ANNALES D'ÉCONOMIE ET DE STATISTIQUE. - N° 6/7 — 1987

The Limits of Counter-Cyclical Monetary Policy: An Analysis Based on Optimal Control Theory and Vector Autoregressions

Robert LITTERMAN *

ABSTRACT. – Optimal control theory can be combined with the probability structure of a vector autoregression to investigate the tradeoffs available to policy-makers. Such an approach obtains results based on a minimal set of assumptions about the economy and the structure of policy actions. This paper takes this approach to analyze the potential effectiveness of countercyclical monetary policy.

Les limites d'une politique monétaire contracyclique : une analyse fondée sur la théorie du contrôle optimal et les modèles auto-régressifs vectoriels

RÉSUMÉ. – La théorie du contrôle optimal peut être combinée avec la structure probabiliste d'un modèle autorégressif vectoriel pour examiner les choix offerts aux responsables de politique économique. Une telle approche obtient des résultats fondés sur des hypothèses minimales portant sur l'économie et la structure des politiques. Cet article utilise cette approche pour analyser l'efficacité potentielle d'une politique monétaire contracyclique

* R. LITTERMAN: Federal Reserve Bank of Minneapolis, MN 55480, USA. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The material contained is of a preliminary nature, is circulated to stimulate discussion, and is not to be quoted without permission of the author.

1 Introduction

It is a commonly stated and apparently widely held belief among economists that vector autoregressions (VARs) cannot be used for policy analysis[1]. At the same time, there are a few economists who defend their use, and a few of us who regularly use VARs in analyzing policy. In this paper I try to clarify some of the issues involved and illustrate an application of the use of a VAR and optimal control theory for formulating monetary policy.

The application asks the question, how much of the cyclical variance of output can we realistically expect to remove with optimal monetary policy. In answering this question we make several optimistic assumptions. We assume, for example, that we know the true reduced from representation of the economy and the true impact of monetary actions on the economy. Further, we assume that we can vary the monetary police rule, and these structures will not change. Of course, it is unrealistic to assume away uncertainty and the possibility of structural change, but even under these ideal conditions it turns out that there are important limitations on the extent to which optimal monetary policy can be expected to reduce the cyclical variance of output and prices. We find that having a time horizon greater than one year is important if counter-cyclical policy is to be effective. In particular, for inflation, where the lags of policy are especially long, very little of the cyclical variance can be removed if a one-year time horizon is considered.

2 Objections to the Use of VARs

There are two basic objections to the use of VARs for policy analysis. The first objection, commonly called the "Lucas critique", (see Lucas [1976]) is that changes in policy will affect the behavior of agents in the economy which will then cause the structure of the economy to change. This criticism was originally directed at standard exercices in policy analysis with structural models, but it clearly applies equally well to policy analysis with VARs. The second objection, which might be called the "correlation versus causation" issue (see Cooley and LeRoy [forthcoming]), is based on the observation that VARs do no more than capture the autocorrelation structure of a vector time series. The argument is that since a VAR does not identify a causal structure, it therefore cannot be used to project the effect of a change in policy, which is part of the causal structure. Unlike the Lucas critique, this second objection is directed not at standard structural models, but only at reduced form models, such as VARs, which do not build in assumptions that identify economic structures.

The debate over these objections has been carried on for a number of years, but often at a level of abstraction that has prevented the main issues from being widely understood. The purpose of this paper is not to renew this debate, but rather to illustrate a simple, concrete example of the use of VARs for policy analysis so that the limitations and benefits of such an approach might be made more apparent. The plan of the paper is first to present a framework for optimal control exercises using VARs, and then to analyze some results based on a simple example.

3 A Framework for Optimal Control with VARs

We begin by assuming that the unknown structure of the economy can be represented by a time-invariant linear difference equation.

$$AY_t = BY_{t-1} + AP_t + A\,u_t \tag{1}$$

where Y_t is an $n \times 1$ vector describing the state of the economy (the first order representation is not restrictive since higher order lags of variables can be included in the state vector), the $n \times n$ matrices A and B describe the structure of the economy, P_t is an $n \times 1$ vector giving the contemporaneous effect on observables of a policy action, and u_t is an $n \times 1$ vector of serially uncorrelated, normally distributed economic shocks with the $n \times n$ variance matrix, Σ_u. We assume that A is invertible and is normalized to have ones along its diagonal.

The direction and magnitude of policy actions are given by the scalar, s_t. We assume that the effects of policy on observables are linearly related to the size of the policy action, that is:

$$P_t = L\,s_t \tag{2}$$

where the $n \times 1$ vector L is a set of weights which gives the contemporaneous impact of a normalized policy action on the observables.

We also assume that policy actions are determined by a structure that includes both a systematic feedback rule and a random component. In other words, policy actions are determined by the rule

$$s_t = FY_{t-1} + v_t. \tag{3}$$

where F is a $1 \times n$ vector which describes the predictable component of the policy action, and v_t is the random component. Let the variance of v_t be Σ_v.

1. For statements to this effect, see Sargent, Lucas and Sargent, Cooley and Leroy, Leamer, and McNees.

The authority that sets policy is assumed to have a loss function given by

$$\mathscr{L}_t = \lim_{T \to \infty} E_t \left\{ \sum_{j=0}^{T} \beta^j [(Y_{t+j} - Y^*_{t+j})' \Phi (Y_{t+j} - Y^*_{t+j}) + g s^2_{t+j}] \right\} \quad (4)$$

where β is a discount rate and the $n \times n$ matrix Φ weights deviations of Y_t from their desired value, Y^*_t.

Suppose the authority knows the form of its loss function and wishes to minimize its loss given known values for Φ, β and g. The authority knows the value of L, but not the economic structure given in A and B. What we show below is that if the structures in A and B remain invariant with respect to alternative values of F, then optimal control theory can be used with a VAR representation for Y to determine the feedback rule for policy actions, that is the vector F*, that minimizes the loss function.

Within the context of this framework the objections to this approach can be made specific. Those who base their objection on the Lucas critique argue that it violates dynamic economic theory to make the assumption that A, B, and L remain invariant with respect to changes in F. Those who object on the basis of the lack of a causal structure in VARs point out that the identification of L requires the specification of an economic structure which is not provided by the VAR.

I will later consider these issues in the context of the example presented below. Before proceeding further, however, it may be desirable to suggest in general terms why I do no not think these objections imply that policy analysis with vector auto-regressions is impossible. With respect to the Lucas critique, my response is that the invariance assumption for A, B and L may be a useful approximation when the change in F is small relative to previous experience. In that case, it may be reasonable to assume that the historical data contain a useful estimate of the how the economy will respond to policy interventions in the future. I think the adequacy of the approximation is the issue, and I don't know how one can address that issue outside the context of a particular application. Moreover, the usefulness of this approach obviously depends on what alternative approaches are available. A highly uncertain local approximation that might be produced by this approach may be useful if there are no better alternatives. I view it as an extreme position to regard the Lucas critique as a theoretical imperative that invalidates the use of VARs for policy analysis, rather than as an caution about the possible inadequacy of an invariance assumption, the relevance of which is context dependent.

Those who object that a VAR does not identify L are correct, but again, that objection does not make the VAR approach invalid. What is required is that we make enough assumptions to identify L. These assumptions, which in effect define what is meant by a policy action, are the minimal assumptions about the structure of the economy which are necessary to conduct policy analysis. As will be shown below, it is not necessary to identify A and B in order to find the optimal policy. Indeed, the VAR approach is motivated in part by the observation that the attempt to identify A and B is not only unnecessary, but also is likely to lead to false restrictions that would bias the results of a policy analysis exercise.

Let us solve the optimal control problem presented above. A more complete version of this argument is given in LITTERMAN [1984]. We consider the class of economies

$$Y_t = DY_{t-1} + w_t \tag{5}$$

where

$$D = C + LG \tag{6}$$

C is the reduced form representation of the economy determined by the unknown structure A, B, L, and F, which is assumed to have generated the historical data. C is given by

$$C = A^{-1}B + LF. \tag{7}$$

The serially uncorrelated random error, w_t, is assumed to be drawn from a normal distribution with the variance matrix Σ_u. We have implicitly limited ourselves to deterministic feedback rules, but it is easy to show that in this context that limitation is not a binding constraint.

G is an arbitrary $n \times 1$ vector. Thus, the problem is to find the point in n-space, G, that generates an economy D which minimizes the expected value of the loss function, $\mathscr{L}_t$. The expectation is taken with respect to the distribution of the w's.

The interpretation of the vector G is that it represents the deviations of the optimal feedback rule from the historical rule, neither of which we identify. That is, G is $(F^* - F)$, where the vector $F^* = (F + G)$ is the optimal policy rule. In order to investigate the characteristics of the economy under the optimal policy it is sufficient to identify G. For the purpose of the discussion in this paper one can assume that F is known by the monetary policy authority (though not by the econometrician), and that therefore, determination of G by the econometrician is sufficient to allow the authority to implement the optimal policy F^*.

Now it should be clear exactly what is the nature of the identification problem. From equation (6) it can be seen that knowledge of C, the VAR representation of the historical data, and of L, the contemporaneous effect of a policy action, is what is needed to solve the optimization problem. It is not necessary to know A, B, or F.

The optimization problem presented here is a standard example of a well known problem, minimizing a discounted quadratic cost functional in a linear system. A useful reference is BERTSEKAS [1976], pages 266 through 268.

4 An Example: Optimal Monetary Policy

The example given here illustrates the application of the above approach to policy analysis to the specific problem of setting monetary policy. While the example is designed to be simple, I think it captures the basic tradeoff facing monetary policy, that is the problem of providing enough money to promote stable real growth in a context of stable prices.

The example includes quarterly data for six variables, real growth, inflation, money growth, a nominal interest rate, the value of the dollar and an index of stock prices. [2] The VAR representation, the matrix C, [3] includes a constant and four lags of each variable, and is estimated using a Bayesian prior following the procedure given in LITTERMAN [1986]. [4]

We identify the vector L by assuming that a monetary policy action has no contemporaneous (current quarter) impact on real growth or inflation and that for each 100 basis point (hundredth of a percent) increase in the nominal interest rate the policy action causes a 1.3 percent decrease in the annualized growth of money, a 2.5 percent decrease in stock prices, and a 3.0 percent increase in the value of the dollar. We normalize L by setting the coefficient on interest rates equal to 1 so that s_t has units given in percentage points. A policy action of size s_t is one that causes the nominal interest rate to move by s_t percentage points. The value of -1.3 for the effect on money growth represents a change in the quarterly average level of the money stock of about 2 billion dollars in the quarter in which the policy action is taken. This value is obtained by averaging the response in monthly data of money to an innovation in interest rates with no contemporaneous effect on money. Such evidence is presented in LITTERMAN [1984 *a*]. The coefficients on stock prices and the value of the dollar are obtained by looking at evidence from daily observations on these series as in LITTERMAN [1984 *a*].

We choose loss functions which trade off deviations of real growth from a desired rate against deviations of inflation from its desired rate. The tradeoff is parameterized so that we can investigate the effect of giving more or less weight to each component.

We investigate loss functions that specify real growth and inflation rates over one, two and three year intervals. Policy effectiveness depends in an important way on this choice of time horizon.

In addition to looking at the effects of changes in the time horizon, we include a cost associated with policy actions so that we can investigate more or less active policies. Penalizing the size of policy actions represents the desire of the monetary authority to minimize its intervention in markets. It also allows us to investigate optimal rules that are as close as desired to the current economic structure. Finally, we include a discount factor so that we can investigate the effect of discounting the future.

In all cases we take the desired real growth and inflation rates to be the average rates captured in the unconditional VAR estimation. These average rates are 2.2 for real growth and 6.1 for inflation. We thus investigate the potential of the optimal policy for reducing variance without affecting the mean growth rates.

In Table 1 we report the estimates of the VAR representation estimated with a Bayesian prior, the covariance/correlation matrix of the residuals, and the unconditional means for each variable. The standard errors of the coefficients and *t*-statistics are not reported because they reflect both prior and sample evidence in this Bayesian VAR estimation and therefore are not readily interpretable. We do report F-statistics for a test of the significance of each set of lags on each variable. Though these statistics suffer from the same problem, they give an indication of the relative importance of each variable in each equation.

The autoregressive representation is not readily interpretable. Usually the dynamics of such a system are described through a series of graphs showing the responses of each variable to a set of orthogonal shocks. In this exercise, however, the primary interest is on one particular response, the response to a policy action. We show this response for each variable in the system in Figures 1-4.

These responses show the pattern of deviations from a baseline forecast that we would expect following an unexpected more toward monetary tightening that raises nominal interest rates by 100 basis points. Real GNP growth drops one quarter after the action and reaches a maximum response two quarters later. The level of Real GNP reaches a maximum response after six quarters. The real growth during the year after the action is taken is about one percent lower because of the action. In the longer run the real growth response is a small rise above the baseline and a slow, damped return to zero. Inflation also falls immediately after the action is taken and reaches a minimum after six quarters. At that time the inflation rate is lowered by .75 of a percent because of the action. Money growth, which is lowered by 1.3 percent in the quarter in which the policy action is taken, returns within a year to its baseline rate.

---

2. The exact measures used are as follows: Real growth, inflation, money growth are defined as first differences of the logs of real GNP, the GNP deflator, and M1. The data is transformed to annual growth rates in this way in order to induce stationarity. We also use the log of the trade weighted Federal Reserve Board index of the value of the U.S. dollar, and the Standard and Poors index of 500 industrial stocks. The nominal interest rate is the yield on 3-month Treasury bills.

3. We solve the optimal control problem assuming that the VAR representation is known. Allowing for the uncertainty of the estimated structure is a much more difficult problem not only because the uncertainty of the structure enters the system multiplicatively, but also because there is a tradeoff between the indirect benefit of shocking the system in order to learn more about its structure and the direct cost of following such a strategy. Some suggested approaches to the solution of this proclem are given in CHOW [1975] and KENDRICK [1981].

4. The Bayesian prior pulls the coefficient estimates toward a random walk representation for each variable (first own lag equal to one, and other coefficients equal to zero). The details are given in DOAN and LITTERMAN [1984]. Following their notation, the prior has an own weight parameter of .2 and a weight on other variables of .5. The lag pattern is a harmonic decay with parameter 1. These "hyperparameters" of the model were chosen on the basis of an informal search designed to optimize out-of-sample forecast performance.

TABLE 1

Equation 1 : Dependent Variable Real GNP Growth (RGNP) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.34445566	RBAR**2	.34445566
SSR	1864.8297	SEE	3.5739063
Durbin-Watson	2.36021305		
Q(36)=31.1361		Significance level	.699058

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1	RGNP	1	.3363383	9	M1	1	.1333126	17	STOCKS	1	7.211450
2	RGNP	2	.2329282E−01	10	M1	2	.4879094E−01	18	STOCKS	2	−2.730200
3	RGNP	3	−.3129764E−01	11	M1	3	−.5521970E−02	19	STOCKS	3	−2.204311
4	RGNP	4	−.4316116E−01	12	M1	4	.2110345E−02	20	STOCKS	4	−.9385809
5	IPD	1	−.3034695E−01	13	TBILLS	1	−.4922771	21	DOLLAR	1	1.719194
6	IPD	2	−.6089598E−02	14	TBILLS	2	−.1080897	22	DOLLAR	2	−2.93693
7	IPD	3	−.3555909E−01	15	TBILLS	3	.2802809E−01	23	DOLLAR	3	−.7104212
8	IPD	4	−.1088508E−01	16	TBILLS	4	.3534038E−01	24	DOLLAR	4	−1.986572
								25	CONSTANT	0	17.27796

F-Tests, Variable	Dependent Variable F-Statistic	RGNP signif. level
RGNP	5.93310	.1881132E−03
IPD	.15517	.9604057
M1	.84338	.4998176
TBILLS	2.39287	.5327784E−01
STOCKS	1.69115	.1551302
DOLLAR	.35739	.8385600

TABLE 1 (continued)

Equation 2 : Dependent Variable Implicit Price Deflator (IPD) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.64709607	RBAR**2	.64709607
SSR	461.29074	SEE	1.7775055
Durbin-Watson	2.33769618		
Q(36) = 34.2871		Significance level	.550202

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1....	RGNP	1	.3920091 E−01	9....	M 1	1	.8118252 E−01	17....	STOCKS	1	−.8758160
2....	RGNP	2	.3116931 E−02	10....	M 1	2	−.3769615 E−02	18....	STOCKS	2	.2896292
3....	RGNP	3	−.4235584 E−02	11....	M 1	3	.1528281 E−02	19....	STOCKS	3	.6445502
4....	RGNP	4	.2320439 E−02	12....	M 1	4	.5148371 E−02	20....	STOCKS	4	.3047304
5....	IPD	1	.4607348	13....	TBILLS	1	.1299975	21....	DOLLAR	1	−4.159099
6....	IPD	2	.7408939 E−01	14....	TBILLS	2	−.3240601 E−01	22....	DOLLAR	2	−1.155149
7....	IPD	3	.2309942 E−01	15....	TBILLS	3	.5467820 E−02	23....	DOLLAR	3	−1.027046
8...	IPD	4	−.5353382 E−01	16....	TBILLS	4	−.2207229 E−01	24....	DOLLAR	4	−.4922653
								25....	CONSTANT	0	31.91498

F-Tests, Variable	Dependent Variable F-Statistic	IPD signif. level
RGNP	.47591	.7533641
IPD	15.32177	.1744920 E−09
M 1	.93843	.4435563
TBILLS	.36010	.8366699
STOCKS	.37234	.8280710
DOLLAR	3.57469	.8195817 E−02

TABLE 1 (continued)

Equation 3 : Dependent Variable Money Supply (M 1) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.49248379	RBAR**2	.49248379
SSR	868.73141	SEE	2.4393063
Durbin-Watson	2.22358652		
Q(36) = 41.6196		Significance level	.239366

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1....	RGNP	1	.5190508 E−02	9....	M 1	1	.4229691	17....	STOCKS	1	2.986856
2....	RGNP	2	−.4683546 E−02	10....	M 1	2	.4371091 E−01	18....	STOCKS	2	−1.1276652
3....	RGNP	3	−.7685761 E−02	11....	M 1	3	.3619745 E−01	19....	STOCKS	3	−.6696003
4....	RGNP	4	−.9600258 E−03	12....	M 1	4	−.1571967 E−01	20....	STOCKS	4	.1379141
5...	IPD	1	−.1963900 E−01	13....	TBILLS	1	−.6056950	21....	DOLLAR	1	−.4588966
6....	IPD	2	.3993841 E−01	14....	TBILLS	2	.4144401	22....	DOLLAR	2	.5576186
7....	IPD	3	−.3192034 E−02	15....	TBILLS	3	.1267585	23....	DOLLAR	3	−1.400561
8....	IPD	4	.5289472 E−02	16....	TBILLS	4	.5932146 E−01	24....	DOLLAR	4	−1.131284
								25....	CONSTANT	0	8.284627

F-Tests, Variable	Dependent Variable F-Statistic	M 1 signif. level
RGNP	.05376	.9945535
IPD	.15196	.9618658
M 1	9.54389	.6974184 E−06
TBILLS	4.41030	.2145377 E−02
STOCKS	1.39272	.2393410
DOLLAR	.29201	.8827952

TABLE 1 (continued)

Equation 4 : Dependent Variable Treasury Bill Rate (TBILLS) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.95731756	RBAR**2	.95731756
SSR	66.889701	SEE	.67686678
Durbin-Watson	1.84789562		
Q(36) = 65.0616		Significance level	.212229 E−02

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1	RGNP	1	.1021421 E−01	9	M 1	1	.4737325 E−01	17	STOCKS	1	.9924165
2	RGNP	2	.2638067 E−02	10	M 1	2	−.5876346 E−03	18	STOCKS	2	.3002318 E−01
3	RGNP	3	.8152543 E−04	11	M 1	3	.5037371 E−02	19	STOCKS	3	−.2600745
4	RGNP	4	.2208652 E−02	12	M 1	4	−.1806986 E−02	20	STOCKS	4	−.2839007
5	IPD	1	−.9271070 E−02	13	TBILLS	1	.9084809	21	DOLLAR	1	−3.197403
6	IPD	2	.1558330 E−01	14	TBILLS	2	−.1054179	22	DOLLAR	2	.3328123
7	IPD	3	−.3722847 E−02	15	TBILLS	3	.7457497 E−01	23	DOLLAR	3	.3220292
8	IPD	4	−.5819143 E−02	16	TBILLS	4	−.2067664 E−01	24	DOLLAR	4	−.2714451
								25	CONSTANT	0	11.74919

F-Tests, Variable	Dependent Variable F-Statistic	TBILLS signif. level
RGNP	.31486	.8676916
IPD	.42880	.7876543
M 1	2.20028	.7179414 E−01
TBILLS	152.45330	.0000000
STOCKS	2.57856	.3986107 E−01
DOLLAR	5.14467	.6611939 E−03

TABLE 1 (continued)

Equation 5 : Dependent Variable S & P 500 Stock Price Index (STOCKS) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.99346015	RBAR**2	.99346015
SSR	.35544486	SEE	.49341198 E−01
Durbin-Watson	1.69699070		
Q(36) = 58.5844		Significance level	.100797 E−01

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1....	RGNP	1	−.4812407 E−03	9....	M1	1	.1497440 E−03	17....	STOCKS	1	1.194142
2....	RGNP	2	−.3354812 E−03	10....	M1	2	.2628849 E−04	18....	STOCKS	2	−.1775374
3....	RGNP	3	−.2131227 E−03	11....	M1	3	.5681882 E−04	19....	STOCKS	3	−.3389712 E−01
4....	RGNP	4	−.2067172 E−03	12....	M1	4	−.6849121 E−04	20....	STOCKS	4	.3621308 E−02
5....	IPD	1	−.3277939 E−03	13....	TBILLS	1	−.6671451 E−02	21....	DOLLAR	1	.4286672 E−01
6....	IPD	2	−.5080301 E−04	14....	TBILLS	2	.3652284 E−02	22....	DOLLAR	2	−.2834495 E−01
7....	IPD	3	−.3683497 E−03	15....	TBILLS	3	.1895069 E−02	23....	DOLLAR	3	−.1612829 E−01
8....	IPD	4	.1661646 E−04	16....	TBILLS	4	.1185347 E−02	24....	DOLLAR	4	.7096639 E−02
								25....	CONSTANT	0	.5165858 E−01

F-Tests, Variable	Dependent Variable F-Statistic	STOCKS signif. level
RGNP	.42462	.7906823
IPD	.07520	.9896485
M1	.01320	.9996528
TBILLS	1.36178	.2500844
STOCKS	1620.59769	.0000000
DOLLAR	.06595	.9919396

TABLE 1 (continued)

Equation 6 : Dependent Variable Value of the Trade Weighted U.S. Dollar (DOLLAR) from 49-2 until 85-4

Observations	147	Degrees of freedom	146
R**2	.96117850	RBAR**2	.96117850
SSR	.67966313 E−01	SEE	.21575975 E−01
Durbin-Watson	1.70425695		
Q(36)=44.5299		Significance level	.155642

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1	RGNP	1	.8754136 E−03	9	M 1	1	.7430616 E−03	17	STOCKS	1	.2024814 E−01
2	RGNP	2	−.4499657 E−04	10	M 1	2	−.2580760 E−03	18	STOCKS	2	−.1609983 E−01
3	RGNP	3	−.1001253 E−03	11	M 1	3	−.2195908 E−04	19	STOCKS	3	−.1031753 E−01
4	RGNP	4	−.4764124 E−05	12	M 1	4	−.1199677 E−03	20	STOCKS	4	−.6423230 E−02
5	IPD	1	−.1395409 E−02	13	TBILLS	1	.3476650 E−02	21	DOLLAR	1	1.130537
6	IPD	2	−.1284121 E−04	14	TBILLS	2	.4336186 E−03	22	DOLLAR	2	−.1402405
7	IPD	3	−.8798746 E−04	15	TBILLS	3	.4350571 E−04	23	DOLLAR	3	−.1654932 E−01
8	IPD	4	−.3697088 E−04	16	TBILLS	4	.4587452 E−03	24	DOLLAR	4	−.1640739 E−02
								25	CONSTANT	0	.1725652

F-Tests, Variable	Dependent Variable F-Statistic	DOLLAR signif. level
RGNP	1.52370	.1982814
IPD	1.14949	.3357393
M 1	.83568	.5045811
TBILLS	3.56665	.8302011 E−02
STOCKS	1.86922	.1189144
DOLLAR	597.73106	.0000000

TABLE 1 (continued)

Covariance/Correlation Matrix
(Correlations are given above the diagonal)

Variable	RGNP	IPD	M1	TBILLS	STOCKS	DOLLAR
RGNP	12.6860	.0042	.24608	.13760	.19000	.08614
IPD	.0267	3.1380	.07211	−.06270	.01469	−.00579
M1	2.1307	.3105	5.90970	.01173	.36637	−.04899
TBILLS	.3306	−.0749	.01923	.45503	−.15800	.22863
STOCKS	.0332	.0013	.04379	−.00524	.00242	−.10864
DOLLAR	.0066	−.0002	−.00256	.00331	−.00011	.00046

Steady State Mean Values

Real GNP growth (annual rate)	2.17 percent
GNP deflator growth (annual rate)	6.13 percent
M1growth (annual rate)	7.55 percent
Treasury Bill Rate (level)	9.59 percent
S and P Stock Price Index (index level)	197.69
Value of the U.S. Dollar (index level)	112.82

FIGURE 1

Responses to Monetary Policy Action

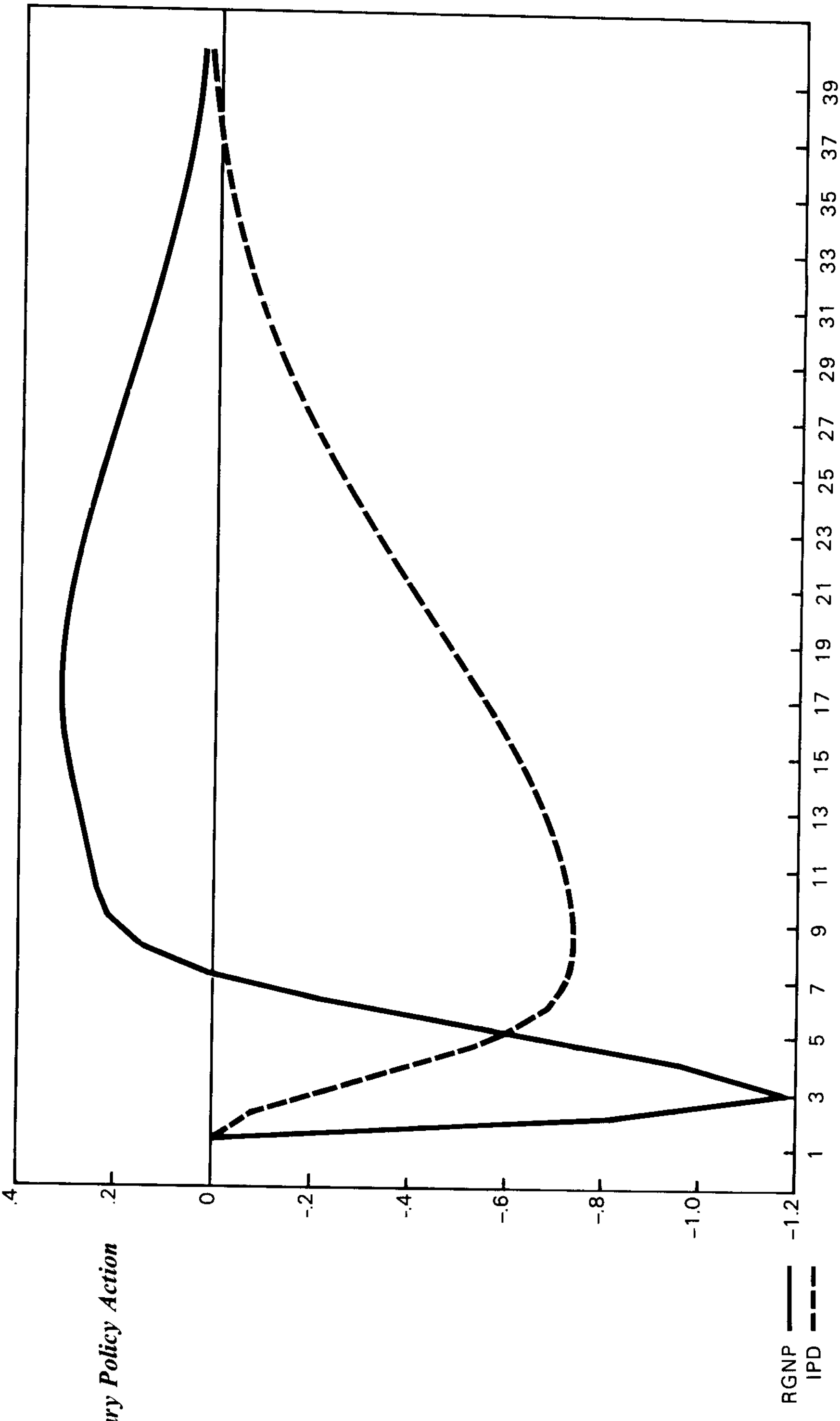


FIGURE 2

Responses to Monetary Policy Action

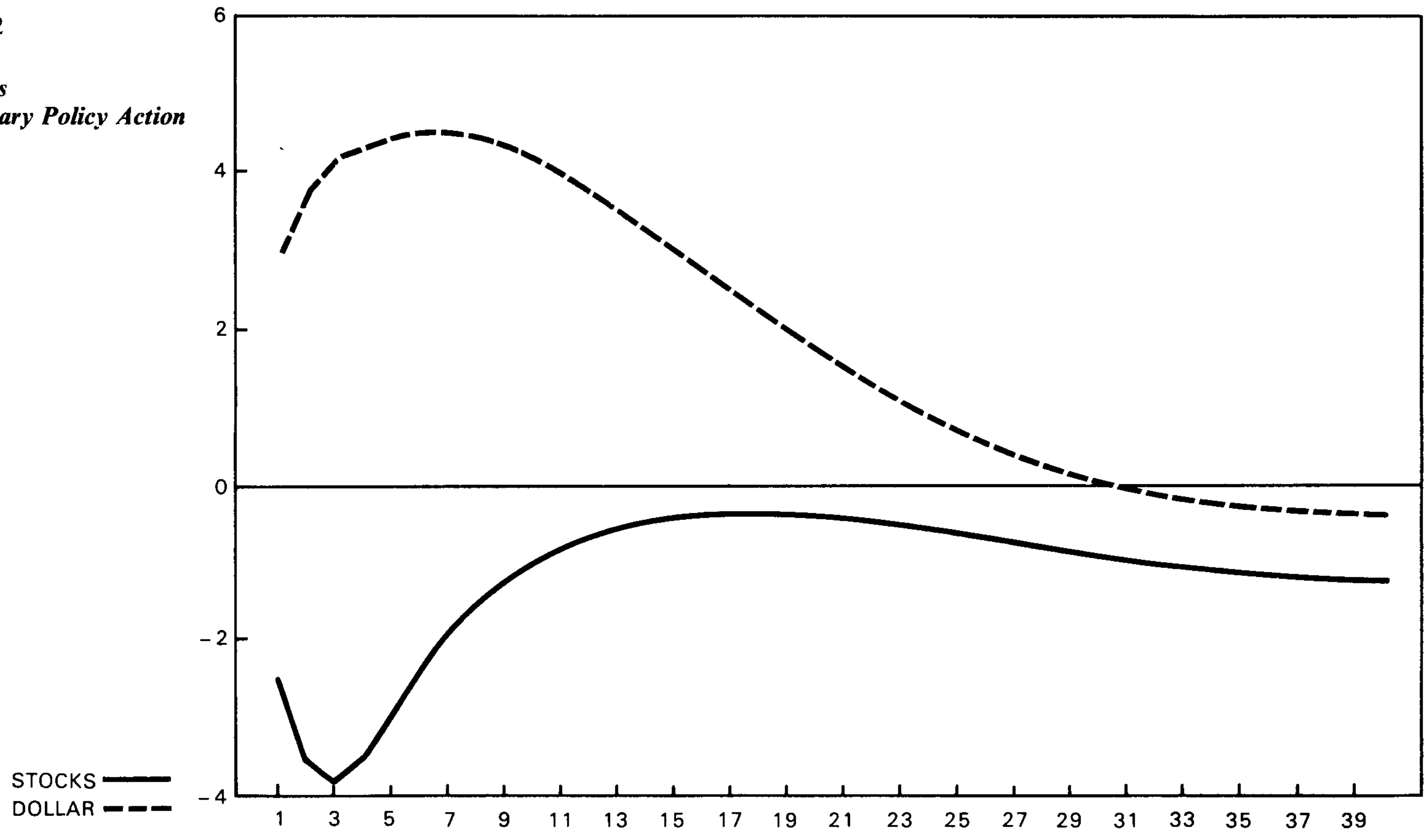


FIGURE 3

Responses to Monetary Policy Action

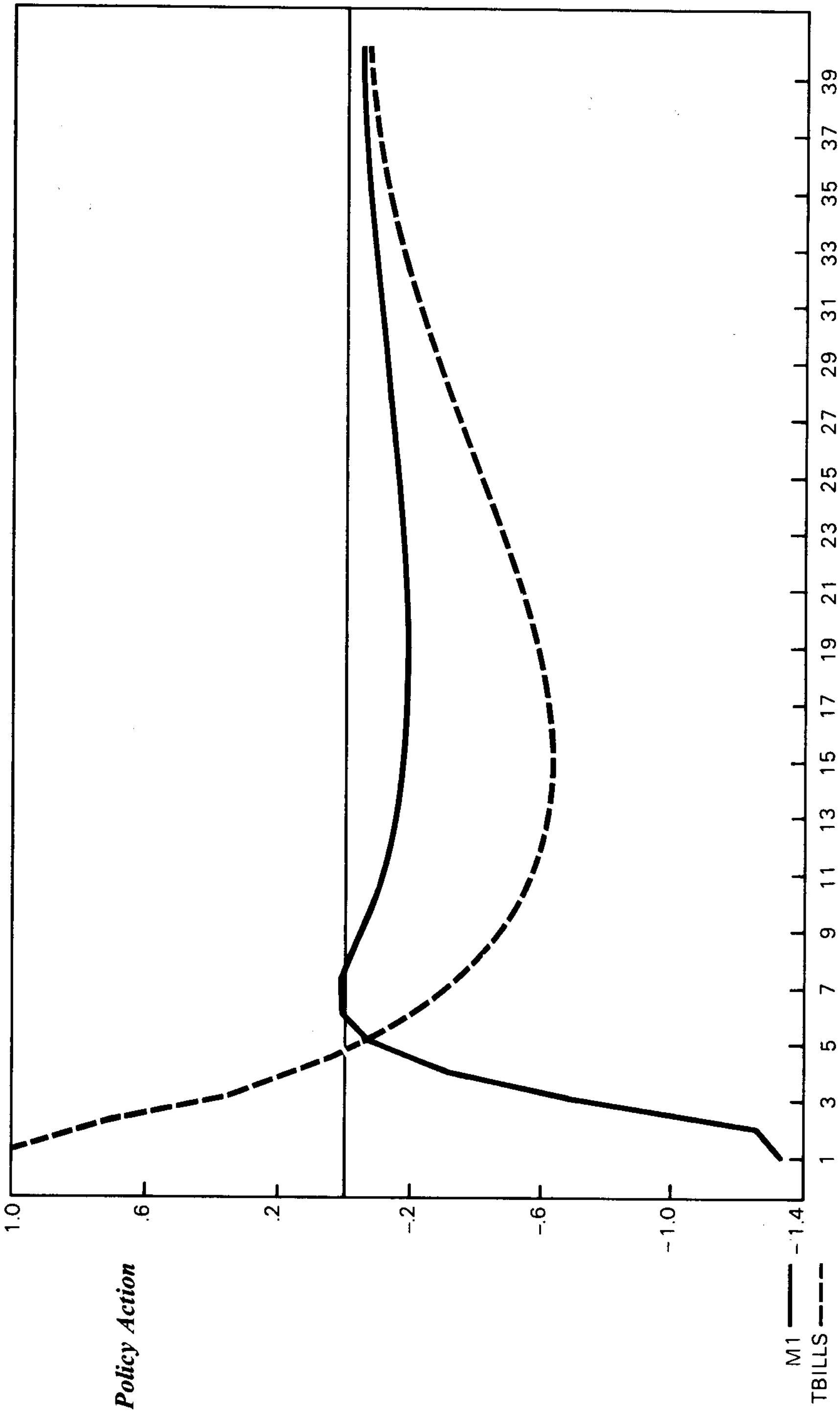


FIGURE 4

Responses to Monetary Policy Action

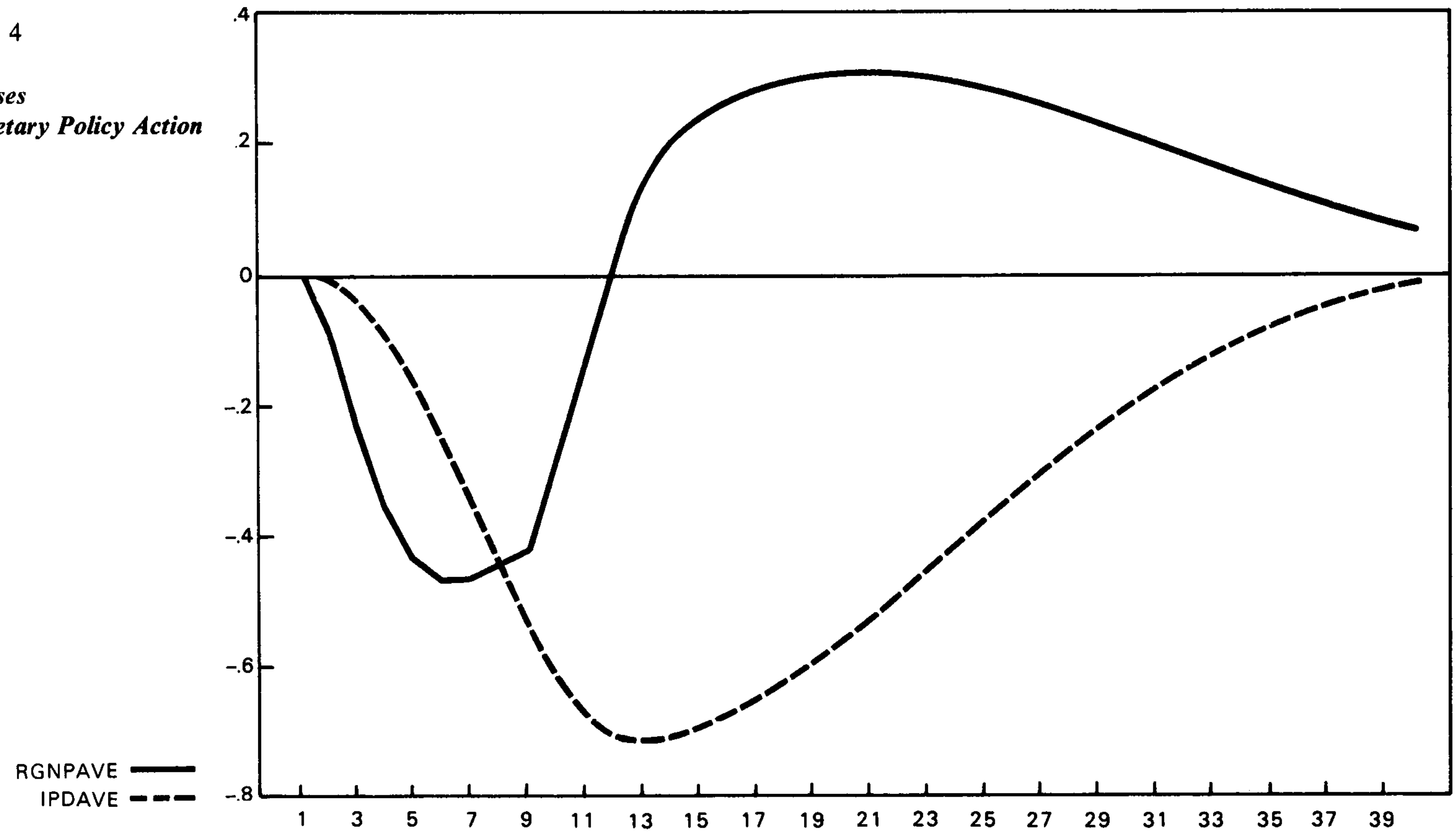
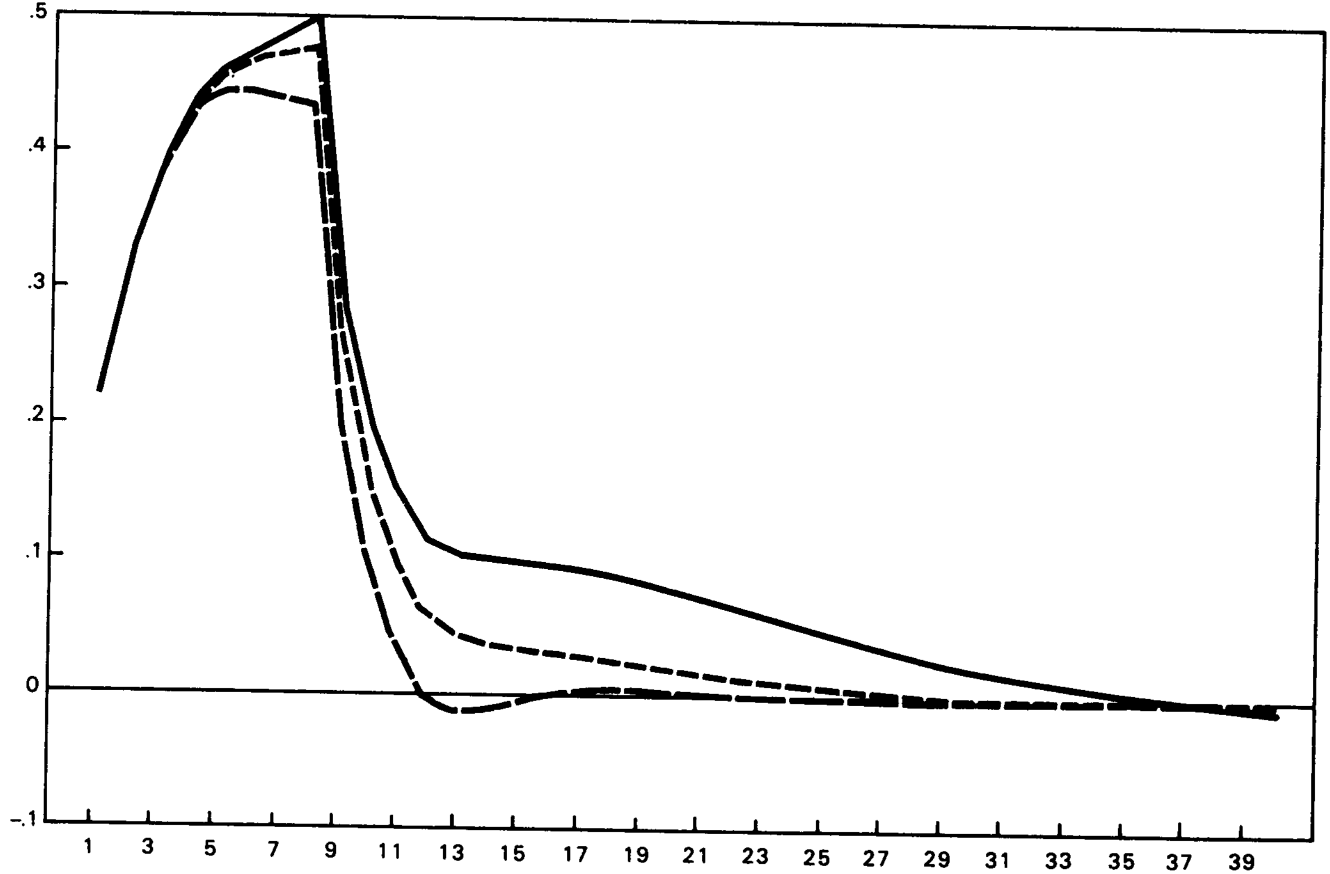


FIGURE 5*a*

Responses of IPDAVE to IPD Shock



Current Policy
Less Activist Response
More Activist Response

FIGURE 5b

Responses of RGNPAVE to IPD Shock

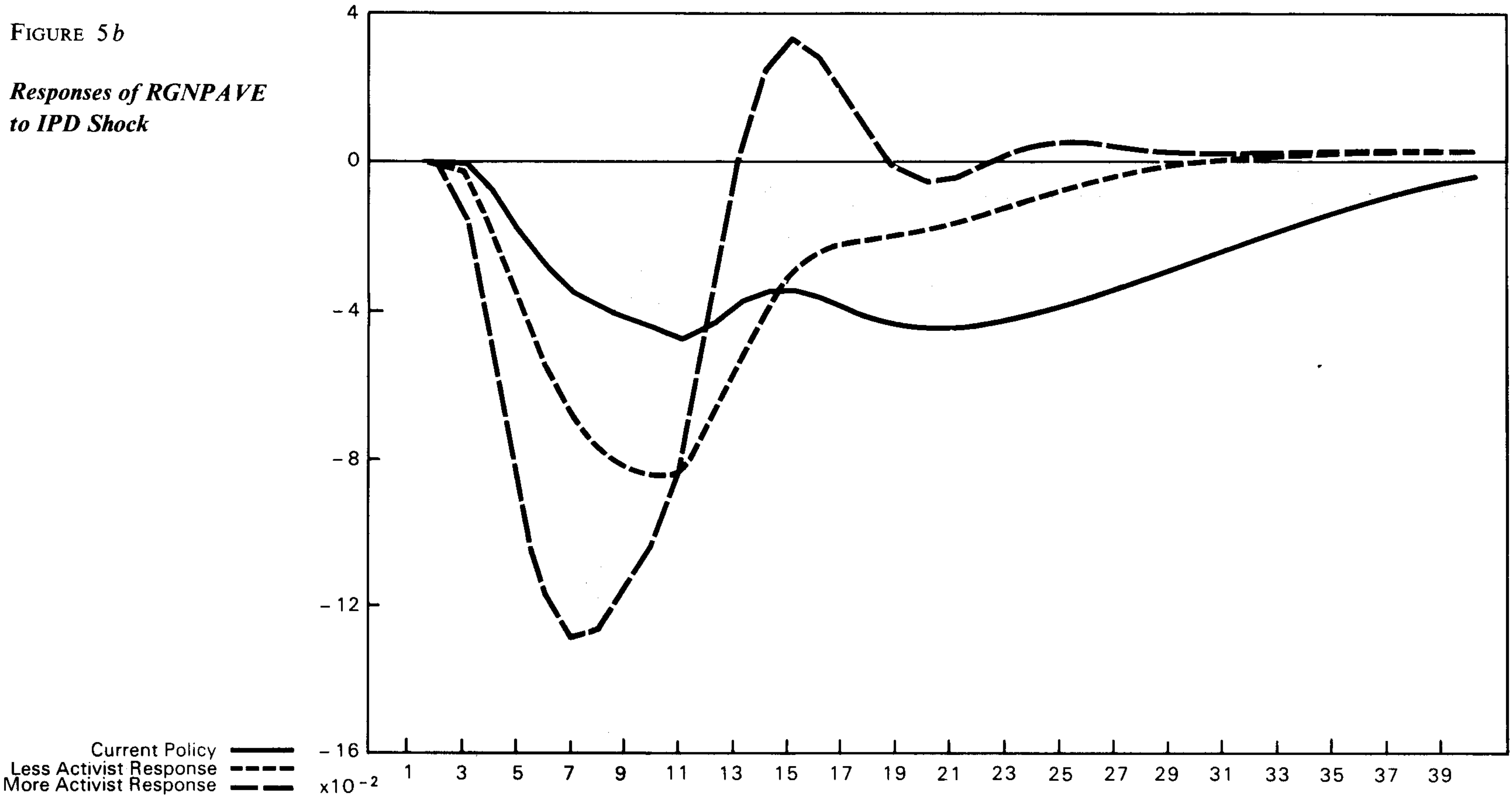


FIGURE 5*c*

Responses of IPD to IPD Shock

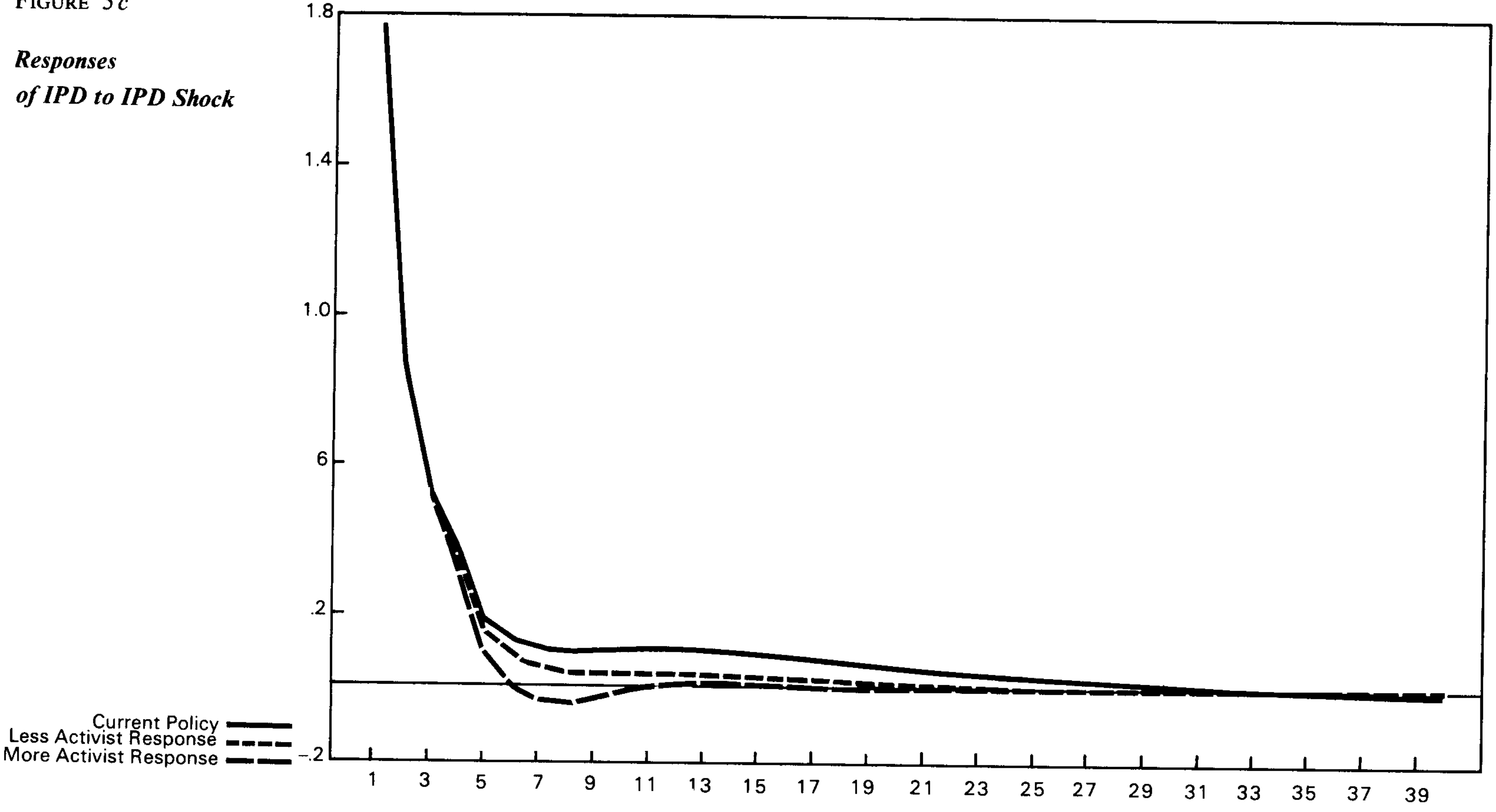


FIGURE 5*d*

Responses of RGNP to IPD Shock

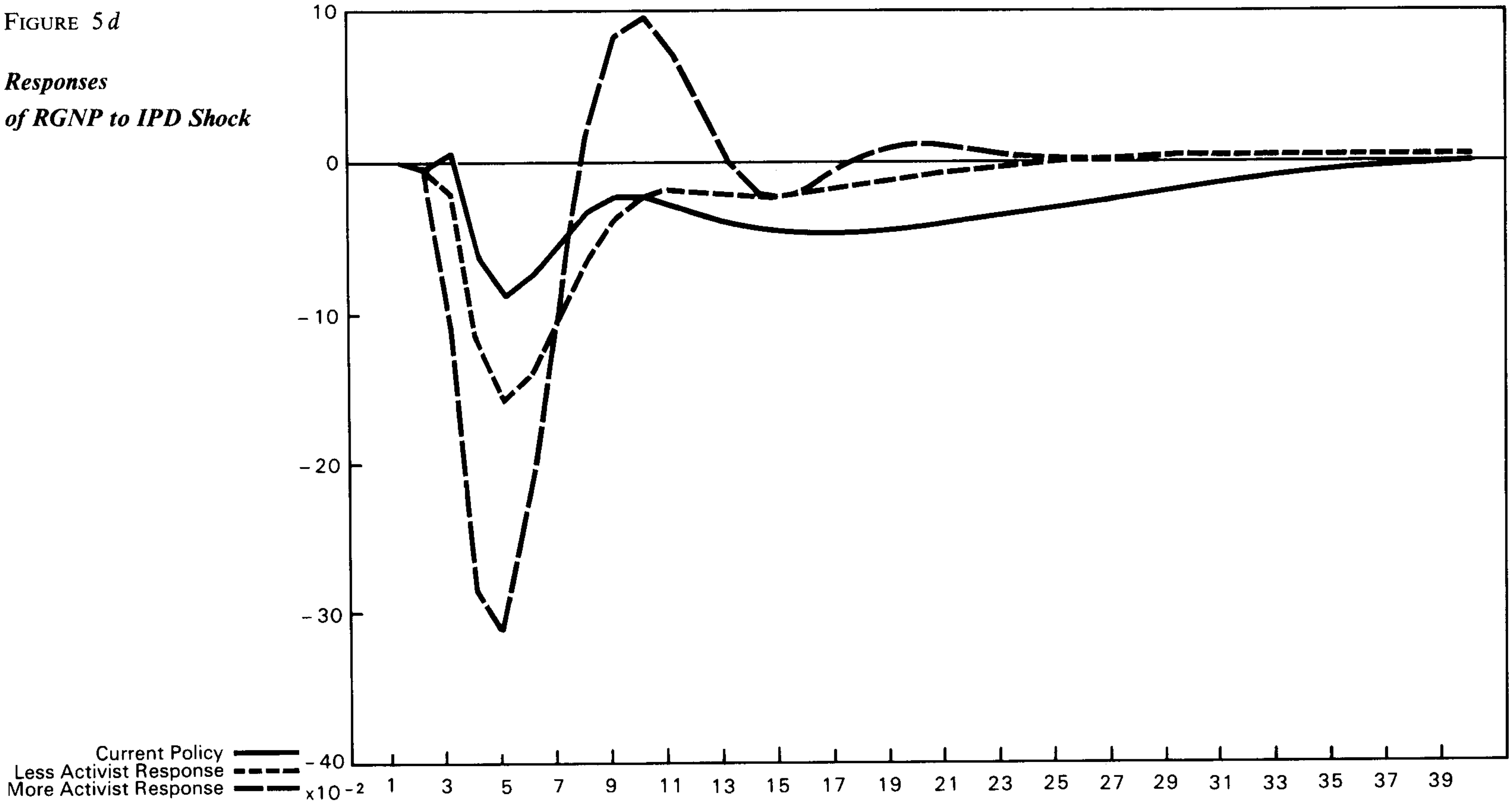


FIGURE 5e

Responses of Dollar to IPD Shock

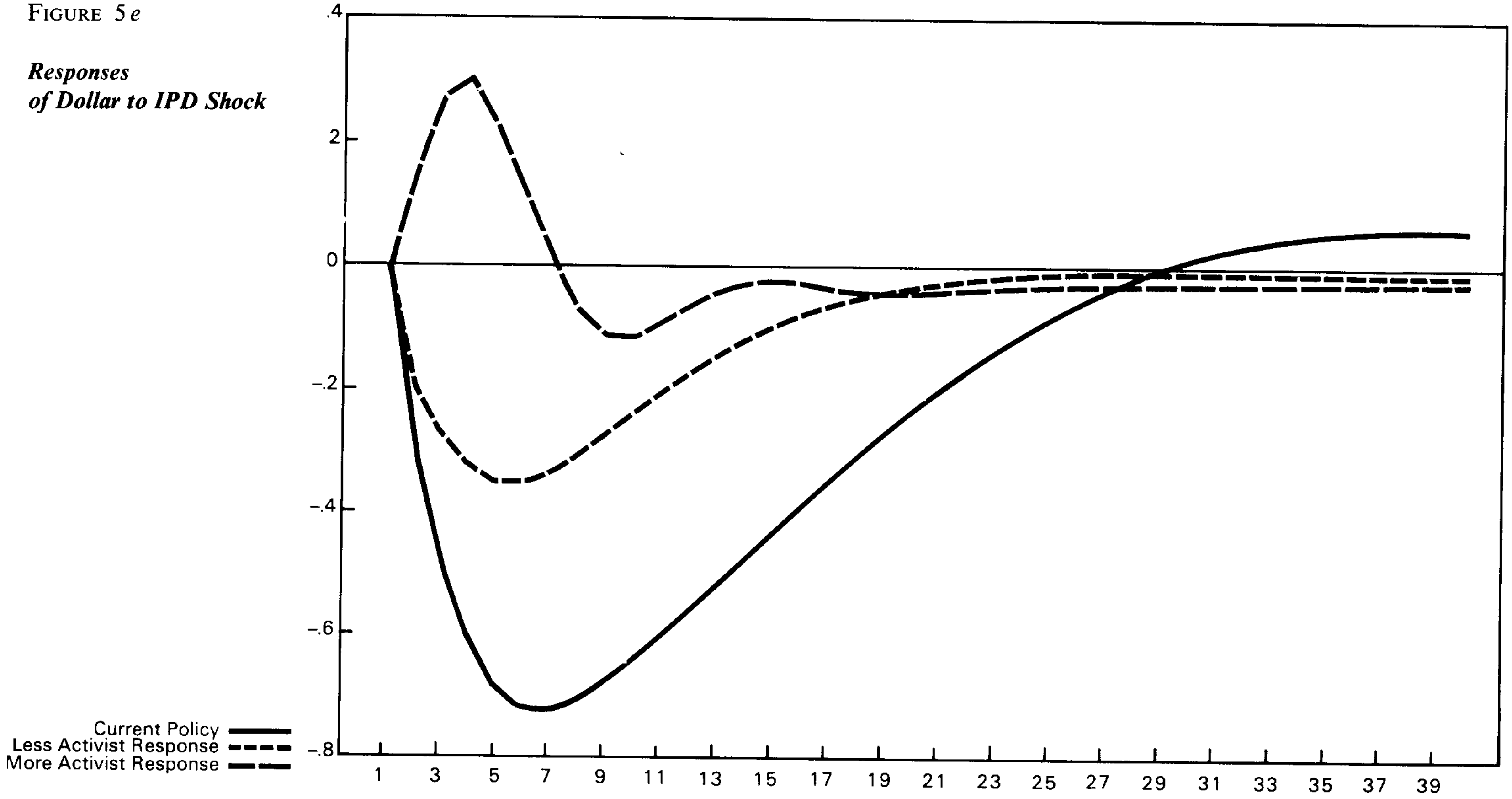


FIGURE 5*f*

Responses of Stocks to IPD Shock

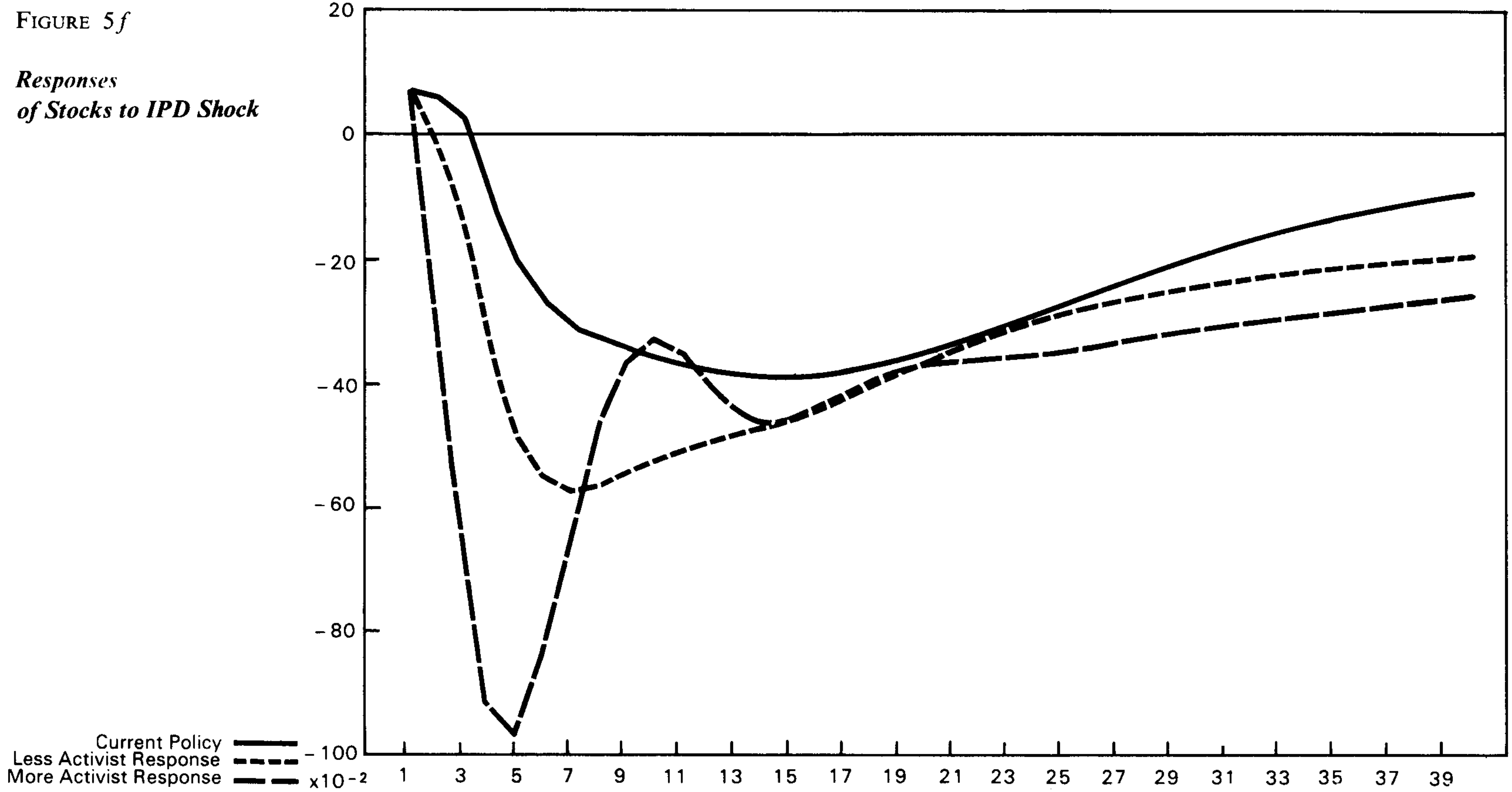


FIGURE 5*g*

Responses of TBILLS to IPD Shock

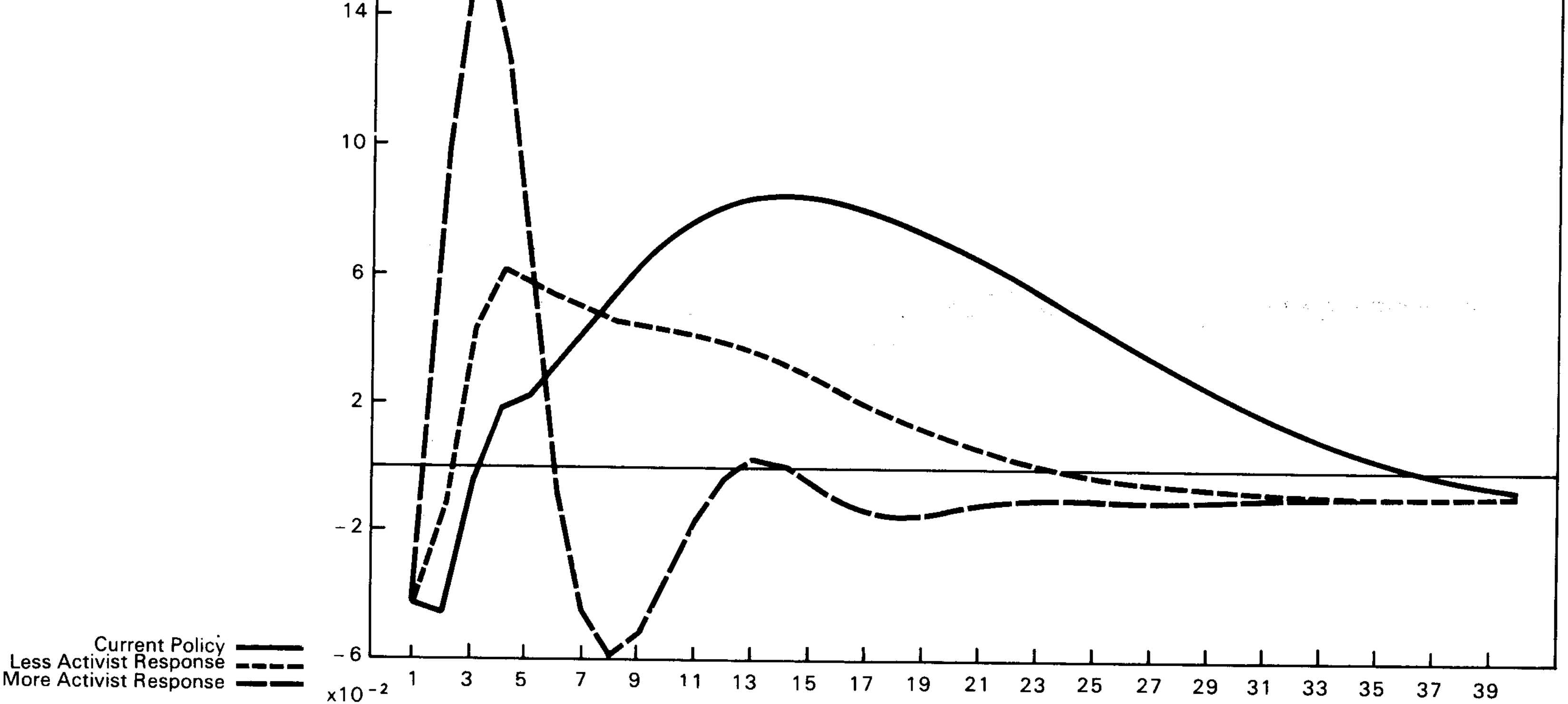


FIGURE 5*h*

Responses of Money to IPD Shock

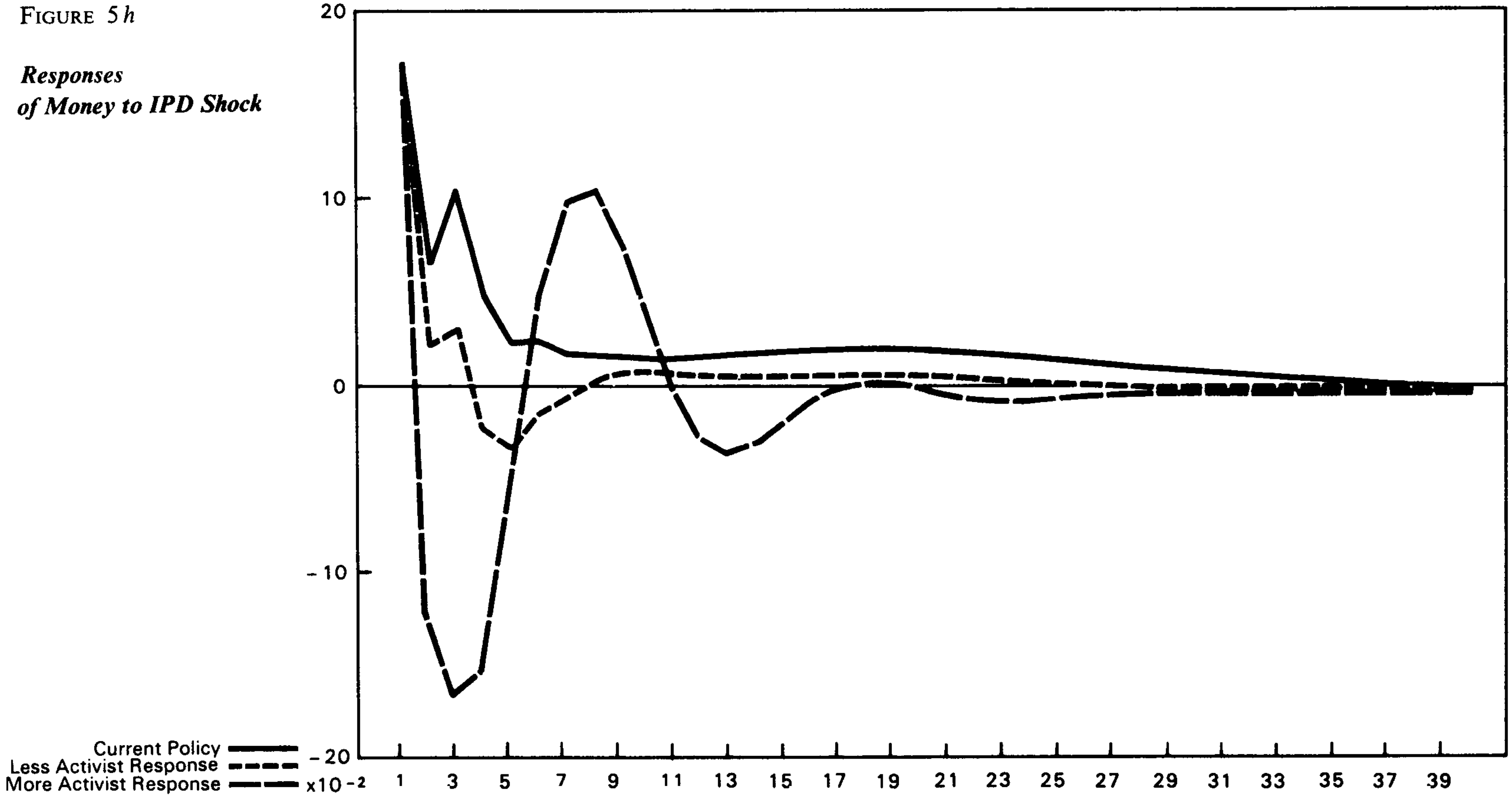
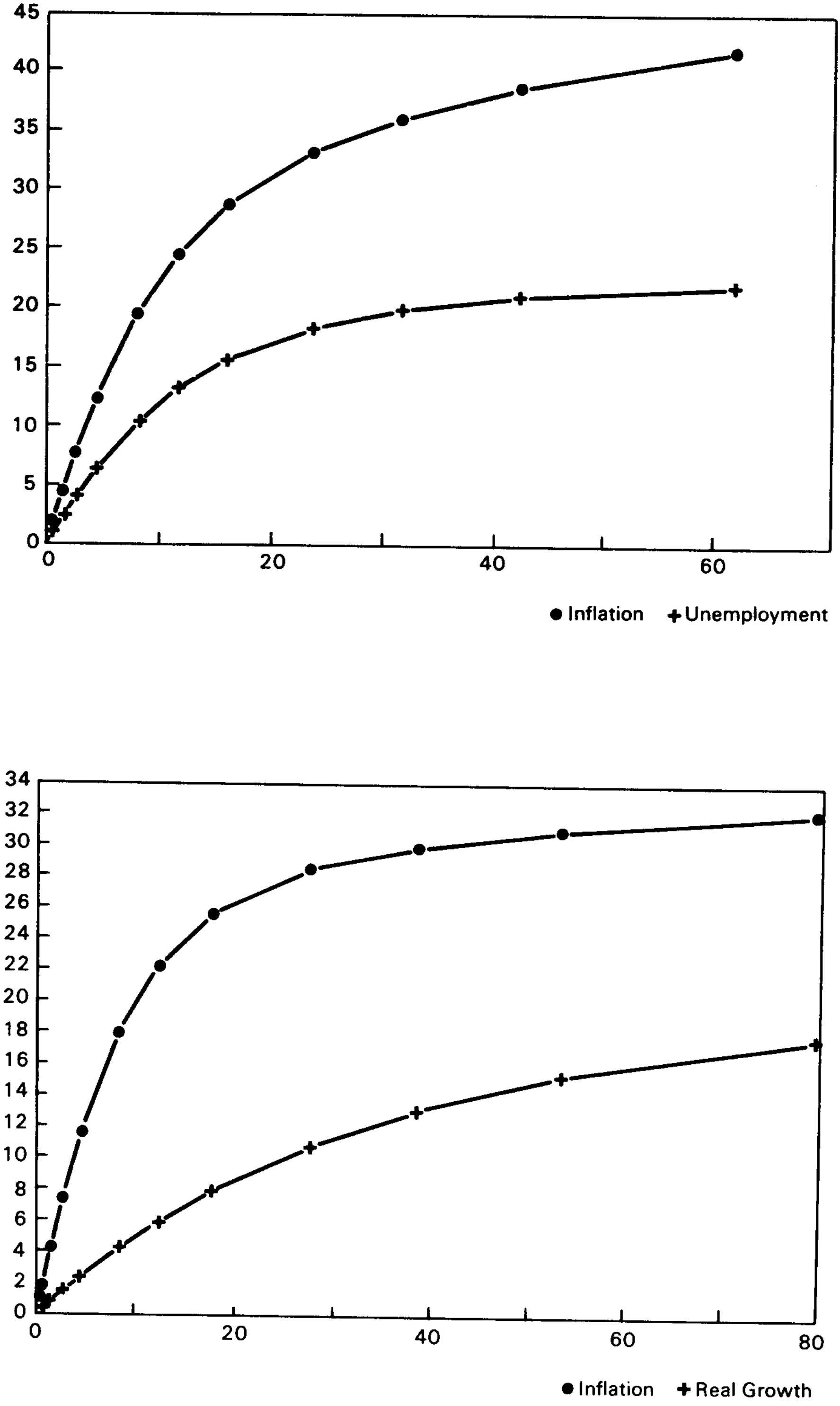


FIGURE 6

Payoff Function. Policy Activism vs. Stabilization



Interest rates return to their baseline rates within a year after the policy action, then fall below baseline and gradually return to it. Stock prices (which respond negatively) and the dollar both show a hump shaped response, increasing in size for a few quarters and then gradually returning to their baseline values.

The real growth and inflation rates averaged over two years in Figure 4 show the lags and smoothing induced by the moving average filter on the quaterly rates shown in Figure 1.

5 The Effectiveness of Optimal Policy

We now begin an investigation of the potential gains through the use of the optimal policy rule. The first step is to calculate the feedback rule for what we will refer to as the "standard" loss function. This is a specific set of parameters for the loss function, from which we will consider various deviations. The standard loss function gives weight of .4 to the squared deviations of real growth and .6 to squared deviations of inflation from target. It uses as targets the unconditional mean values for each. In the standard loss function the discount factor is .95, and the square of policy actions are penalized with a weight of 1 (i.e., $g=1$). Real growth and inflation rates are averaged over two year intervals. The optimal policy feedback vector is shown in Table 2.

The next step in evaluating the optimal policy is to compute the steady state variances of real growth and inflation under the historical and optimal policy rules. Before we can do this, however, we have to make one further assumption concerning the variance matrix of the economic shocks. We have to decide how much of the historical variation in the innovations to variables affected by policy is due to the random component of policy, and how much is due to economic shocks. For a given variance of the random component to policy, Σ_v, the variance matrix of economic shocks is given by

(8) $$\Sigma_u = \Sigma_w - L\,\Sigma_v\,L'$$

Note that the inclusion of a random component to policy in this model is not meant to criticize the historical performance of the Federal Reserve. It could be simply a reflection of a rational, systematic Fed reaction to information not included in the current model. Note also that we are not completely free to choose Σ_v, since for large enough values Σ_u will not be positive definite. Intuitively, we cannot attribute more of the variance of money, interest rates, stock prices, or the dollar (or of their covariances) to random policy actions than the total variance observed in the historical data. In fact, given our assumption about L we find that the variance must be less than .28. We will start by assuming that the variance is .1, and later we will investigate the implications of making other assumptions. The

TABLE 2

Optimal Feedback Rule for Standard Loss Function

No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient	No.	Label	Lag	Coefficient
1. . . .	RGNP	1	−.3867592 E−01	13. . . .	M 1	1	−.3465787 E−01	21. . . .	STOCKS	1	−1.159749
2. . . .	RGNP	2	−.2446631 E−01	14. . . .	M 1	2	−.6637270 E−02	21. . . .	STOCKS	1	−1.159749
3. . . .	RGNP	3	−.1917166 E−01	15. . . .	M 1	3	−.1732787 E−02	23. . . .	STOCKS	3	−.3188543 E−01
4. . . .	RGNP	4	−.1457635 E−01	16. . . .	M 1	4	−.7818282 E−03	24. . . .	STOCKS	4	−.5470154 E−01
5. . . .	RGNP	5	−.9476955 E−02	17. . . .	TBILLS	1	.1109076	25. . . .	DOLLAR	1	6.570948
6. . . .	RGNP	6	−.3167541 E−02	18. . . .	TBILLS	2	−.1812482 E−01	26. . . .	DOLLAR	2	−.3627365
7. . . .	IPD	1	−.7409333 E−01	19. . . .	TBILLS	3	−.2205534 E−02	27. . . .	DOLLAR	3	.2465454
8. . . .	IPD	2	−.1897673 E−01	20. . . .	TBILLS	4	−.1401766 E−02	28. . . .	DOLLAR	4	.1282505
9. . . .	IPD	3	−.6652640 E−02					29. . . .	CONSTANT	0	−25.05447
10. . . .	IFD	4	−.1546308 E−02								
11. . . .	IPD	5	−.2306343 E−02								
12. . . .	IPD	6	−.4812231 E−03								

assumption of .1 implies that in the historical period the random component to monetary policy raised the standard error of a one-quarter ahead forecast of interest rates from 69 to 76 basis points. In Table 3 we give the covariance/correlation matrix of the economic shocks under this assumption.

Given these estimates of the economic shocks we can begin by asking how much of the steady state variance of real growth and inflation can be eliminated by following the optimal control strategy. This steady state variance calculation is easily obtained by generating the moving average representations:

$$Y_t = \sum_{s=0}^{\infty} M_s u_{t-s} \tag{9}$$

and then taking the limit for large N of the sum:

$$\sum_{s=0}^{N} M_s \Sigma_u M_s' \tag{10}$$

Note that we use the Σ_u in order to focus on the effects of optimal feedback rules rather than on the improvment due to a reduction in the policy induced random noise.

We focus attention on the elements of this matrix that give the variance of components of the state vector, Y, that correspond to average real growth and inflation over one, two, and three year horizons.

We find that for the historical period the steady state standard error using the original covariance matrix of residuals is 2.11 for real growth and 2.06 for inflation. We also calculate that the steady state variance using the covariance matrix of economic shocks would be 2.04 for real growth and 1.88 for inflation. The latter variances may be a better benchmark for comparison with the optimal policy since the difference between them reflects the gains available through the active use of policy for control purposes. The gains relative to the historical period reflect, in addition, the gains due to the elimination of the assumed random components to policy.

TABLE 3

Covariance/Correlation Matrix of Economic Shocks
(Correlations are given above the diagonal)

Variable	RGNP	IPD	M1	TBILLS	STOCKS	DOLLAR
RGNP	12.6860	.0042	.24985	.15579	.19251	.09599
IPD	.0267	3.1380	.07322	−.07099	.01488	.00646
M1	2.1307	.3105	5.73285	.10671	.34827	.03093
TBILLS	.3306	−.0749	.15223	.35503	−.09478	.02750
STOCKS	.0332	.0013	.04047	−.00274	.00236	.04258
DOLLAR	.0066	−.0002	.00143	.00031	−.00004	.00037

Under our standard optimal policy, as defined above, we find the steady state standard error is 1.82 for real growth and 1.27 for inflation. Thus, the optimal policy achieves an 11 percent reduction of the standard error of real growth as well as a decrease of 33 percent in the standard error of inflation. We use the standard error, σ_v, of the policy action, s_t, as a measure of the degree of activism of the feedback rule. For this policy the policy standard error is .37, which given the units assumption means that policy will on average impact interest rates by 37 basis points each quarter relative to where they would be predicted to be under historical policy. By varying the weights attached in the loss function to real growth versus inflation we can trace out a possibility frontier facing the monetary authority. We show three such frontiers in Table 4.

We see that there are policies for which the standard errors of both real growth and inflation can be reduced. Notice, however, that even under the optimistic assumptions made here the optimal policies achieve only relatively small reductions in the cyclical variance of output. This result appears to be relatively robust to the horizon over which the real growth target is defined. For inflation, on the other hand, if a one-year horizon is the target, then very little of its variance can be reduced by an optimal policy. If the horizon is two or three years, however, then the standard error can be reduced by 30 percent or more, three to four times as much as with the one-year horizon.

We next investigate the impact on this possibility frontier of changes in the weight associated with the size of policy actions in the loss function. This investigation allows us to look at more versus less active policies. In Table 5 we compare the tradeoff frontier associated with the standard policy with a less active policy.

The less active policy puts a weight of 10. on squared policy actions. Now the possibility frontier moves in the direction of more variance for real growth and inflation, but with the benefit that interest rates are only required to move no more than 15 basis points on average from where they would be expected to be under the historical policy.

In order to illustrate the effect of following the optimal policy rule, in figures 5*a* and 5*h* the responses of the target variables in the system to a shock to inflation are compared under current policy to that under the more and less activist optimal policies.

TABLE 4

Possibility Frontiers Facing the Monetary Authority Steady State Standard Errors (Percent reduction from current policy)

Current Policy—1 year growth horizon

	Standard Error
Real GNP growth.	2.81
GNP Deflator growth.	2.12

Possibility Frontier—1 year growth horizon

Weight on Real GNP	Weight on GNP Deflator	Real GNP	GNP Deflator	σ_v
1.0	.0	2.44 (17.9)	3.79 (****)	.730
.8	.2	2.47 (16.5)	1.84 (5.1)	.577
.6	.4	2.52 (14.0)	1.68 (8.0)	.494
4	.6	2.58 (11.1)	1.61 (9.3)	.419
.2	.8	2.66 (7.2)	1.57 (10.1)	.363
.0	1.0	2.78 (1.2)	1.55 (10.5)	.361

Current Policy—2 year growth horizon

	Standard Error
Real GNP growth.	2.04
GNP Deflator growth.	1.88

Possibility Frontier—2 year growth horizon

Weight on Real GNP	Weight on GNP Deflator	Real GNP	GNP Deflator	σ_v
1.0	.0	1.62 (20.8)	4.28 (****)	.672
.8	.2	1.66 (18.7)	1.66 (11.6)	.472
.6	.4	1.74 (15.0)	1.39 (26.1)	.409
.4	.6	1.82 (11.0)	1.27 (32.7)	.373
.2	.8	1.91 (6.5)	1.19 (36.5)	.365
.0	1.0	2.03 (.5)	1.16 (38.6)	.399

Current Policy—3 year growth horizon

	Standard Error
Real GNP growth.	1.68
GNP Deflator growth.	1.74

Possibility Frontier—3 year growth horizon

Weight on Real GNP	Weight on GNP Deflatoe	Real GNP	GNP Deflator	σ_v
1.0	.0	1.34 (20.3)	4.40 (****)	.570
.8	.2	1.38 (18.0)	1.52 (12.5)	.349
.6	.4	1.45 (13.7)	1.18 (31.9)	.321
.4	.6	1.52 (9.5)	1.03 (40.5)	.323
.2	.8	1.60 (5.1)	.95 (45.3)	.349
.0	1.0	1.68 (.2)	.91 (47.8)	.394

TABLE 5

Possibility Frontiers Comparison of a More, Versus a Less, Active Policy Steady State Standard Errors

Optimal Policies

More active policy: $g=1$.
Less active policy: $g=10$.
Current policy: $g \to \infty$.

Weight on Real GNP	Weight on GNP Deflator	Real GNP		GNP Deflator		σ_v	
		More	Less	More	Less	More	Less
1.0	.0	1.62	1.86	4.28	2.10	.672	.157
.8	.2	1.66	1.88	1.66	1.65	.472	.126
.6	.4	1.74	1.91	1.39	1.51	.409	.122
.4	.6	1.82	1.94	1.27	1.43	.373	.125
.2	.8	1.91	1.97	1.19	1.39	.365	.134
.0	1.0	2.03	2.00	1.16	1.35	.399	.148
Current policy.		2.04		1.88		.000	

We next investigate the effect of changes in the discount rate included in the loss function. For very large rates of discounting the main effect is the degree of policy activism is reduced as the loss associated with current actions outways the highly discounted benefits in the future. These results (with weights in the loss function on real growth, inflation and policy actions set at .4, .6, and 1., respectively) are shown in Table 6.

We next consider the effect of changes in our assumption about the degree of policy randomness in historical data. As noted above, we have

TABLE 6

Effects of Changes in the Rate of Discounting Steady State Standard Errors (percent reduction from current policy)

Optimal Policies

Rate of Discounting	Real growth	Inflation	
1.00	1.81 (11.3)	1.26 (33.1)	.390
.99	1.82 (11.1)	1.26 (32.9)	.378
.95	1.82 (11.0)	1.27 (32.7)	.373
.9	1.82 (11.0)	1.27 (32.6)	.368
.8	1.82 (11.0)	1.27 (32.6)	.367

assumed that the random component of historical policy had a variance of .1. It turns out the results are rather insensitive to this assumption. In Table 7 we show these results, which again are based on a loss function with weights on real growth, inflation and policy actions of .4, .6, and 1., respectively.

TABLE 7

Effects of Changes in the Policy Randomness Assumption (percent reduction)

Historical σ_v	Real growth Current	Real growth Optimal	Inflation Current	Inflation Optimal	Optimal Policy σ_y
.000	2.11	1.83 (13.3)	2.06	1.28 (38.0)	.408
.025	2.09	1.83 (12.8)	2.02	1.27 (36.8)	.399
.050	2.08	1.82 (12.2)	1.97	1.27 (35.5)	.390
.100	2.04	1.82 (11.0)	1.88	1.27 (32.7)	.373
.200	1.97	1.81 (8.5)	1.69	1.26 (25.5)	.334
.270	1.92	1.80 (6.6)	1.53	1.25 (18.6)	.304

6 Concluding Remarks

The empirical results presented here are intended to be suggestive of the potential returns and limitations of an optimal monetary policy. They illustrate an approach which requires a minimal number of a priori assumptions, but an approach which, nonetheless, has been strongly criticized. Given this context, let us return to the two criticisms of the approach mentioned above.

First, consider the criticism that the VAR is not an identified structural model and therefore cannot form the basis for policy analysis. Here the identifying assumptions are clearly sufficient for the exercise at hand. The key assumption is the identification of L, the contemporaneous impact of a policy action on the variables in the system. Some readers may want to question the particular assumption made here, and the uncertainty about this assumption is clearly a limitation on this analysis. Such questioning is not a criticism of this approach, however. Some assumption about L is required in any structural analysis. Unfortunately, in typical uses of structural models for policy analysis the assumption about L is lost in an unnecessary and potentially misleading discussion of the economic structure in A, B, and F. I think it is a positive aspect of the approach that it focuses attention on this particular assumption, because it alone is the critical one.

The Lucas critique is less easy to dismiss. As argued above, the response to it is that we make an invariance assumption that may be useful in some contexts. Here the nature of the invariance assumption is that real growth and inflation will respond to policy induced changes in interest rates, money growth, the value of the dollar and stock prices in the way that they have responded to random variations in these variables observed in the historical data. In my opinion, whether that is a reasonable assumption depends on the size and nature of the policy actions being considered.

To illustrate the context dependence of this criticism, I mention another tradeoff suggested by the model which I do not believe would remain invariant in response to attempts to exploit it. Given the linear nature of the system and our assumption about the effect of policy actions, one could entertain policies designed to change the mean real growth and inflation rates by changing the mean policy action. According to the model, on average, by taking policy actions that raise interest rates and lower money growth, the monetary authority could lower both the inflation and real growth rates. The tradeoff is estimated by the model to be that real growth rates would fall by .47 of a percent for each 1. percent reduction in inflation. In my view, such a tradeoff, involving a change in the mean real growth rate based on a change in monetary policy would seem to be a good candidate for the lack of invariance suggested by the Lucas critique, especially if large changes were contemplated.

In this exercise, we ask about changing the variance of cyclical fluctuations as a function of the timing of policy actions. By changing the weight in the loss function associated with policy actions we can design policies that are as close as desired to current policy. We can, in effect, discuss policies that are so close to current policy that the invariance assumption can be taken as a foregone conclusion.

• References

BERTSEKAS, D. P. (1976). — *Dynamic Programming and Stochastic Control*, Academic Press.

CHOW, G. C. (1975). — *Analysis and Control of Dynamic Economic Systems*, John Wiley & Sons.

COOLEY, T. F. and LEROY, S. (forthcoming). — "A Theoretical Macroeconometrics: A Critique", *Journal of Monetary Economics.*

DOAN, T. and LITTERMAN, R. (1984). — *Regression Analysis of Time Series*, VAR Econometrics

KENDRICK, D. A. (1981). — *Stochastic Control For Economic Models*, McGraw-Hill.

LEAMER, E. (1985). — Vector "Autoregressions for Causal Inference?", In *Understanding Monetary Regimes*, Carnegie Rochester Series Volume 22.

LITTERMAN, R. B. (1984*a*). — "Forecasting and Policy Analysis With Bayesian Vector Autoregression Models", *Federal Reserve Bank of Minneapolis Quaterly Review* (Fall).

LITTERMAN, R. B. (1984*b*). — "The Costs of Intermediate Targeting", *Federal Reserve Bank of Minneapolis Working Paper*, 254

Litterman, R. B. (1986). – "Forecasting with Bayesian Vector Autoregressions: Five Years of Experience", *Journal of Business and Economic Statistics* (January).

Lucas, R. E. Jr. (1976). – "Econometric Policy Evaluation: A Critique", Brunner K. and Meltzer, A. H. Eds., *The Phillips Curve and Labor Markets*, Amsterdam, North-Holland.

Lucas, R. E. and Sargent, T. (1978). – "After Keynesian Macroeconomics", *After The Phillips Curve*, Federal Reserve Bank of Boston.

McNees, S. (1986). – "The Forecasting Accuracy of VARs and CMs: The Evidence and Its Interpretation", *Journal of Business and Economic Statistics* (January).

Sargent, T. J. (1979). – "Estimating Vector Autoregressions Using Methods Not Based on Explicit Economic Theories", *Federal Reserve Bank of Minneapolis Quaterly Review* (Summer).

Sargent, T. J. (1985). – "Autoregressions, Expectations and Advice", *American Economic Review*, 74, No. 2.

Sims, C. A. (forthcoming). – "Are Forecasting Models Usable for Policy Analysis", *Federal Reserve Bank of Minneapolis Quaterly Review*.