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Growth, Desirability, Profitability and Unemployment

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ABSTRACT. – In this paper, we consider how the standard tools of micro-economic analysis can be used to study comparative statics and normative dynamics of macroeconomic analysis.

Croissance, désirabilité, profitabilité et chômage

RÉSUMÉ. – Dans ce papier, nous envisageons comment les outils usuels de l'analyse micro-économique peuvent être utilisés dans l'étude de la statique comparative et de la dynamique normative en analyse macroéconomique.

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1 Introduction

In a simple neoclassical model, profitability, defined by marginal willingness to produce, and desirability, defined by variation in consumer's welfare, are identical criteria. They are not identical even in the simple Keynesian model where they can be conflictual. Consequently, if one interprets a Keynesian policy as a local action to promote growth, one has to carefully distinguish between its profitability and its desirability in order to be able to assess such a policy and measure its social relevance. MALINVAUD [1980, 1983] suggests that the failures of the Keynesian policies during the seventies were effectively due to the fact that these policies were not implementable in a world of underprofitability. Accordingly, he proposes to insert profitability as an explanatory variable in macroeconomic dynamics. In his proposition, desirability is left implicit (although, it is considered in MALINVAUD [1977]).

In this paper, our first purpose is to explicit both desirability and profitability and to characterize their possible conflicts. Working in the context of a temporary supply-constrained equilibrium model, [1] we will exhibit its multiplier matrix and use this matrix to measure the effect of a local Keynesian policy on growth, profitability and desirability.

Our second purpose is to study the positions of a desirability requirement and a profitability requirement in the dual space of supply-constrained equilibria. The reason is that repressed inflation can be explained in a supply-constrained economy by these positions. Moreover, pressures on prices can begin before full employment, as advocated by KEYNES [1936] and contrarily to the Hansen-Samuelson presentation.

Our third purpose is to link up the previous points into a dynamical process, in fact, in a MDP process converging towards a first best or a second best allocation. [2]

Our fourth purpose is to analyse the multipliers. This is a work that can be done simultaneously with the second purpose and is illustrated for the case of the simple Keynesian model.

Of course, we have to consider generalized multipliers. In the synthesis between second best and disequilibrium (PICARD [1982], BRONSARD et WAGNER [1982], DRÈZE [1984]), DRÈZE [1984] exhibited some atemporal multipliers. They are mixed up, in this paper, with Keynes' temporary multipliers to form our generalized multipliers. They do not change the nature and scope of Keynesian macroeconomics.

But the consideration of expectations can do that and our fifth and last purpose is to illustrate this point. The consideration of arbitrary expectation functions leads to a result similar in its spirit to the one obtained by POLEMARCHAKIS [1983] in consumption theory. The consideration of rational expectations (in RADNER's sense [1972]) *when added* to special assumptions either on the consumption functions or on the production functions effectively destroys the device of the multipliers. As expectations are neither so arbitrary nor so rational, as the special assumptions used are not often mentioned, our analysis can help to remove some confusion in this field of macroeconomics.

The paper is organized as follows. Section 2 is devoted to consumers and to the concept of desirability (in a temporary framework (GRANDMONT [1983]) under quantity rationings). Section 3 is devoted to producers and to the characterization of profitability. In section 4, the supply-constrained equilibrium defines a smooth function and its derivative is the multiplier matrix. In section 5, we recover and reinterpret the simple Keynesian model with the help of profitability and desirability. Section 6 is devoted to normative dynamics and section 7 to expectations.

2 The Consumer

The intertemporal indirect utility function is denoted S*. It depends on current prices, p, the nominal discount factor, γ, future prices, $\tilde{p}$, (discounted at the period $t+1$), current income, R, and the flow of future incomes, $\tilde{R}$, (discounted at the period $t+1$). When there are no futures markets, $\tilde{p}$ and $\tilde{R}$ are expected future prices and incomes. Let us first assume that these expectations are exogenous. [3] In this case, we will write:

$$(1) \qquad v(p, \gamma, R) \equiv S^*(p, \gamma \tilde{p}, R + \gamma \tilde{R})$$

Now, let us show that the derivative of v with respect to the nominal discount factor, γ, leads to the introduction of a financial asset. First, denoting $\bar{p} = \gamma \tilde{p}$ and $\bar{R} = R + \gamma \tilde{R}$ and using Roy identities, one has

$$(2) \qquad \begin{aligned} v_\gamma &\equiv S_{\bar{p}} \tilde{p} + S_{\bar{R}} \tilde{R} \\ &\equiv -S_{\bar{R}} [\tilde{x} \tilde{p} - \tilde{R}] \end{aligned}$$

where $\tilde{x}$ is the vector of future commodities. Let x_0 be an artificial commodity introduced to bridge the gap between future expenditures $\tilde{p}\tilde{x}$ and future incomes $\tilde{R}$. By construction, x_0 is a commodity to be delivered in period $t+1$ and supposes the existence of a financial market on which

1. The concept of supply-constrained equilibria was studied by KURZ [1982], DEHEZ and DRÈZE [1984] in a slightly different context.

2. The considered MDP process will be an M-72 process in the terminology of d'ASPREMONT and TULKENS [1980].

3. Other assumptions about expectations will be considered in section 7.

the consumer can buy x_0 at the cost γx_0. (γx_0 is then the cost at period t of an asset whose nominal value in period $t+1$ is x_0). Equation (2) can then be written

$$v_\gamma \equiv -S_{\bar{R}} x_0 \equiv -v_R x_0. \tag{3}$$

Of course, one has

$$v_p + v_R x \equiv 0 \tag{4}$$

where x is the vector of current commodities.

Relations (3) and (4) are the temporary Roy identities and lead to a complete system of commodity and asset demands. As usual, this complete system is characterized by a Slutsky Local Structure.

Let us now assume that the consumer is subject to quantity constraints and let x_2 be the vector of rationed commodities. The temporary indirect utility function can then be written

$$v\,(p, \gamma, R, x_2)$$

and leads to the additional characterization

$$v_{x_2} \equiv -v_R (p_2 - \pi_2) \tag{5}$$

where π_2 is the shadow price of x_2 for the consumer. Let us consider that x_2 is the vector of labor services and that they are negatively measured. The vector $p_2 - \pi_2$ is, therefore, the vector of marginal propensities to work. It can be positive, negative or null. It is positive when there is underemployment, null when there is full employment and negative when there is overemployment. It is often assumed that the law of voluntary exchange applies even at the margin so that overemployment is not possible. In this paper, on the contrary, it is assumed that the worker does not reject a job as soon as the wage rate of an extra hour is below his marginal propensity to be paid. [4]

The indirect temporary utility function extended to the quantity rationing situation leads to a complete temporary physical and financial consumer demand system which can be written as follows:

$$x_1 = x_1^* (p_1, p_2, \gamma, R, x_2) \tag{6}$$

$$x_0 = x_0^* (p_1, p_2, \gamma, R, x_2) \tag{7}$$

These functions are characterized by a Slutsky Local Structure. The spill-over effects $\partial x_1/\partial x_2$ and $\partial x_0/\partial x_2$ have the additive property

$$p_1 \frac{\partial x_1}{\partial x_2} + \gamma \frac{\partial x_0}{\partial x_2} \equiv -p_2. \tag{8}$$

Finally, assuming that prices associated to commodity demands and to labor supplies are fixed, the total differential of the temporary indirect utility function with Quantity Rationing can be written

(9) $$\frac{dv}{\lambda} = -x_0\, d\gamma + d\mathrm{R} - (p_2 - \pi_2)\, dx_2$$

where $\lambda \equiv v_{\mathrm{R}}$.

This differential measures the variations in consumer welfare associated to changes in the nominal discount factor, the current income and the current level of quantity rationings. We shall refer to this measure as the *desirability* of these changes. That such a measure can be used within a macroeconomic framework will be illustrated in the sequel, linking together cost-benefit analyses and macroeconomic models.

Remark that in the case of underemployment $(p_2 - \pi_2 > 0)$, an increase in employment $(dx_2 < 0)$ is desirable $\left(\frac{dv}{\lambda} > 0\right)$.

3 The Producer

If expectations are exogenous, the intertemporal profit function, P, can be written:

(10) $$\mathrm{P}(p, \gamma) \equiv p\, \eta\, (p, \gamma\tilde{p}) + \gamma\tilde{p}\, \tilde{\eta}\, (p, \gamma\tilde{p})$$

where η and $\tilde{\eta}$ are current and planned supply functions. The behaviour of this intertemporal profit function is not altered if we add or substract some exogenous variables. Let D and $\tilde{\mathrm{D}}$ denote respectively current and expected dividends. In order to define and characterize a complete temporary system of net supplies, let us consider the relation

(11) $$\mathrm{V}(p, \gamma, \mathrm{D}) \equiv p\, \eta\, (p, \gamma\tilde{p}) + \gamma\tilde{p}\, \tilde{\eta}\, (p, \gamma\tilde{p}) - \mathrm{D} - \gamma\tilde{\mathrm{D}}.$$

Because the profit function is homogeneous of degree one, the differentiation of V with respect to γ immediately defines a financial asset:

(12) $$\mathrm{V}_\gamma \equiv \tilde{p}\, \tilde{\eta}\, (p, \gamma\tilde{p}) - \tilde{\mathrm{D}} = \eta_0\, (p, \gamma\tilde{p}).$$

Differentiating with respect to p, one has

(13) $$\mathrm{V}_p \equiv \eta\, (p, \gamma, \tilde{p}).$$

4. This assumption allows us to extend the Benassy-Drèze-Younès (BENASSY [1975], DRÈZE [1975], YOUNÈS [1975]) set of Keynesian equilibria. This is an extension with respect to the contemporary theoretical and econometric literature on non-Walrasian equilibrium (see ARTUS, LAROQUE and MICHEL [1984] and SNEESSENS [1983] on the estimation of equilibria with quantity rationing) but not necessarily with respect to Keynes. In fact, his definition of an equilibrium is general enough to encompass our viewpoint (see book I for example).

Equations (12) and (13) define a complete system of commodity and asset supplies in a temporary framework. This system is homogeneous of degree zero in p and γ and its Jacobian matrix is symmetric and positive semidefinite.

Let us now assume that the producer is subject to quantity constraints. Before writing the new function V two considerations must be done. First, in this producer's model a price effect is a substitution effect, so that there is no substitution effect through prices among rationed commodities or between rationed and unrationed commodities. Consequently, the prices of rationed commodities disappear from our equations. Second, we want to consider a simple economy in which investment projects are exogenous relative to the current economic variables as in the Samuelson-Hansen presentation of the basic macroeconomic model. Moreover, the production of capital goods can be rationed by market demands. In order to capture this idea with a single producer, it must be admitted that the profit function is not parameterized by net productions of new capital goods (y_3) but by both outputs (b_3) and inputs (a_3) of new capital goods. Therefore, let y_1 and b_3 be the rationed commodities, y_1, being the vector of net supplies of consumer goods and services. The new function V can then be written:

$$(14) \quad V(p_2, \gamma, D, y_1, b_3, a_3) \equiv p_1 y_1 + p_2 y_2^* (p_2, \gamma, y_1, b_3, a_3) + p_3 b_3 - p_3 a_3 + \gamma y_0^* (p_2, \gamma, y_1, b_3, a_3) - D$$

where y_2^* and y_0^* are respectively labor service demands and asset supply.

Differentiating this function relative to p_2 and γ leads to the complete system

$$(15) \quad y_2 = y_2^* (p_2, \gamma, y_1, b_3, a_3)$$

$$(16) \quad y_0 = y_0^* (p_2, \gamma, y_1, b_3, a_3).$$

This system is homogeneous of degree zero in p_2 and γ and its derivative with respect to p_2 and γ forms a symmetric and positive semidefinite matrix.

Moreover, rejecting temporal reversibility, that is to say, assuming

$$(17) \quad \frac{\partial \tilde{y}_2}{\partial y_1} = 0$$

and

$$(18) \quad \frac{\partial \tilde{y}_2}{\partial b_3} = 0,$$

one has

$$(19) \quad \frac{\partial y_0}{\partial y_1} = 0$$

and

$$(20) \quad \frac{\partial y_0}{\partial b_3} = 0$$

because $y_0 \equiv \tilde{p}\tilde{y} - \tilde{D}$, where $\tilde{y}$ are expected rationings or planned net supplies.

The derivatives of V with respect to y_1, b_3 and a_3 can then be written:

(21) $$V_{y_1} \equiv p_1 + p_2 \frac{\partial y_2}{\partial y_1} \overset{\text{def}}{=} p_1 - \pi_1$$

(22) $$V_{b_3} \equiv p_3 + p_2 \frac{\partial y_2}{\partial b_3} \overset{\text{def}}{=} p_3 - \pi_3$$

(23) $$V_{a_3} \equiv p_3 + p_2 \frac{\partial y_2}{\partial a_3} + \gamma \frac{\partial y_0}{\partial a_3} \overset{\text{def}}{=} -p_3 + \varphi_3$$

where π_1, π_3 and φ_3 are the shadow price of respectively consumer goods and services, outputs of new capital goods and inputs of new capital goods.

Finally, let us note that the total differential of the temporary profit function with quantity rationing coincides with the total differential of our function V if D and $\tilde{D}$ are fixed. This is written:

(24) $$dV = dP.$$

The real part of dP is denoted dP_r and measures the variation in the producer's real profit associated to changes in the levels of quantity rationing. By equations (21) to (23), this can be written:

(25) $$dP_r = (p_1 - \pi_1)\, dy_1 + (p_3 - \pi_3)\, db_3 - (p_3 - \varphi_3)\, da_3.$$

We shall refer to this measure as the *profitability* of these changes.

A profitability requirement for an investment project can be expressed by the relation:

(26) $$\frac{dP_r}{p_3\, da_3} = \beta$$

where β is some positive scalar.

4 Equilibrium

Growth will be defined as the real variation in global supply or Gross National Product (Y). As

(27) $$Y = p_1 y_1 + p_1 w_1^H + p_3 b_3$$

where w_1^H represents consumer's initial endowments in consumption goods, growth is measured by

(28) $$dY_r = p_1\, dy_1 + p_3\, db_3$$

where the subscript r indicates that only real variations are considered.

At this stage, we can characterize any transformation of an economic state by its consequences, that is to say, by its profitability, its desirability and the growth associated to it. As an equilibrium is an economic state in which individual behaviors are consistent, we shall restrict our attention to equilibria and to their transformations.

Let us consider excess supply functions at date t:

(29) $$\zeta_1 = y_1 + w_1 - x_1^*(p_1, p_2, \gamma, R, x_2)$$

(30) $$\zeta_2 = y_2^*(p_2, \gamma, y_1, b_3, a_3) - x_2$$

(31) $$\zeta_3 = b_3 - a_3 + w_3$$

(32) $$\zeta_0 = y_0^*(p_2, \gamma, y_1, b_3, a_3) + \frac{w_0}{\gamma} - x_0^*(p_1, p_2, \gamma, R, x_2)$$

where w represents initial endowments, $w_1 = w_1^F + w_1^H$ and $w_0 = w_0^F + w_0^H$, w_1^F being the producer's initial endowments while w_1^H are the consumer's initial endowments. [5]

Let

(33) $$R \overset{\text{def}}{=} Y + p_2 y_2 - S^F$$

where S^F is an exogenous financial amount transferred from the consumer to the producer. Within the framework defined in sections 2 and 3, the excess supply functions are characterized by the Walras law

(34) $$p_1\zeta_1 + p_2\zeta_2 + p_3\zeta_3 + \gamma\zeta_0 = p_3 a_3 - p_1 w_1^F - \gamma y_0^*(.) - w_0^F - p_3 w_3 - S^F.$$

By equation (34), the consequences of the Walras law, in the considered economy, is not the existence of a linear combination between the market-clearing conditions (we do not have $p_1\zeta_1 + p_2\zeta_2 + p_3\zeta_3 + \gamma\zeta_0 \equiv 0$). Instead, equilibrium ($\zeta' = (\zeta_1', \zeta_2', \zeta_3', \zeta_0') = 0$) leads to

(35) $$p_3 a_3 - p_1 w_1^F - \gamma y_0^*(.) - w_0^F - p_3 w_3 - S^F = 0.$$

A supply-constrained equilibrium is defined as an equilibrium in which the levels of rationings, y_1, x_2, b_3, and the nominal discount factor, γ, are determined by the equilibrium. These variables are, therefore, the endogenous variables of the equilibrium. Equilibrium conditions ($\zeta = 0$) may determine the endogenous variables in such a way that quantity rationings are not binding. The supply-constrained equilibrium is then equivalent to a Walrasian equilibrium. There is no paradox in that situation: in the Walrasian equilibrium, the institutional parameters are prices while, in the supply-constrained equilibrium, the institutional parameters are quantities. The constrained quantities may correspond to Walrasian prices.

Equation (34) implies that our supply-constrained equilibrium can be regular, that is to say, the matrix of the derivatives of the excess supply functions relative to the endogenous variables, $e = (y_1', x_2', b_3', \gamma)'$ can be of full rank at the equilibrium (excess supply functions are continuously differentiable by sections 2 and 3). Let us denote B this matrix and let $\bar{e}$ be the vector of the exogenous variables of the equilibrium. In particular, $\bar{e}$

contains current prices, expected prices, expected incomes, initial endowments, inputs of new capital goods and S^F. Then from the implicit function theorem, it follows that there exists a continuously differentiable function e^* such that

$$e = e^*(\bar{e}) \tag{36}$$

on a relevant neighborhood of an equilibrium point. Moreover, the Jacobian matrix of e^* is denoted M and one has

$$\mathrm{M} = -\mathrm{B}^{-1}\Gamma \tag{37}$$

where Γ is the matrix of the derivatives of the excess supply functions relative to $\bar{e}$.

These relations define and characterize the set of regular supply-constrained equilibria. The matrix M is called the multiplier matrix.

Within the framework defined in sections 2 and 3 and if

(i) B has full rank,

(ii) p_1, p_2, p_3 and initial endowments are kept constant,

it follows from (36) and (37) that there exists a multiplier matrix M such that growth, profitability, desirability associated to the marginal transformation of a supply-constrained equilibrium are measured by

$$dY_r = (p_1,\ 0,\ p_3,\ 0)\,\mathrm{M}\,d\bar{\bar{e}} \tag{38}$$

$$dP_r = (p_1 - \pi_1,\ 0,\ p_3 - \pi_3,\ 0)\,\mathrm{M}\,d\bar{\bar{e}} - (p_3 - \varphi_3)\,da_3 \tag{39}$$

$$\frac{dv}{\lambda} = (p_1,\ \pi_2,\ p_3,\ -x_0)\,\mathrm{M}\,d\bar{\bar{e}} - dS^F \tag{40}$$

where $d\bar{\bar{e}}$ is the vector $d\bar{e}$ in which p_1, p_2, p_3 and initial endowments are kept constant.

Let us remark that equations (38), (39) and (40) measure the consequences of changes in exogenous variables when equilibrium is preserved. Because these changes are such that equilibrium is preserved, it follows that they are subject to the "budget constraint" (35), that is to say, the financial

5. This model is defined in a temporary framework; at date t, there exist only spot markets and these markets can be partitioned into

$n_1 \geqslant 1$ consumer goods and services;

$n_2 \geqslant 1$ labor services;

$n_3 \geqslant 1$ capital goods;

$n_0 = 1$ financial asset.

This corresponds to the partition of WALRAS [1874-77], PARETO [1896-97], BARONE [1935]. (The temporary general equilibrium has a long history before HICKS [1946]. Hick's contribution was to introduce expectations into such a framework). In this partition, there is no market for used capital goods and services rendered by them. One can read the paper under the assumption $n_1 = n_2 = n_3 = n_0 = 1$.

consequences of a change in an exogenous variable appear in the total differential of equation (35). For example, let us assume that there is a change in inputs of new capital goods, a_3, and let us consider the total differential of equation (35). One has

$$p_3\, da_3 = \gamma\, dy_0 + y_0\, d\gamma.$$

In particular, if the determination of endogenous variables is such that $d\gamma = 0$, one has

$$p_3\, da_3 = \gamma \frac{\partial y_0}{\partial a_3} da_3.$$

This means that each new capital good is financed by financial asset issues, which have effects on future decisions by equation (12). In that example, financial asset issues are considered instead of money issues. Of course, it would be possible to allow for a change in w_0, that is to say, for money issues, leading, in that case, to a rupture of past financial contracts.

In this section, equations (38), (39) and (40) have asserted that a variation in exogenous variables can be tracked down in its ultimate consequences on consumption units, and production units. This is a delicate matter. Consequently, the rest of this paper will be devoted to the analysis of these relations. In section 5, we analyse the implied comparative statics, in section 6, we discuss the suggested normative dynamics and in section 7, we consider the effects of endogenous expectations.

5 Comparative Statics of Profitability and Desirability: the Case of the Simple Multipliers

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In order to discuss the comparative statics of the supply-contrained equilibrium, let us assume that among the exogenous variables, only a_3 is varying. da_3 is then considered as the vector of the macroeconomic policy parameters [6] and our single producer includes the public sector. In fact, our producer decision-making problem, being a problem of cost minimization, is appropriate for the public sector.

The multipliers of such a policy on net productions of consumer goods and services (y_1) and on the nominal discount factor (γ) are presented in Appendix A [relations (60) and (61)] and are used to measure growth, desirability and profitability associated with that policy [relations (62), (63) and (65)].

These complex relations are greatly simplified when the nominal discount factor remains constant. For example, in (60), it can be seen that the nominal discount factor does not vary when the direct effect of new capital goods on the financial asset market is equal to its indirect effect through the market of consumer goods and services. In that simplified case, relations (62), (63) and (65) reduce to (66), (67) and (68).

Finally, when the consumption function is the simple Keynesian one, that is, when spill-over effects of labor services reduce to their income effects because marginal propensities to consume out of labor incomes and non-labor incomes are equal, the effect of da_3 on growth, desirability and profitability can be written:

(41) $$dY_r = \frac{1}{1-\alpha} p_3 \, da_3$$

(42) $$dP_r = \left[(p_1 - \pi_1) \frac{k_1}{1-\alpha} p_3 + (p_3 - \pi_3) - (p_3 - \varphi_3) \right] da_3$$

(43) $$\frac{dv}{\lambda} = \left\{ [(p_1 - \pi_1) - (p_2 - \pi_2) Y_{21}] \frac{k_1}{1-\alpha} p_3 - (p_2 - \pi_2)(Y_{23} + Z_{23}) + (p_3 - \pi_3) \right\} da_3$$

where $k_1 = \frac{\partial x_1}{\partial R}$, $\alpha = p_1 k_1$, $Y_{21} = \partial y_2 / \partial y_1$, $Y_{23} = \partial y_2 / \partial b_3$, $Z_{23} = \partial y_2 / \partial a_3$. (See Appendix A for a proof).

In the same way, the multipliers of a_3 on y_1, x_2, b_3 and γ can be written:

	da_3
dy_1	$\frac{k_1}{1-\alpha} p'_3$
dx_2	$Y_{21} \frac{k_1}{1-\alpha} p'_3 + Y_{23} + Z_{23}$
db_3	1
$d\gamma$	0

Let us assume simultaneously that $k_1 > 0$ and $\alpha < 1$. Moreover, let us assume that $Y_{21} \leqslant 0$ and $Y_{23} + Z_{23} \leqslant 0$, that is to say, an increase in the production of consumption goods and capital goods supposes, in the short run, an increase in the different qualities of labor services (negatively measured). We then have the following sign structure:

	da_3
dy_1	+
dx_2	−
db_3	+
$d\gamma$	0

6. New capital goods inputs are the only exogenous variable that can be considered as macroeconomic policy parameters in this simplified model. The explicit introduction of the public sector in the equilibrium model would allow to consider many other macroeconomic policy parameters but this generalization would not be really useful here because our purpose is the illustration of a methodology.

This means that an increase in new capital goods has a positive effect on the production of consumption goods and of capital goods, a positive effect on employment (x_2 is measured negatively) and a null effect on the nominal discount factor.

From equation (41), it appears that if the signs of the multipliers are constant, there is no limit to the growth of Gross National Product and, from this viewpoint, one has to advocate capacity constraints to stop a sequence of policies $da_3 > 0$ (for example, dx_2 is bounded by population). But this viewpoint is too narrow. Even in the case of the simple Keynesian model, one has to consider the motivation of the agents. The criteria of profitability and desirability take into account marginal propensitities to produce and work and, consequently, agents' reactions to any policy $da_3 > 0$.

The simultaneous consideration of growth, profitability and desirability completes the Keynesian model by indicating the consequences of any macroeconomic policy on the economy and on real profits and utility. These consequences allow to know when a policy must be stopped or altered. Moreover, consideration of desirability and profitability is sufficient to explain that repressed inflation and stagflation start before full employment.

To illustrate this point, let us first consider that $p_3 = \pi_3 = \varphi_3$ (prices of new capital goods are equal to their marginal costs and marginal revenues).

Equation (42) becomes

$$(44) \qquad dP_r = (p_1 - \pi_1)\frac{k_1}{1-\alpha}p_3\, da_3$$

Furthermore, let us assume that there is no substitution between capital and labor in the short run, i.e. $Z_{23} = 0$. It follows that equation (43) can be written as

$$(45) \qquad \frac{dv}{\lambda} = (p_1 - \pi_1)\frac{k_1}{1-\alpha}p_3\, da_3 - (p_2 - \pi_2)\left(Y_{21}\frac{k_1}{1-\alpha}p_3 + Y_{23}\right)da_3$$

In these relations, we can interpret the wedges $(p_1 - \pi_1)$ between market prices and shadow prices (here the marginal cost of consumption goods) as the marginal propensities to produce. In the same way, the wedges $(p_2 - \pi_2)$ can be seen as the marginal propensities to work.

As suggested by MALINVAUD [1980], to be effective, a macroeconomic policy (da_3) should satisfy a profitability requirement $\beta = \frac{dP_r}{p_3\, da_3} \geqslant 0$. As a matter of fact, this cannot be an absolute requirement: a producer can accept a $dP_r < 0$ in order to save his positive profits. But, of course, in this case, he will claim for a revision of prices.

With equation (45), we complete Malinvaud's representation by introducing a desirability requirement. A macroeconomic policy should be desirable too and, this means that it should increase welfare. Our welfare criterion reduces to the profitability criterion only when $p_2 = \pi_2$. Consequently, profitability and desirability can conflict. A consumer can accept a $\frac{dv}{\lambda}<0$ to save his job but, in this case, he calls for a revision of prices.

For simplicity purposes, let us assume $\beta=0$. We then see that a macroeconomic policy is naturally bounded by $dP_r=0$ and $dv=0$, that is to say, by

$$(p_1-\pi_1)k_1=0 \tag{46}$$

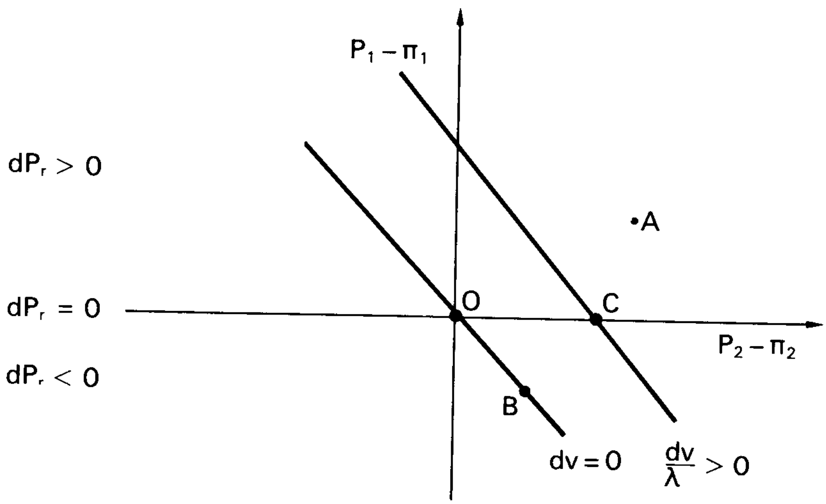
$$(p_1-\pi_1)\frac{k_1}{1-\alpha}p_3=(p_2-\pi_2)\left(Y_{21}\frac{k_1}{1-\alpha}p_3'+Y_{23}\right). \tag{47}$$

Let us consider simultaneously equations (45) and (47). These equations are parameterized by prices (and by the other exogenous variables). Consequently, the optimal level of capital goods characterized by equation (47) is parameterized by prices. Therefore, optimality is defined relatively to the level of exogenous variables. To each price system $[p_1 p_2 p_3]$ consistent with the existence of an equilibrium, corresponds an optimal level a_3. We shall consider that this level a_3 is unique. If in (47) each wedge is equal to zero, we get a first best, otherwise we find a second best and the distance between a second best and a first best depends on the level of the exogenous prices.

In Diagram I, we consider the wedge space in the case $n_1=n_2=1$. In this space, each point is the image of an extended supply-constrained equilibrium as defined in our model. Indeed for each given price system and each vector of capital goods a_3, the markets determine γ and the quantities of y_1, x_2 and b_3. The marginal rates of substitution and the marginal rates of transformation are then uniquely determined. It follows that the wedges $p_2-\pi_2$ and $p_1-\pi_1$ are uniquely determined.

To the curve $dv=0$ corresponds the optimal level of capital goods. The origin is a first best, the point B is a second best and point A is not optimal, neither in a first best nor in a second best sense.

DIAGRAM I



Let us denote p_1^0, p_2^0, p_3^0, and a_3^0 the levels of exogenous variables (the other exogenous variables being kept constant) which correspond to the first best allocation. If $a_3 \neq a_3^0$, while $[p_1, p_2, p_3] = [p_1^0, p_2^0, p_3^0]$ one cannot be at the point 0 nor at the point B since the optimal level of capital goods is unique. In that case, one can be, for example, at the point A. Let p_1^1, p_2^1, p_3^1, and a_3^1 be the level of exogenous variables associated to the point B. If $a_3 \neq a_3^1$, one can still be at the point A.

Now let us assume that we are at the point A and let us imagine a sequence of policies $da_3 > 0$. Then, if the price system is p_1^1, p_2^1, p_3^1, these policies will possibly lead us to the point B which is an optimal one. In the next section, we will study such a dynamical system.

The preceding remarks on desirability of a policy can be transposed in order to discuss the profitability of a policy. At the point C, real profits are maximized, profitability is null but these profits cannot correspond to a first best allocation.

Finally, let us remark that the point A corresponds to a Keynesian equilibrium. By extending the Benassy, Malinvaud, Muellbauer and Portès terminology, the point B corresponds to an equilibrium of classical unemployment. But the point B can be desirable for the consumer without being desirable for the producer because profitability stops at the point C. We then have a potential conflict. This fact corresponds to the Keynesian idea that pressures on prices can start before full employment since, in C, the producer will ask for a revision of prices.

Recall that the curves $\overline{OB}$ and $\overline{OC}$ in diagram I were obtained under the assumption $p_3 = \pi_3 = \varphi_3$. By dropping this assumption these curves will no longer pass through the origin.

When spill-over effect of labor services do not reduce to their income effect, one has generalized multipliers. In that generalized case, a similar analysis can be done [7] and leads to similar results.

6 MDP Processes in Macroeconomics

In this section, we will use the preceding comparative static results in order to sketch an MDP process based on desirability and an MDP process based on profitability (taking for granted that the convergence towards a maximal Goss National Product is not interesting). Then, we will interpret these processes in terms of the information they presuppse.

Let us start with the equations (67) and (68) of the Appendix and write them into the following compact form

(48) $$dP_r = q_3\, da_3$$

(49) $$\frac{dv}{\lambda} = r_3\, da_3$$

Let $\hat{\theta}_1$, and $\hat{\theta}_2$ be two positive diagonal matrices and let us consider that

(50) $$da_3 = \hat{\theta}_1 \, q_3$$

or

(51) $$da_3 = \hat{\theta}_2 \, r_3.$$

These relations define two kinds of dynamical systems. Both of them will lead to an equilibrium since they are monotonic. Indeed if we substitute (50) and (51) into (48) and (49), we find

(52) $$dP_r = q_3 \, \hat{\theta}_1 \, q_3 \geqslant 0$$

(53) $$\frac{dv}{\lambda} = r_3 \, \hat{\theta}_2 \, r_3 \geqslant 0.$$

With respect to Diagram I, these equations mean that if we are at the point A, q_3 and r_3 are different from zero. Applying equation (51) will lead us to the point 0 if the initial price system is p^0 and to the point B if the initial price system is p^1. In the same way, applying equation (50) will lead us to a point on $dPr = 0$, this point varying according to the initial price system.

Equations (50) and (51) are interesting because they give the direction for the whole vector da_3 such that profitability or desirability can become null. But these equations suppose that q_3 and r_3 associated to each policy are known, that is to say, at each step, one must compute π_1, π_2, Y_{21}, k_1 and so on. This is a requirement of micro-economic information. Besides, it can be dangerous to only use a macroeconomic policy: it seems that what should be done is to use a nested process where a policy on da_3 is accompanied by a policy on r_3 (for example, by policies on prices).

Moreover, let us remark that the first process corresponds to the producer's marginal willingness while the second process corresponds to the consumer's marginal willingness. Of course, they are not equivalent and can be conflictual. One can study a compromise by considering a convex linear combination of both processes. One can also consider both processes simultaneously.

7. The case of the generalized multipliers can be analysed according to the lines of section 5 if one uses the decomposition

$$(I - A_{11})^{-1} = N + \frac{N k_1 (p_1 - \pi_1) N}{1 - (p_1 - \pi_1) N k_1}$$

where $N = \left[I - \frac{\partial x_1}{\partial x_2} \frac{\partial y_2}{\partial y_1} \right]^{-1}$ and A_{11} is defined in Appendix A.

7 The Case of Rational Expectations

In fact, it is not realistic to assume that a change in the inputs of new capital goods can be implemented without changing agents' expectations. In this section, we will consider the effects of the modifications ($d\tilde{R}$, $d\tilde{x}_2$, $d\tilde{y}_1$) associated with da_3.

Of course, if we assume that expectations are arbitrary functions of da_3, it is not possible to characterize growth, profitability and desirability. Besides, taking the opposite viewpoint, we assume that when expectations are rational, growth, profitability and desirability can be fully characterized. By "rational" expectations, we mean that both agents have identical expectations on future prices and on any other exogenous variables. Furthermore, they assume that there will be an extended supply–constrained equilibrium on every market. (Usually, it is considered that agents expect a Walrasian equilibrium, which is not the proper context to study the often asserted disappearance of the multiplicative effects of da_3).

In the case of rational expectations, it can be seen that when the nominal discount factor is not sensitive to the inputs of new capital goods, a change in these inputs has no effect on desirability, no multiplicative effect on Gross National Product, the effect on profitability being equal to the difference between marginal costs and marginal revenues associated to these inputs. Therefore, this difference is null with a neoclassical production function. This general statement must be qualified. As the proof is in Appendix B, we will just sketch it here.

First, the multiplicand associated to the simple "Keynesian" multipliers will vanish and, consequently, we will lose this kind of multiplicative effect not because the multipliers are altered but because the multiplicand vanishes. A reason for this result is that the consumer and the producer expect identical $\tilde{S}^F$ so that the consumer is conscious that even if he does not finance the investment today, he will have to do it tomorrow. The rest is a matter of actualization. Remark that such a phenomenon depends on the assumption of a finite horizon.

Even if we have lost "Keynesian" multipliers, we still have some physical multipliers of the kind $(I - S_{12} Y_{21})^{-1}$. In general, they will not disappear under the rational expectation hypothesis. In the particular case where the consumption functions are "Keynesian" i.e. when $\frac{\partial x_1}{\partial x_2} = -\frac{\partial x_1}{\partial R} p_2$, they will disappear. They will also disappear if we assume that the production function is neoclassical ($\pi_3 = \varphi_3$). But, on prior grounds, these last two assumptions have nothing to do with the rational expectation hypothesis. Consequently, in spite of its remarkable bill of requirements, the rational expectations hypothesis is not as implosive as what is usually believed. It is a mixture with other seemingly technical assumptions that does the job.

8 Conclusion

In this paper, we used an extended supply-constrained equilibrium model within a temporary framework consistent with National Accounts in order to measure the desirability and profitability of a macroeconomic policy and to utilize then in order to define normative macroeconomics as an MDP process. Multipliers appeared in a natural way and were considered at various levels of generality. On the whole, our analysis was the result of a cross-fertilization between macroeconomics and microeconomics rather than a new exposition of the microfoundations of macroeconomics.

If we had to design some research avenues, we would say that the endogenisation of investment is a minor affair. The consideration of many financial assets should start from a macroeconomic viewpoint while the consideration of many consumers (in order to study macroeconomic reforms as GUESNERIE [1977] studied fiscal reforms) should start from a microeconomic viewpoint. As for the analysis of expectations, it seems unclear if it is an empirical question or if it should necessitate a relevant axiomatization.

APPENDIX A

Let us consider the differentials of equations (29) to (32) when $\zeta=0$ and when the only varying exogenous variable is a_3.

First, the differentials of equations (30) and (31) can be written

(54) $$dx_2 = Y_{21}\,dy_1 + Y_{20}\,d\gamma + (Y_{23}+Z_{23})\,da_3$$

(55) $$db_3 = da_3$$

where $Y_{20} = \partial y_2/\partial\gamma$.

Substituting these differentials into equations (29) and (32) leads to

(56) $$(I - A_{11})\,dy_1 + A_{10}\,d\gamma = A_{13}\,da_3$$

(57) $$A_{01}\,dy_1 + A_{00}\,d\gamma = A_{03}\,da_3$$

where

$$A_{11} = \frac{\partial x_1}{\partial R}p_1' + \left[\frac{\partial x_1}{\partial R}p_2' + \frac{\partial x_1}{\partial x_2}\right]Y_{21}$$

$$A_{10} = -\frac{\partial x_1}{\partial \gamma} - \left[\frac{\partial x_1}{\partial R}p_2' + \frac{\partial x_1}{\partial x_2}\right]Y_{20}$$

$$A_{01} = \frac{\partial y_0}{\partial y_1} - \frac{\partial x_0}{\partial R}p_1' - \left[\frac{\partial x_0}{\partial R}p_2' + \frac{\partial x_0}{\partial x_2}\right]Y_{21}$$

$$A_{00} = \frac{\partial y_0}{\partial \gamma} - \frac{\partial x_0}{\partial \gamma} - \left[\frac{\partial x_0}{\partial R}p_2' + \frac{\partial x_0}{\partial x_2}\right]Y_{20} - \frac{w_0}{\gamma^2}$$

$$A_{13} = \frac{\partial x_1}{\partial R}p_3' + \left[\frac{\partial x_1}{\partial R}p_2' + \frac{\partial x_1}{\partial x_2}\right][Y_{23}+Z_{23}]$$

$$A_{03} = \frac{\partial x_0}{\partial R}p_3' - \frac{\partial y_0}{\partial b_3} - \frac{\partial y_0}{\partial(-a_3)} + \left[\frac{\partial x_0}{\partial R}p_2' + \frac{\partial x_0}{\partial x_2}\right][Y_{23}+Z_{23}]$$

Equation (56) can be written

(58) $$dy_1 = (I - A_{11})^{-1}A_{10}\,d\gamma + (I - A_{11})^{-1}A_{13}\,da_3$$

In the same way, equation (57) can be written

(59) $$d\gamma = -A_{00}^{-1}A_{01}\,dy_1 + A_{00}^{-1}A_{03}\,da_3$$

Solving equations (56) and (57) leads to

(60) $$d\gamma = [A_{00} - A_{01}(I - A_{11})^{-1}A_{10}]^{-1}[A_{03} - A_{01}(I - A_{11})^{-1}A_{13}]\,da_3$$

(61) $$dy_1 = \{(I - A_{11})^{-1}A_{10}[A_{00} - A_{01}(I - A_{11})^{-1}A_{10}]^{-1} \times [A_{03} - A_{01}(I - A_{11})^{-1}A_{13}] + (I - A_{11})^{-1}A_{13}\}\,da_3$$

Substituting (60) and (61) into (54) allows to express dx_2 in terms of da_3. Finally, the effect of a change in a_3 on growth of Gross National Product, on profitability and on desirability is obtained by substituting (54), (55), (60) and (61) into equations (38), (39), (40):

$$(62)\quad dY_r = \{p_1\{(I-A_{11})^{-1}A_{10}[A_{00}-A_{01}(I-A_{11})^{-1}A_{10}]^{-1} \times [A_{03}-A_{01}(I-A_{11})^{-1}A_{13}]+(I-A_{11})^{-1}A_{13}\}+p_3\}\,da_3$$

$$(63)\quad dP_r = \{(p_1-\pi_1)\{(I-A_{11})^{-1}A_{10}[A_{00}-A_{01}(I-A_{11})^{-1}A_{10}]^{-1} \times [A_{03}-A_{01}(I-A_{11})^{-1}A_{13}]+(I-A_{11})^{-1}A_{13}\} + (p_3-\pi_3)-(p_3-\varphi_3)\}\,da_3$$

$$(64)\quad \frac{dv}{\lambda} = [p_1+\pi_2 Y_{21}]\,dy_1+[\pi_2 Y_{20}-x_0]\,d\gamma+[\pi_2(Y_{23}+Z_{23})+p_3]\,da_3$$

$$= \{-[p_1+\pi_2 Y_{21}](I-A_{11})^{-1}A_{10}+[\pi_2 Y_{20}-x_0]\}\,d\gamma + [p_1+\pi_2 Y_{21}](I-A_{11})^{-1}A_{13}+[\pi_2(Y_{23}+Z_{23})+p_3]\}\,da_3$$

$$(65)\quad = \{\{-[p_1+\pi_2 Y_{21}](I-A_{11})^{-1}A_{10}+[\pi_2 Y_{20}-x_0]\} \times [A_{00}-A_{01}(I-A_{11})^{-1}A_{10}]^{-1}[A_{03}-A_{01}(I-A_{11})^{-1}A_{13}] + [p_1+\pi_2 Y_{21}](I-A_{11})^{-1}A_{13}+[\pi_2(Y_{23}+Z_{23})+p_3]\}\,da_3.$$

These relations (62), (63), (64) and (65) express the effect of a change in the inputs of new capital goods on Gross National Product, real profits and utility.

In order to analyse these complex relations, let us make a few simplifying assumptions. Let us assume that the nominal discount factor does not vary when there is a change in new capital goods, that is to say, the coefficient relating $d\gamma$ to da_3 in (60) is equal to zero. It follows that (62), (63) and (64) become

$$(66)\quad dY_r = \{p_1(I-A_{11})^{-1}A_{13}+p_3\}\,da_3$$

$$(67)\quad dP_r = \{(p_1-\pi_1)(I-A_{11})^{-1}A_{13}+(p_3-\pi_3)-(p_3-\varphi_3)\}\,da_3$$

$$(68)\quad \frac{dv}{\lambda} = \{[p_1+\pi_2 Y_{21}](I-A_{11})^{-1}A_{13}+\pi_2(Y_{23}+Z_{23})+p_3\}da_3.$$

By using the following assumption already done in section 3

$$p_2 Y_{21}+\pi_1=0$$
$$p_2 Y_{23}+\pi_3=0$$

desirability can be expressed as

$$(69)\quad \frac{dv}{\lambda} = \{[(p_1-\pi_1)-(p_2-\pi_2)Y_{21}](I-A_{11})^{-1}A_{13} -(p_2-\pi_2)(Y_{23}+Z_{23})+p_3-\pi_3\}\,da_3$$

that is to say in the *wedge space.*

Now, let us assume that the consumption functions are Keynesian, that is to say, that spill-over effects of labor services reduce to their income effects. This can be written

$$(70)\quad \frac{\partial x_1}{\partial x_2} = -\frac{\partial x_1}{\partial R}p_2'$$

(71) $$\frac{\partial x_0}{\partial x_2} = -\frac{\partial x_0}{\partial R} p_2'$$

Let us define

$$k_1 = \frac{\partial x_1}{\partial R}$$

$$k_0 = \frac{\partial x_0}{\partial R}$$

$\alpha_1 = p_1 k_1$ (marginal propensity to consume). Then, one has

$$(I - A_{11}) = I - k_1 p_1',$$

$$(I - A_{11})^{-1} = I + \frac{k_1 p_1'}{1-\alpha}$$

$$A_{13} = k_1 p_3'$$

and

(72) $$dY_r = \frac{1}{1-\alpha} p_3 \, da_3$$

(73) $$d\mathbf{P}_r = \left\{ (p_1 - \pi_1) \frac{k_1}{1-\alpha} p_3 + (p_3 - \pi_3) - (p_2 - \varphi_3) \right\} da_3$$

(74) $$\frac{dv}{\lambda} = \left\{ [(p_1 - \pi_1) - (p_2 - \pi_2) Y_{21}] \frac{k_1}{1-\alpha} p_3 \right.$$

$$\left. - (p_2 - \pi_2)(Y_{23} + Z_{23}) + (p_3 - \pi_3) \right\} da_3$$

APPENDIX B

Let us consider that when a_3 is varying, expectations $\tilde{y}_1$, $\tilde{x}_2$ and $\tilde{R}$ are also varying.

The differentials of equations (29)-(32) can be written

(75) $$dy_1 = k_1\, dR + S_{12}\, dx_2 + S_{10}\, d\gamma + l_1\, d\tilde{R} + S_{1\tilde{2}}\, d\tilde{x}_2$$

(76) $$dx_2 = Y_2\, dy_1 + Y_{20}\, d\gamma + (Y_{23} + Z_{23})\, da_3 + Y_{2\tilde{1}}\, d\tilde{y}_1$$

(77) $$db_3 = da_3$$

(78) $$Y_{00}\, d\gamma + Y_{01}\, dy_1 + (Y_{03} + Z_{03})\, da_3 + Y_{0\tilde{1}}\, d\tilde{y}_1 - \frac{w_0}{\gamma^2}\, d\gamma - k_0\, dR - S_{02}\, dx_2 - S_{00}\, d\gamma - l_0\, d\tilde{R} - S_{0\tilde{2}}\, d\tilde{x}_2 = 0$$

where $k_i = \dfrac{\partial x_i}{\partial R}$, $l_i = \dfrac{\partial x_i}{\partial \tilde{R}}$, $S_{i\,2} = \dfrac{\partial x_i}{\partial x_2}$, $S_{i\,\tilde{2}} = \dfrac{\partial x_i}{\partial \tilde{x}_2}$, $S_{i\,0} = \dfrac{\partial x_i}{\partial \gamma}$, $i = 0$, 1, and where $Y_{00} = \partial y_0/\partial \gamma$, $Y_{01} = \partial y_0/\partial y_1$, $Y_{03} = \partial y_0/\partial b_3$, $Z_{03} = \partial y_0/\partial a_3$.

The differentials of equations (27) and (33) lead to

(79) $$dR = p_1\, dy_1 + p_3\, da_3 + p_2\, dx_2.$$

Then, (75) to (79) lead to

(80) $$(I - A_{11})\, dy_1 + A_{10}\, d\gamma = A_{13}\, da_3 + [k_1 p_2 + S_{12}]\, Y_{2\tilde{1}}\, d\tilde{y}_1 + S_{1\tilde{2}}\, d\tilde{x}_2 + l_1\, d\tilde{R}$$

(81) $$A_{01}\, dy_1 + A_{00}\, d\gamma = A_{03}\, da_3 + [Y_{0\tilde{1}} + [k_0 p_2 + S_{02}]\, Y_{2\tilde{1}}]\, d\tilde{y}_1 + S_{0\tilde{2}}\, d\tilde{x}_2 + l_0\, d\tilde{R}$$

where A_{11}, A_{10}, A_{13}, A_{01}, A_{00} and A_{03} are already defined in Appendix A.

Now let us assume that $d\tilde{R}$, $d\tilde{y}_1$ and $d\tilde{x}_2$ are endogenous and obtained by solving the expected markets, that is to say, agents have identical expectations and these expectations are such that $\tilde{\zeta}_1 = 0$, $\tilde{\zeta}_2 = 0$ and $\tilde{\zeta}_3 = 0$.

Then, we have

(82) $$d\tilde{y}_1 = k_{\tilde{1}}\, dR + S_{\tilde{1}2}\, dx_2 + S_{\tilde{1}0}\, d\gamma + l_{\tilde{1}}\, d\tilde{R} + S_{\tilde{1}\tilde{2}}\, d\tilde{x}_2$$

(83) $$d\tilde{x}_2 = Y_{\tilde{2}1}\, dy_1 + Y_{\tilde{2}0}\, d\gamma + (Y_{\tilde{2}3} + Z_{\tilde{2}3})\, da_3 + Y_{\tilde{2}\tilde{1}}\, d\tilde{y}_1$$

(84) $$d\tilde{b}_3 = d\tilde{a}_3 = 0$$

Transposing (27) and (33) into the future leads to

(85) $$d\tilde{R} = \tilde{p}_1\, d\tilde{y}_1 + \tilde{p}_2\, d\tilde{x}_2 - d\tilde{S}^F.$$

By the consumer's expected budget contraint introduced in (2), we have

(86) $$d\tilde{S}^F = dx_0.$$

In order to analyse the complete system, we shall make the same simplifying assumptions as before; that is to say, we will assume that the nominal discount factor does not react to a variation in a_3 and that spill-over effects reduce to income effects. It follows that from (34) and (32)

(87) $$p_3\, da_3 = \gamma\, dy_0 = \gamma\, dx_0$$

Therefore, one has

(88) $$\gamma\, d\tilde{S}^F = p_3\, da_3$$

(82) can then be written

$$d\tilde{y}_1 = k_{\tilde{1}}\,[p_1\, dy_1 + p_3\, da_3 + p_2\, dx_2] + S_{\tilde{1}2}\, dx_2 + S_{\tilde{1}0}\, d\gamma$$

$$+ l_{\tilde{1}}\left(\tilde{p}_1\, d\tilde{y}_1 + \tilde{p}_2\, d\tilde{x}_2 - \frac{1}{\gamma} p_3\, da_3\right) + S_{\tilde{1}\tilde{2}}\, d\tilde{x}_2$$

$$= k_{\tilde{1}}\,(p_1\, dy_1 + p_3\, da_3) + l_{\tilde{1}}\tilde{p}_1\, d\tilde{y}_1 - \frac{l_{\tilde{1}}}{\gamma} p_3\, da_3$$

$$(I - \gamma\, k_{\tilde{1}}\tilde{p}_1)\, d\tilde{y}_1 = k_{\tilde{1}} p_1\, dy_1$$

because $l_{\tilde{1}} = \gamma\, k_{\tilde{1}}$.

(80) becomes

$$(I - A_{11})\, dy_1 = A_{13}\, da_3 + l_1\, \tilde{p}_1\, d\tilde{y}_1 - \frac{l_1}{\gamma} p_3\, da_3$$

$$(I - A_{11})\, dy_1 = A_{13}\, da_3 + l_1\, \tilde{p}_1 \frac{k_{\tilde{1}}}{a - \gamma\, \tilde{k}_1\, k_{\tilde{1}}} p_1\, dy_1 - \frac{l_1}{\gamma} p_3\, da_3$$

$$\left[I - k_1 p_1 - \frac{l_1 \tilde{p}_1 k_{\tilde{1}}}{a - \gamma\, \tilde{k}_1\, k_{\tilde{1}}} p_1\right] dy_1 = \left[k_1 p_3 - \frac{l_1}{\gamma} p_3\right] da_3$$

$$= 0$$

because $l_1 = \gamma\, k_1$.

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