèle de l'économie britannique est présentée. On en conclut que le modèle est instable mais que la stabilité peut être atteinte par de légères modifications de structures.

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1 Introduction

The long-run or steady-state properties of dynamic econometric models are often of considerable interest. Relevant economic theory is typically of an equilibrium or comparative static kind and has relatively little to say about dynamic adjustment, thus in checking the consistency of an empirical model with the economic theory, only its long-run implications are pertinent. In dynamic single-equation models, such comparisons are commonplace: having first checked the stability of the model, attention is concentrated on its long-run or steady-state properties and their relation to economic theory. Procedures for carrying out the same analysis in dynamic linear simultaneous equation models are also well-established. Practical large-scale macroeconometric models are nonlinear in variables, however, and in this area researchers are less well-equipped, apart from the proposal that the methods of linear analysis be applied to a local linearization of the nonlinear model. In models with rational expectations, the steady-state properties may be of assistance not only in understanding the theoretical structure of the model but also in providing terminal conditions for dynamic solutions.

In this paper we consider two approaches to the evaluation of the long-run properties of dynamic nonlinear models, namely those of extended simulation and direct solution. In the first approach a dynamic solution path is obtained over a suitably lengthened simulation or forecast database. In the second approach steady-state versions of each structural equation are derived, and the resulting structure is solved by standard numerical methods. This is seen to be relatively straightforward to apply, and a comparison of the direct solution with dynamic simulation results aids the interpretation of the latter, particularly with respect to the question of stability. We discuss some important issues that arise in defining an appropriate long run or steady state of a practical macroeconomic model, and present an application to the City University Business School (CUBS) model of the UK economy. In its standard form this model is unstable, but this can be corrected, so the analysis can be illustrated by both a stable and an unstable model.

In Section 2 we briefly review the long-run implications of single-equation models, paying attention to some developments in the recent literature. In Section 3 we turn to simultaneous equation models, first generalizing the preceding material for the linear case, then presenting the two approaches to the analysis of the nonlinear case. Some questions concerning the definition of an appropriate steady state that arise whichever approach is employed are considered in Section 4, and the application to the CUBS model is presented in Section 5. Concluding comments are contained in Section 6.

2 Single-Equation Dynamic Model

The long-run implications of a single-equation dynamic model are usually summarized most simply by the corresponding static equilibrium model, bearing in mind that this is of little interest unless the dynamic model is stable. In static equilibrium the exogenous variables are assumed to have constant values and dynamic adjustment is complete, so that the dependent variable is also constant. For the simple autoregressive-distributed lag model

(1) $$y_t = a + \sum_{j=1}^{r} b_j y_{t-j} + \sum_{j=0}^{s} c_j x_{t-j} + u_t$$

the deterministic static equilibrium is

$$y^e = \frac{\sum c_j}{1 - \sum b_j} x^e.$$

Alternatively, defining the polynomials $b(\mathrm{L})$ and $c(\mathrm{L})$ in the lag operator L as follows,

$$b(\mathrm{L}) = 1 - \sum_{j=1}^{r} b_j \mathrm{L}^j, \qquad c(\mathrm{L}) = \sum_{j=0}^{s} c_j \mathrm{L}^j,$$

the model may be written

$$b(\mathrm{L})\, y_t = a + c(\mathrm{L})\, x_t + u_t.$$

Then the stability condition is that the roots of the equation $b(z) = 0$ satisfy $|z| > 1$, and the long-run coefficient or multiplier is $c(1)/b(1)$.

A model containing current and lagged values of variables can be rearranged into a model expressed mainly in terms of their first differences, and various possibilities arise, depending on how the remaining levels term is handled. In effect, the polynomials $b(\mathrm{L})$ and $c(\mathrm{L})$ are divided by $\Delta = 1 - \mathrm{L}$, and the question is where to leave the remainder term in this polynomial division. BEWLEY [1979] leaves it on the constant term, and obtains

(2) $$\left(1 - \sum b_j\right) y_t = a + \sum_{j=0}^{r-1} b_j^* \Delta y_{t-j} + \left(\sum c_j\right) x_t + \sum_{j=0}^{s-1} c_j^* \Delta x_{t-j} + u_t,$$

where $b_j^* = -(b_{j+1} + \ldots + b_r)$ and $c_j^* = -(c_{j+1} + \ldots + c_s)$, so that on renormalizing the equation, a direct estimate of the long-run coefficient and its standard error can be obtained by regressing y_t on x_t and current and lagged first differences of y and x (using an instrumental variable estimator). Alternatively, leaving the remainder term on the highest power of L gives

(3) $$\Delta y_t = a + \sum_{j=1}^{r-1} b'_j \Delta y_{t-j} + b'_r y_{t-r} + \sum_{j=0}^{s-1} c'_j \Delta x_{t-j} + c'_s x_{t-s} + u_t$$

where $b'_j = -(1-b_1-\ldots-b_j)$ and $c'_j=(c_0+\ldots+c_j)$, so that the remaining levels variables appear with the maximum lags in the model. Then the long-run coefficient is $-c'_s/b'_r$, supporting the familiar interpretation that an equation expressed entirely in first differences determines no long-run static equilibrium.

A common justification for dynamic models is that economic agents react to forecasts of a variable, or estimates of an unobserved component of a variable, with these being formed as an extrapolation or linear filter of past observations. For example, consider the model

(4) $$b(\mathrm{L})\, y_t = a + \theta\, \hat{x}_{t+1,\,t} + u_t,$$

where $\hat{x}_{t+1,\,t}$ is the one-step-ahead forecast of x_{t+1} formed at time t. If x_t obeys the autoregressive process

$$x_t = \varphi_1 x_{t-1} + \ldots + \varphi_p x_{t-p} + \varepsilon_t$$

then the forecast of x_{t+1} based on current and past values of x alone is simply

$$\hat{x}_{t+1,\,t} = \varphi_1 x_t + \ldots + \varphi_p x_{t-p+1}.$$

Substituting this into the initial model gives equation (1) with $c_j = \theta\varphi_{j+1}$, $j=0,\ldots,s$, $s=p-1$. In a static, consistent expectations equilibrium, however, model (4) has a long-run coefficient of $\theta/b(1)$, which may differ from the quantity calculated directly from (1) ignoring the role of expectations as a source of dynamics, namely $c(1)/b(1)$, as KELLY [1985] points out. The two coincide if $\varphi_1+\ldots+\varphi_p=1$, that is, if the autoregression has a unit root, representing an integrated or difference-stationary process, which is often observed in economic time series analysis.

A dynamic equilibrium in which variables have common constant growth rates is often of interest, corresponding to situations of constant inflation or steady-state growth, for example. Defining the growth rate g by $x_t/x_{t-1}=1+g$ and using the approximation $\Delta \log x_t = g$, valid for small g, gives a convenient interpretation of the dynamic model (1)-(3) when, as is often the case, this is specified as a log-linear equation. If the theory postulates that, in equilibrium, y and x are in constant proportionality $y=kx$, say, then in a logarithmic equation there is a long-run coefficient of one. This restriction can be readily imposed on version (3) of the model, taking $r=s$, to give

$$\Delta \log y_t = a + \sum b'_j \Delta \log y_{t-j} + \sum c'_j \Delta \log x_{t-j}$$

$$- c'_s(\log y_{t-s} - \log x_{t-s}) + u_t$$

which is a model of the form used by SARGAN [1964] and DAVIDSON *et al.* [1978], subsequently popularized as the « error correction » model. In dynamic equilibrium the coefficient of proportionality, k, depends on the growth rate, g, which in the present setting must be common to both variables. This restriction can be relaxed in more general models, and if the equilibrium relation is required to be independent of the growth rates, the model may be further restricted (CURRIE [1981]).

The connection between these models and commonly-observed features of economic time series is noted by GRANGER [1981], in introducing the concept of « co-integrated » series. If y and x are integrated or difference-stationary processes, but there exists a constant A such that $w_t = y_t - A x_t$ is stationary, then y and x are said to be co-integrated. Broadly speaking, y and x move together in the long run, providing a statistical interpretation of the previous notion of an equilibrium relation. There is then a direct correspondence to a dynamic model of the error correction form, in which Δy_t is related to Δx_t, lagged values of these differences, and the « equilibrium error » w_{t-1}.

3 Simultaneous Equation Systems

The dynamic linear simultaneous equation model relating a vector of endogenous variables y_t to exogenous variables x_t and disturbances u_t is written in structural form as

$$\mathrm{B}(\mathrm{L})\, y_t + \mathrm{C}(\mathrm{L})\, x_t = u_t$$

where $\mathrm{B}(\mathrm{L})$ and $\mathrm{C}(\mathrm{L})$ are matrices of polynomials in L. Individual structural equations are multivariate generalizations of the equations discussed above, but the obvious carrying forward of notation should not be allowed to obscure the fact that the « equilibrium » relation in an error correction equation, for example, might involve variables that are each endogenous to the simultaneous equation model. The stability condition is that the roots of the determinental equation $|\mathrm{B}(z)| = 0$ satisfy $|z| > 1$. The corresponding deterministic static equilibrium relationships are

$$\mathrm{B}(1)\, y^e + \mathrm{C}(1)\, x^e = 0.$$

The final form solution

$$y_t = -\mathrm{B}(\mathrm{L})^{-1}\, \mathrm{C}(\mathrm{L})\, x_t + \mathrm{B}(\mathrm{L})^{-1}\, u_t$$

expresses each endogenous variable as an infinite distributed lag function of exogenous variables, together with an error term comprising moving averages of the original disturbances. The coefficients in the expansion of $-\mathrm{B}(\mathrm{L})^{-1}\, \mathrm{C}(\mathrm{L})$ are the interim multipliers describing the response of y_{it} to a unit shock in $x_{j,t-k}$, and cumulate to give the long-run multipliers $-\mathrm{B}(1)^{-1}\, \mathrm{C}(1)$.

In practice, especially in view of the nonlinearities described below, these questions are studied by comparing two solution paths for the endogenous variables. For given initial conditions and a sequence of exogenous variable values x_t, $t=1,\ldots,n$ the model is solved period-by-period, with $u_t=0$, to yield the control solution or base run $\hat{y}_t$. Then the vectors x_t are replaced by $x_t+\delta$, $t=1,\ldots,n$ and the exercise is repeated, giving the perturbed solution or simulation $\tilde{y}_t$. If the vector δ has a single element δ_j equal to one, with all other elements zero, then the differences $\tilde{y}_t-\hat{y}_t$, $t=1,\ldots,n$ are simply the dynamic multipliers describing the response of the endogenous variables to a unit step in x_j at $t=1$; as $n\rightarrow\infty$ these converge to the long-run multipliers given as the jth column of $-\mathrm{B}(1)^{-1}\,\mathrm{C}(1)$, if the model is stable, although convergence will be slow if $|\mathrm{B}(z)|$ has roots close to the unit circle. Stability is often assessed by observing whether or not the solution path converges, but in relatively small linear models the roots of $|\mathrm{B}(z)|$ may be examined directly.

Practical large-scale macroeconometric models are nonlinear in variables. For example, behavioural equations are often specified to be linear in the logarithms of variables, whereas identities relate the same variables linearly; also real magnitudes, the corresponding nominal magnitudes and the relevant price deflators typically appear in different equations of the model, representing separate variables linked by nonlinear identities. A convenient general notation for the structural form is then

$$f(y_t,\ z_t,\ \alpha)=u_t,$$

where f is a vector of functions having as many elements as the vector y_t, all predetermined variables (exogenous, current and lagged, and lagged endogenous) are collected in the vector z_t, and α is a vector of parameters; this represents greater generality than is necessary, however, since nonlinearity in parameters is rare. It is assumed that there exists, implicitly, a single inverse relationship.

$$y_t=g(u_t,\ z_t,\ \alpha),$$

valid for relevant z-values, analogous to the reduced form in the linear case. Forecasting and policy analysis exercises usually ignore the impact of the random error term and utilize the deterministic solution of the model, written implicitly as

$$y_t^0=g(0,\ z_t,\ \alpha)$$

and obtained explicitly as the numerical solution, to the desired degree of accuracy, of the equations

$$f(y_t^0,\ z_t,\ \alpha)=0.$$

Likewise dynamic multipliers are computed as the difference between base and perturbed deterministic solution paths. These are obtained, as in the previous paragraph, by solving the model period-by-period for given sequences of exogenous variable values and given initial conditions, the model generating its own lagged endogenous variables as the solution proceeds. The long run is deemed to be attained once the dynamic multipliers have settled down to constant values; highly unstable behaviour is readily identifiable.

Such simulation exercises may be conducted over a historical period or a forecast period, with the exogenous variable values comprising an observed database or a projected database. If a historical base in chosen, then it is common practice to impose residual adjustments to ensure that the model solution coincides with the actual data, giving a "perfect tracking" solution. Perturbed solutions are then obtained subject to the same residual adjustments, thus multipliers are obtained as deviations from actual data. Running simulations over a period whose economic and political history is known might be felt to aid interpretation. However a practical difficulty in studying long-run properties is simply that a sufficiently long database over which the complete model can be solved may not be available, due to regime changes and structural breaks or, at a more mundane level, inconsistencies and other vagaries in official statistics. Also the historical record usually has greater variance than a forecast projection, which may hinder the study of the model's dynamics. A forecast base avoids this difficulty by projecting relatively smooth exogenous variable trajectories and expecting relatively smooth endogenous variable solution paths, once short-run dynamic effects work themselves out. In principle, a forecast database can be continued into the indefinite future, or at least sufficiently far to allow an assessment of the model's stability and of its convergence or otherwise in the absence of shocks. In practice statistical models are local approximations, valid within the range of sample experience, so there may be problems in setting up an extended forecast database that yields an acceptable base solution, since the model is being driven far outside that experience. We return to these problems in the next section.

The alternative direct solution approach is to specify a long-run version of the model, equation by equation, and then solve this using standard nonlinear techniques to obtain a long-run solution and associated multipliers. The dynamics of individual structural equations are collapsed as discussed in Section 2 to give a corresponding steady-state relation. Converting the model to steady state form in this way increases the degree of dependence among the endogenous variables, for what might originally be a lagged relationship between endogenous variables becomes a "contemporaneous" one in the steady-state version. Thus simplifications of the solution procedure that result from a block recursive structure of the dynamic model are likely to disappear in the steady state version. Explicit expectations variables can be accommodated by appropriate definitions of the steady state, but implicit expectations mechanisms embedded in distributed lags are ignored by this procedure. Questions concerning the treatment of identities, stock-flow relations, and so forth, also depend on the definition of an appropriate steady state, to which we now turn.

4 Specifying the Steady State

Whichever method is used to elicit the long-run or steady-state properties of a model, a number of matters on which the underlying dynamic model may be relatively silent need to be resolved before the exercise can proceed. First, there is the nature of the steady state itself. To define a completely static long-run equilibrium presents difficulties for the typical dynamic model, which in particular contains various stock-flow relationships represented by identities or near-identities, together with various behavioural relationships whose dynamic specification reflects sample experience and so may not conform to the classical equilibrium paradigm. Thus a long-run concept that is more appropriate technically and more realistic is that of a steady-state growth path, exhibiting constant growth in real variables, constant inflation and a constant unemployment rate.

Secondly, the more detailed specification of the steady state may require conditions that ensure that economic magnitudes remain within their admissible ranges, for example various homogeneity and neutrality properties, which may or may not be reflected in model formulation and estimation. In some cases a model-builder may pay much attention to such requirements and ensure that the appropriate equation satisfies the relevant restrictions, whereas in other cases the model specification may be dependent more on statistical evidence and less on economic theory, so that such restrictions are not satisfied. This would become apparent once one attempted to elucidate steady-state properties by either approach, which would be of interest in itself and may require model respecification. For example, to rely solely on statistical evidence, ignoring theoretical restrictions, may result in an unstable model. Then the long-run response of the model to given shocks is not uniquely defined, except perhaps in terms of ever-accelerating or decelerating growth. However this cannot endure forever without other, perhaps implicit, mechanisms coming into play – ceilings, floors, and so forth – and in this sense such a model has an inadequate steady-state representation, even though it may adequately capture short-term developments.

In the extended simulation approach, it is of course an empirical question whether the model settles onto a steady-state growth path. In this connection the construction of the extended forecast database requires a certain amount of care. The exogenous variables in models of national economies usually fall into three main groups: variables describing the economic environment in the rest of the world, domestic economic policy variables, and various "external trends" including demographic factors, technical progress and natural resources. These must all be projected forward at chosen growth rates in an internally consistent manner, obeying relevant identities and restrictions on the range of variables. In a relatively open model, with a relatively large number of exogenous variables, it is necessary to ensure that any implicit relations among these variables are satisfied in projected data; such relations are satisfied automatically, in principle, in historical data and so they are not explicitly included in a model's specification. Assumptions about the prevailing fiscal and monetary policy stance are required, noting theoretical results about potential intrinsic dynamic instability under bond finance.

In a relatively closed model, some variables treated as exogenous in the previous case are now explicitly modelled as endogenous variables. For example, in the policy area, public sector reaction functions and other automatic rules might be specified. While reaction functions are frequently ill-determined empirically and unstable, they are often used to achieve macroeconomic closure of a model for simulation purposes, and may have an important bearing on the model's stability. At a minimum, adding equations and endogenizing variables simply serves to make explicit the relations among variables referred to in the previous paragraph.

In the direct solution approach, an exogenous variable dataset at a single point in time is all that is required. An actual dataset relating to a recent past period will often be convenient, and also ensure internal consistency, as noted above. The resulting solution is not of great interest in itself, for example it does not generate growth estimates, these being imposed in the conversion of structural equations to steady-state form; again, it is important to ensure that these are internally consistent. Features of interest that might emerge include the steady-state unemployment rate and other "great ratios". In order to extract further features of the steady-state model, it is necessary to conduct simulation experiments as in standard analysis of nonlinear macroeconomic models, noting however that in this case only a single-period calculation is needed.

Finally, we note the potential distinction between the long-run or steady-state properties of a macroeconometric model and the concept of the long run in economic theory. Both represent a state in which all adjustments are complete, but the extent to which all factors are adjustable in the model may be less than complete, particularly when dealing with short-term models used regularly for forecasting. In a quarterly forecasting model a variable that is difficult to model, such as the exchange rate, may be treated as exogenous and projected in accordance with the forecaster's judgement, whereas in long-run analysis the complete interaction of exchange rates, interest rates and asset holdings is important. Factor prices have only recently begun to play an important role in short-term models, whereas factor mobility and substitutability are emphasized in long-run analysis. Likewise attention has only recently shifted from cyclical variations in productivity growth around an autonomous trend, to trend productivity growth itself. Thus running a quarterly forecasting model unamended over an extended simulation period does not necessarily provide answers to the questions posed in economic analysis of the long run. To deal with these, various off-model calculations and additional assumptions may be required. For example, quarterly models often ignore the determination of full-capacity output, which may be of relatively little importance in short-term analysis but which is fundamental to long-run change. In this case potential capacity constraints could be allowed to influence the model through a calculation utilizing evidence from other sources.

5 Application to the CUBS Model

In this section we apply the procedures discussed above to study the long-run properties of the City University Business School (CUBS) model of the UK economy. This is one of the models regularly deposited at the

ESRC Macroeconomic Modelling Bureau and included in the Bureau's annual reviews (WALLIS *et al.*, [1984, 1985, 1986]). It is the newest and smallest of the UK models, and was developed to emphasize supply-side considerations. It is an annual model, used regularly for medium-term projections. These features of the model make it a suitable test-bed for the present exercise.

The CUBS model differs from most models in the absence of an income-expenditure framework and in the emphasis on supply-side factors in the determination of output. The model distinguishes four factors of production: capital, labour, energy, and raw materials. Demands for these are based on profit maximization within a perfectly competitive framework, and therefore labour demand depends upon the real wage to employers, capital stock, and energy and materials prices. The labour market includes a labour supply schedule which depends on the real employee wage and population. In the long run the natural rate of unemployment is determined by the level of unemployment benefits, however the adjustment of the real wage to this long-run market-clearing position is very sluggish. The factor demand equations and the assumed production function determine the supply of total private sector output for given factor prices and output price. The supply curve is assumed to be vertical in the long run, but aggregate demand can have a short-run influence on the level of output. The exchange rate is influenced by relative prices and the difference between the UK oil balance and that of other industrialized countries. The real exchange rate balances the current account of the balance of payments. There is no explicit modelling of the monetary sector. Official foreign exchange reserves are assumed to be exogenous and a given public sector borrowing requirement (PSBR) is assumed to be financed by money (the M0 measure is used) or government debt.

An extended simulation database is first constructed; this comprises smooth trajectories for the exogenous variables over the period 1984-2039. This represents a state in which exogenous real variables grow at 3 per cent per annum, with 3 per cent inflation. Thus world trade, all world price variables, population, and real government expenditure increase by 3 per cent annually. The nominal world interest rate is then 6 per cent throughout, and tax rates remain constant. The time trend in the capital stock equation is continued linearly.

On solving the model over this period its instability is immediately apparent. Plots of GDP growth, inflation and unemployment presented in Figures 1-3 clearly demonstrate explosive behaviour. Study of the complete solution for all variables leads one to suspect the linkages between money, prices and the PSBR as the principal source of the model's instability. Given the highly condensed nature of the demand side of the model, prices are determined not through the interaction of demand and supply but in a quasi-reduced form equation, whose single equation long-run solution relates the price level to the level of output and the quantity of money; the elasticity with respect to the latter variable is 1.41. Moreover, in the relation describing the financing of the PSBR, money grows faster than bonds.

With respect to the price equation, other research (WREN-LEWIS, [1984]) has indicated that estimates of the influence of money are highly sensitive

FIGURE 1

CUBS Dynamic Model—Output Growth (% p.a.)

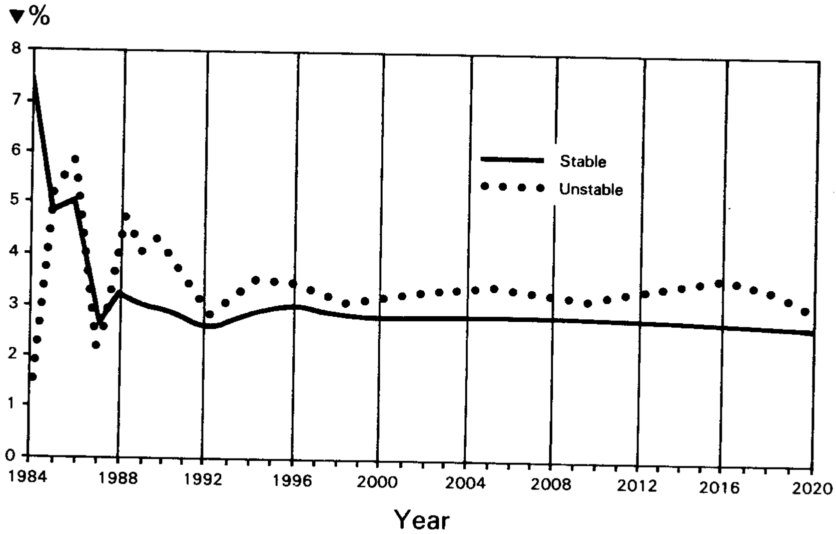


FIGURE 2

CUBS Dynamic Model—Inflation (% p.a.)

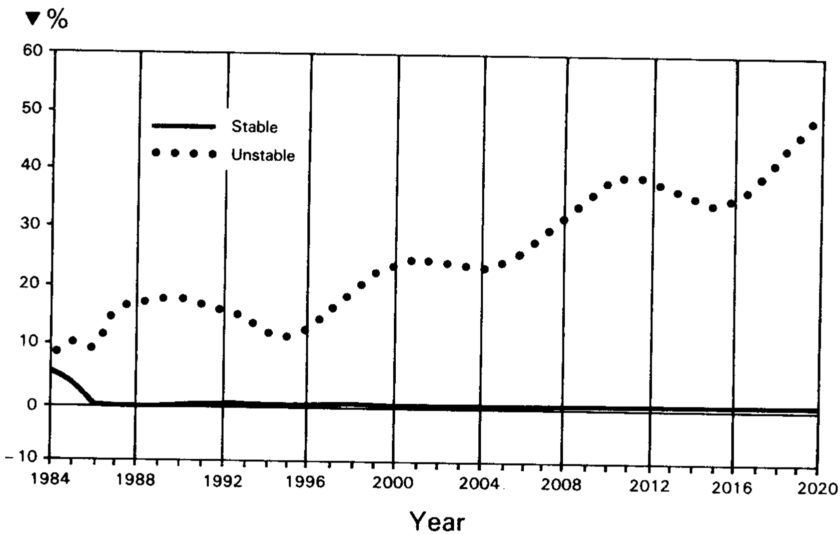
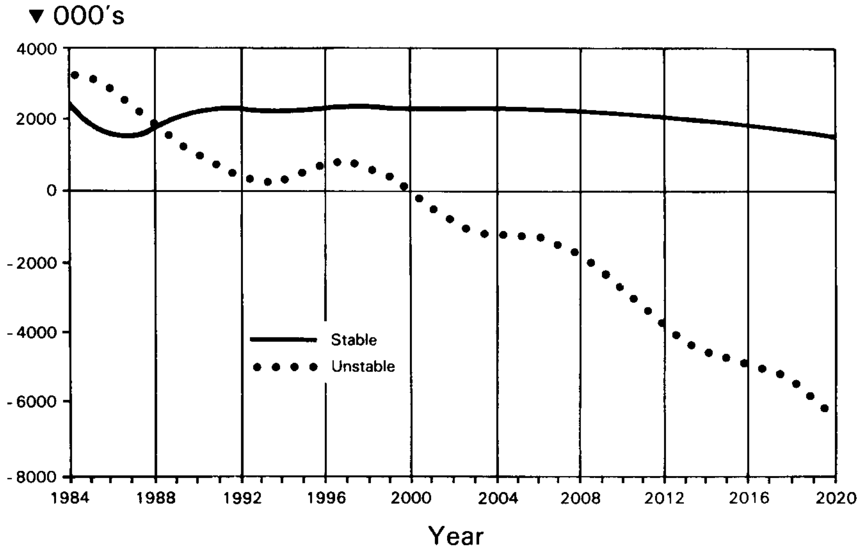


FIGURE 3

CUBS Dynamic Model—Unemployment



to the menu of other variables appearing in the equation, with world trade and exchange rates being of a special significance when included in the equation. Such variables are clearly admissible within the theorical framework described above, and their inclusion in the CUBS price equation substantially reduces the long-run monetary elasticity. For our present purposes, without embarking on a complete re-specification of this equation, we simply reduce this elasticity to one-half of its previous value. Likewise, we adjust the PSBR relation to give "balanced finance", in which money and bonds grow at the same rate. On solving the modified model over the extended simulation database a stable solution is obtained, as indicated by the alternative trajectories of GDP growth, inflation and unemployment also plotted in Figures 1-3.

On turning to the steady-state model other problems emerge. The steady-state model is computed by recoding the dynamic model in a form which eliminates lagged variables by expressing them in terms of current values of the level of the variable and its steady-state growth rate. The steady-state growth rates are chosen to represent a balanced growth path. However the resulting steady-state version of the CUBS model is not consistent with balanced growth. Accordingly, we have undertaken a limited amount of respecification in an attempt to overcome these various problems.

First, we note that the model includes a KLEM production function specifying a relationship between output and capital, labour, energy and raw material inputs, as follows

$$Y = F(K, L, E, M).$$

A production function of this form implies that the output variable should be defined as *gross* output, in contrast to the net output or value-added measure that is relevant in more traditional production functions dealing with only capital and labour inputs. However a net output data series is used in the CUBS model, and whereas no explicit treatment is given of raw materials, energy is included. The relative price terms in the production relation are also defined in terms of net output, implying that relative prices affect the capital/labour ratio. Although the model has constant returns to scale in respect of capital and labour, the presence of a term in energy demand results in increasing returns to scale overall. As a quick solution to these problems we delete the term in energy demand in the production function, and so ensure the possibility of balanced growth in (net) output, capital and labour.

Secondly, the specification of a balanced growth path requires a modification to the coefficient of the trend variable in the capital stock equation, which represents embodied technical progress. Since the factor demand equations for capital and labour are rearranged to give structural equations in which each depends on the other, in their reduced form this trend variable also influences labour demand. As labour supply depends on population growth and the real wage we need to consider these three equations jointly to determine the value of the trend coefficient that is consistent with balanced growth in labour and capital and hence output, given population growth of 1 per cent per annum. We accordingly alter the long-run trend coefficient from its estimated value implying 3.7 per cent growth per annum to a calculated value of 0.9. Since the only technical progress in the model is capital-augmenting, real wages fall on this balanced growth path.

Thirdly, we rewrite the price equation so that the growth rate of prices is identically equal to the rate of growth of money less the rate of growth of output. In local analysis of the steady state relationships between money, prices and the PSBR we calculate the corresponding coefficient on money in the model's price equation, namely 0.94, in place of an estimated coefficient of 1.41.

Finally, we consider the financing of the PSBR, and rewrite the relation between the PSBR and the growth of money and bonds so that both grow at the same rate, as above. This illustrates a difficulty in completely eliminating all dynamic relationships in a steady state model since the PSBR is related to the absolute change in the stock of money, but it is the proportionate change that influences the inflation rate.

The steady state version of the amended model is then solved from the 1984 values of the exogenous variables, with the following results:

output growth: 0.95% p.a.;

price inflation: 6.7% p.a.;

money growth: 7.7% p.a.;

unemployment rate: 13%;

ratio of PSBR/GDP: 3.9%.

A revised simulation database consistent with balanced growth is then constructed, and by solving the steady state model over a sequence of periods we verify that the steady state solution does in fact produce balanced growth. However the solution is base dependent for the reasons outlined above.

We then make the same changes to the dynamic model, namely eliminate the energy demand variable, alter the value of the trend coefficient in the capital stock equation, change the values of the coefficient on money in the price equation, and assume balanced finance of the PSBR. Running this model over the revised simulation database gives a stable solution, with similar output growth and inflation, but with a higher long-run level of the unemployment rate. Clearly, complete correspondence has not been achieved.

Having computed a solution for both the steady-state and adjusted dynamic models we illustrate their properties by means of simulation analysis. This consists of perturbing three exogenous variables in turn. The experiments are: a permanent reduction of 5 per cent in the standard rate of income tax (approximately 1 1/2 percentage points); a permanent increase of 5 per cent in the volume of government procurement spending; a 10 per cent reduction in the rate of unemployment benefit. The comparative long-run responses of output, inflation, unemployment and the PSBR-GDP ratio relative to the base unperturbed solution are shown in Table 1. The steady-state responses confirm the earlier descriptions of the model, with demand expansion having no long-run effect on output or unemployment. This is supported by the tax-rate and government spending simulations. The long-run or natural rate of unemployment can be shifted by the rate of unemployment benefit, however. The responses from the dynamic model are similar to those of the steady-state model, but are not identical. Thus there are still some differences to be resolved, and although some of these may be explained by a different base trajectory between the steady-state

TABLE 1

Responses of the Steady-State and Adjusted Dynamic Models (differences from base run)

	GDP level*	Inflation rate (% p.a.)	Unemployment rate (%)	PSBR-GDP ratio (%)
Reduction in standard rate of income tax (5%)				
steady-state model	0.08	0.57	0.01	0.28
dynamic model**	0.01	0.26	−0.10	0.18
Increase in government spending (5%)				
steady-state model	0.27	0.73	0.00	0.35
dynamic model**	−0.07	0.19	−0.11	0.15
Reduction in rate of unemployment benefit (10%)				
steady-state model	0.99	−4.02	−1.40	−1.97
dynamic model**	2.70	−0.35	−1.21	−3.22

* Difference expressed as a percentage of base solution.
** End period.

and dynamic models, a greater part is probably due to the dynamics of the equations. An illustration of this occurs in the unemployment benefit simulation where the dynamic model at first produces a positive response of real wages. This results from the inclusion of a term in the growth of money earnings, lagged, in the earnings equation, and elimination of this influence produces results which have the expected negative response of real wages to a benefit shock. For this simulation, the results in Table 1 refer to the dynamic model with this further modification. Other differences between the steady-state and dynamic models persist, most markedly in the responses of inflation.

The dynamic adjustment of output, inflation and unemployment are given in Figures 4-6. In general, a cyclical response is obtained for the tax rate and government spending simulations. This oscillatory adjustment is much weaker than in the original model. The unemployment response is also oscillatory in the benefit simulation. In this experiment there is an impact fall in inflation, as lower benefits reduce the PSBR and hence money growth. This effect is then partially offset by the subsequent impact on output in later periods. Unemployment takes a long time to reach its new long-run equilibrium, and this has only barely been achieved by the end of the solution period (over 50 years). This reveals the very sluggish adjustment of the labour market in the CUBS dynamic model The new long-run positions for the other two shocks are more rapidly achieved and could be approximated from a far shorter simulation period.

FIGURE 4

Responses in the Adjusted Dynamic Model—GDP
(per cent difference from base solution)

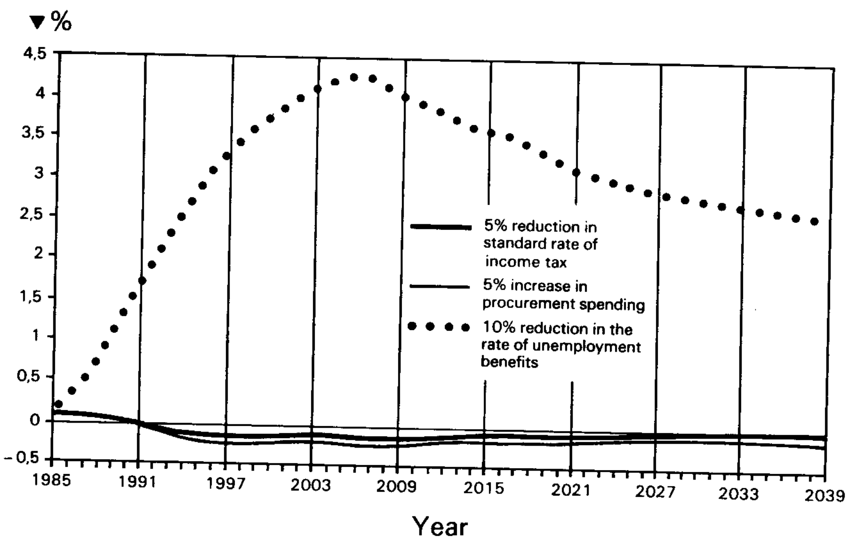


FIGURE 5

Responses in the Adjusted Dynamic model—Inflation
(difference from base solution)

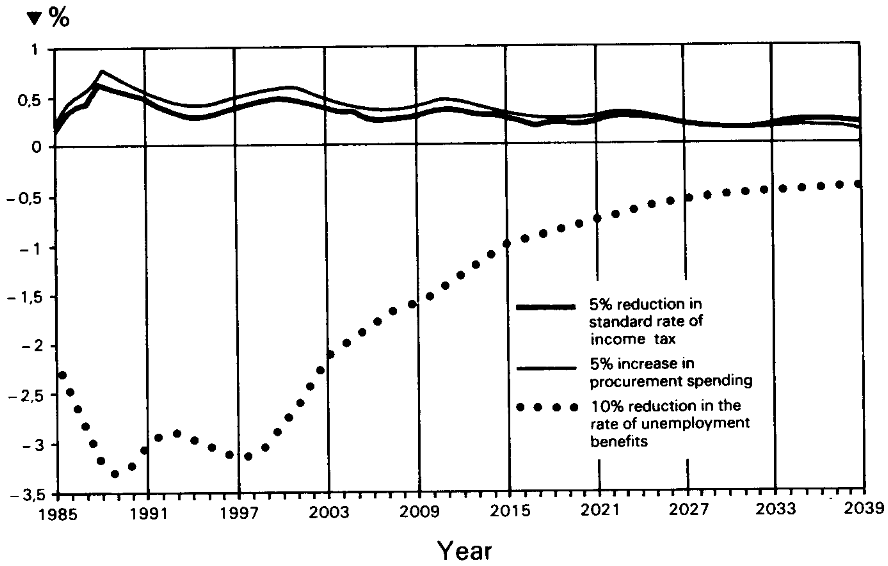
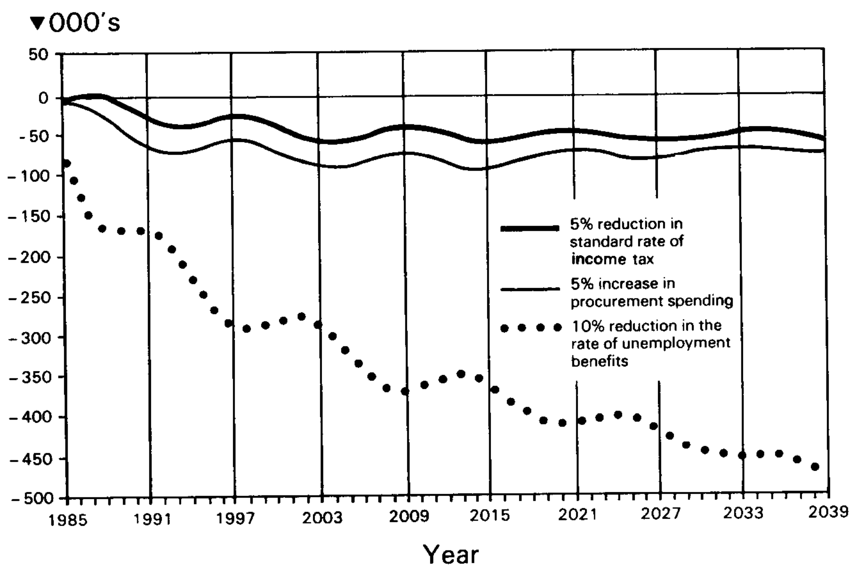


FIGURE 6

Responses in the Adjusted Dynamic Model—Unemployment
(difference from base solution)



6 Conclusion

In this paper we consider the construction and solution of the steady-state version of a dynamic nonlinear macroeconometric model, as a direct route to the elucidation of its long-run properties. This is seen to represent an attractive alternative to the dynamic simulation of the model over a period sufficiently long to allow short-run adjustments to be complete. A practical problem in dynamic simulation is to determine when a model has "settled down", and if its dynamic adjustment is slow a long simulation database may be required, taking the model far beyond its historical precedents and possibly introducing further difficulties. Direct evaluation of the long-run position avoids this.

Similar exercises have been reported by Malgrange [1983, 1985], Masson [1986] and Murphy *et al.* [1986], working with models for whose construction they themselves have been partly or wholly responsible. A distinguishing feature of our work is that we are not the proprietor of the model used as an example; rather, one interest has been to see whether the procedures considered can be reasonably advocated as evaluation devices to independent researchers interested in models and their properties. In the event a particular difficulty has been that the model studied is not only unstable but also lacks internally consistent steady state implications in its original form, despite having a more coherent theoretical structure than some other UK macro-models. Some modifications were therefore necessary, and rewriting the original model in its steady state form was of considerable assistance in determining the nature of its long-run properties, both actual and desired. While the example is specific, the required modifications arose in three particular areas of the model, and these are areas of difficulty in some other UK models, in our experience. These concern the specification of, first, financing rules and the long-run asset position, secondly, factor growth rates and the implied production function, and thirdly, the determinants of the long-run rate of inflation. An adequate treatment of each of these three areas is necessary before any contemporary national-economy model has a satisfactory long-run solution.

In the present exercise exact correspondence between the long-run properties of the modified dynamic model and its steady-state version was not achieved, given the limited amount of respecification that was undertaken. In this connection we endorse the view of Murphy *et al.* [1986] that if there is a strong emphasis on a model's long-run or steady-state properties, perhaps through a desire to use the model for medium-term policy analysis, then these should receive attention from the beginning of the model's construction. Statistical evidence alone does not often lead to a clear-cut choice among competing specifications, and the range of possibilities must be limited in some way. At the model-building stage, to consider only those specifications which survive econometric testing and which deliver sensible long-run implications seems a modest yet commendable requirement. This paper has shown how such long-run implications can be assessed.

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