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# A Multiple Time Series Approach to Analyzing and Forecasting the Major French Monetary Aggregates

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**ABSTRACT.** – The seasonally adjusted French monetary aggregates M1, M2, and M3 are analyzed and forecasted by a multiple time series model. Two principal features of the analysis are the modelling of the variance-covariance structure of the joint distribution of the aggregates and the allocation of the aggregates to permanent and transient sources by means of an allocation parameter which is estimated from the data. The forecasting performance of the multiple model is compared to that of univariate random walk and ARIMA models.

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## Une approche de séries temporelles multiples pour analyser et prévoir les principaux agrégats monétaires français

**RÉSUMÉ.** – Les agrégats monétaires français M1, M2 et M3, corrigés des variations saisonnières, sont analysés et prévus par un modèle de séries temporelles multivarié. Deux traits principaux de l'analyse sont la modélisation de la structure de variance-covariance de la distribution jointe des agrégats et l'allocation des agrégats en une partie permanente et une partie transitoire à l'aide d'un coefficient d'allocation estimé à partir des données. La performance en prévision du modèle multivarié est comparée à celle de marches aléatoires univariées et de modèle ARIMA.

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# 1 Introduction

In this article a multiple time series model is introduced and applied to the analysis and forecasting of the seasonally adjusted French M1, M2, and M3 monetary aggregates. The model has four desirable features. First, in addition to accommodating the accounting and definitional relationships between these aggregates, the model provides a variance-covariance structure for the joint distribution of them which, when appropriately estimated, leads to a stochastic and dynamic modelling of the flows of assets across the aggregates. These flows have grown in importance since the mid 1970s in the United States, France, and other west European nations as market participants have responded to financial innovation, deregulation, changing interest rates, floating exchange rates, continuing government deficits and other influences. Such flows of assets have also been associated with instabilities in money demand, particularly for narrow aggregates, which have adversely influenced the forecast simulation performance of the standard single equation money demand models for a variety of countries. These models have led in most cases to consistent overprediction, even when known values of the exogenous variables in the equation are used to generate what are called dynamic forecast simulations. [1] Second, the model provides a separation of each of the constituent time series into two components, one representing what we call the unobserved, continuing or permanent influences and the other the transient influences. This separation is particularly useful for the application in this paper because it is natural to think of the monetary aggregates, which are observed, as being influenced by the continuing but unobserved actions of monetary authorities and economic agents and also by unanticipated random shocks or disturbances which strike the system through time as authorities and agents pursue their respective goals.

Third, the model features an allocation parameter which serves to allocate the total variance-covariance of the monetary aggregates to the permanent and transient sources. This parameter changes over time and these changes also have interesting interpretations with respect to the monetary aggregates. Fourth, the model can be used for forecasting the constituent time series jointly and it forecasts French M1, M2, and M3 as well as or better than alternate forecasting models. In the study of forecasting performance shown below, we compare the multiple model forecasts with those of univariate random walks and univariate ARIMA models. The latter are included in the comparison because Den Butter and Fase [1980] have shown that a univariate ARIMA model forecasted French M2 remarkably well during the period from the first quarter of 1977 through the fourth quarter of 1978. Indeed, these authors referred to their forecasts for France as "almost perfect" because they deviated on average only 0.4 percent from the realized values over the forecast period. They also showed that univariate ARIMA models performed better than dynamic forecast simulations from money demand equations developed by them for France and for five other EEC countries. Lastly, this multiple model does not require the difficult subjective step of identification of the ARIMA stochastic processes underlying the time series data and can be routinely and easily applied to both analyzing and forecasting the monetary aggregates.

# 2 Model formulation

It has been asserted that money supply changes (or some other transformation of the corresponding levels) are a random walk despite the continuing influences of monetary authorities and market participants. PIERCE and PORTER [1973, p. 541] concluded after extensive analysis that "... it is reasonable to say that over the 1968-1973 sample period seasonally adjusted money supply [U.S. M1] consists of a linear time trend plus a random walk". More recently, MELTZER [1982] investigated U.S. M1 growth from 1975 to 1981 and asserted that quarterly money growth rates can be described as a random walk with a large error term. In considering a closely related concept, GOULD, MILLER, NELSON, and UPTON [1978] concluded that a quarterly random walk model was appropriate for the velocity of M1 for the period 1955-1973. In an earlier paper, GOULD and NELSON [1974] found that the velocity series constructed by Friedman and Schwartz for the U.S. for 1869-1972 is "... well characterized by a simple random walk". As a result of these findings and related work of other researchers, we have chosen to incorporate a random walk hypothesis into our multiple time series approach, but we utilize it to model the permanent component rather than the observations on the aggregates themselves (or some transformation of them). As will be shown, this produces various advantages for both analysis and forecasting; in particular, forecasting performance is enhanced relative to univariate random walk models of the observations on the aggregates.

# 3 Model description

Because of the overlapping in the definition of M1, M2, and M3, it is convenient to work with M1, M2 − M1 (*disponibilités quasi monétaires*), and M3 − M2 (*placements en caisses d'épargne, bons du Trésor, des PTT et de la CNE*). We introduce the notation

$$X_1(t) = M_1(t)$$

1. Overprediction resulted for France in the money demand studies of DOOLEY and SPINELLI [1985], FROCHEN [1983, 1984], *INSEE* [1984] and *Ministère de l'Économie* [1984]; see also BOUGHTON [1979, 1981], HENDRY [1979], CALLIARI, SPINELLI, and VERGA [1984], ROWLEY [1985], and THORNTON [1985]. Earlier and seminal works are GRANDMONT [1973], GOLDFIELD [1973, 1976] and ENZLER, JOHNSON, and PAULUS [1976]. Good surveys appear in JUDD and SCADDING [1983] and HAFER and HEIN [1980 and 1982].

$$X_2(t)=M_2(t)-M_1(t)$$
$$X_3(t)=M_3(t)-M_2(t),$$

where $X_1(t)$, $X_2(t)$ and $X_3(t)$ are referred to as level variables. We are interested in rates of change in these time series (and in M1, M2, and M3) but for convenience we work with

(1) $$Y_1(t)=\log(X_1(t))-\log(X_1(t-1))$$
(2) $$Y_2(t)=\log(X_2(t))-\log(X_2(t-1))$$
(3) $$Y_3(t)=\log(X_3(t))-\log(X_3(t-1)),$$

which provide a satisfactorily close approximation for the data used in this paper and which are, as will be shown in Figures 2, 3, and 4, adjusted approximately for trend. Following the discussion in the previous section, a random walk model for these observations is

(4) $$Y_1(t)=Y_1(t-1)+\eta_1(t)$$
(5) $$Y_2(t)=Y_2(t-1)+\eta_2(t)$$
(6) $$Y_3(t)=Y_3(t-1)+\eta_3(t),$$

where $\eta_1(t)$, $\eta_2(t)$ and $\eta_3(t)$ are serially uncorrelated, jointly normally distributed disturbance terms each having mean 0.

We link (4), (5), and (6) through the variance-covariance structure of the vector $\eta(t)=[\eta_1(t), \eta_2(t), \eta_3(t)]'$, so that

$$V_\eta(t)=\begin{bmatrix} \mathrm{Var}[\eta_1(t)] & \mathrm{Cov}[\eta_1(t), \eta_2(t)] & \mathrm{Cov}[\eta_1(t), \eta_3(t)] \\ \mathrm{Cov}[\eta_2(t), \eta_1(t)] & \mathrm{Var}[\eta_2(t)] & \mathrm{Cov}[\eta_2(t), \eta_3(t)] \\ \mathrm{Cov}[\eta_3(t), \eta_1(t)] & \mathrm{Cov}[\eta_3(t), \eta_2(t)] & \mathrm{Var}[\eta_1(t)] \end{bmatrix}$$

In addition to $V_\eta(t)$, one can consider the correlation matrix associated with the vector $\eta(t)$,

(7) $$R_\eta(t)=\begin{bmatrix} 1 & R[\eta_1(t), \eta_2(t)] & R[\eta_1(t), \eta_3(t)] \\ R[\eta_2(t), \eta_1(t)] & 1 & R[\eta_2(t), \eta_3(t)] \\ R[\eta_3(t), \eta_1(t)] & R[\eta_3(t), \eta_2(t)] & 1 \end{bmatrix},$$

where as usual

$$R[\eta_i(t), \eta_j(t)]=\frac{\mathrm{Cov}[\eta_i(t), \eta_j(t)]}{\sqrt{\mathrm{Var}[\eta_i(t)]\,\mathrm{Var}[\eta_j(t)]}}$$

for $1 \leqslant i, j \leqslant 3$. We will make use of properties of $R_\eta(t)$ in analyzing the French monetary aggregates below.

The model can be generalized by assuming that each of the time series (1), (2), and (3) is composed of an unobserved permanent component and a transient component, where it is the permanent component instead of the observed series that is modeled as a random walk. If we let $Y(t)=[Y_1(t), Y_2(t), Y_3(t)]'$, the resulting model can be written as

(8) $$Y(t)=P(t)+\varepsilon(t)$$
(9) $$P(t)=P(t-1)+\eta(t),$$

where $P(t)=[P_1(t), P_2(t), P_3(t)]'$ is the vector of permanent components while $\varepsilon(t)=[\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t)]'$ is the vector of serially uncorrelated transient components having mean 0 and variance-covariance matrix $V_\varepsilon(t)$. Moreover, it is assumed that $\varepsilon(t)$ and $\eta(t)$ are contemporaneously uncorrelated. Note that (9) can be written as $P(t)-P(t-1)=\eta(t)$, emphasizing that the first differences of the permanent component series are independent (i. e., form a white noise process). Indeed, this is the only sense in which equation (9) is to be interpreted in this paper, as will be made clear below.

We note that the term "random walk" as used to refer to the processes $P_1(t)$, $P_2(t)$, and $P_3(t)$ in (8) and (9) has a slightly different interpretation in our model than the one that has come to be standard in the stochastic processes literature. In the latter, a random walk is understood to possess independent increments (i. e., the first differences are a white noise process), and starting values are usually specified for time $t=0$ [i. e., one specifies $P_1(0)$, $P_2(0)$, and $P_3(0)$ as initial conditions]. In this paper the phrase, random walk, means that we have independent increments but that we specify $P_1(T+1)$, $P_2(T+1)$, and $P_3(T+1)$ as initial conditions rather than $P_1(0)$, $P_2(0)$, and $P_3(0)$, where T denotes the last time period for which data on Y is observed. We choose this specification because we wish to use the model for forecasting and because it is reasonable to regard earlier observations as less informative for forecasting than more recent observations. As a result of our choice of initial conditions, the conditional variances of the observations will grow larger as one moves backward in time from time $t=T+1$. This point will be considered in detail in the Appendix to the paper, where a full discussion of the model's properties is presented.

Finally, we introduce an allocation parameter $\gamma(t)$, $0\leqslant\gamma(t)\leqslant 1$, which serves to allocate the total variance-covariance matrix $V(t)=V_\eta(t)+V_\varepsilon(t)$ to the two sources of randomness in (8) and (9), namely the transient component $\varepsilon(t)$ in (8) and the random influence $\eta(t)$ associated with the permanent component in (9). This is accomplished by means of

(10) $$V_\varepsilon(t)=(1-\gamma(t))\,V(t)$$

(11) $$V_\eta(t)=\gamma(t)\,V(t)$$

where $V(t)$ is nondiagonal.

From (10) and (11) we see that the parameter $\gamma(t)$ can be expressed as

(12) $$\gamma^3(t)=\frac{|V_\eta(t)|}{|V(t)|}=\frac{|V_\eta(t)|}{|V_\eta(t)+V_\varepsilon(t)|},$$

so that $\gamma^3(t)$ can be interpreted as the ratio of the generalized variance $|V_\eta(t)|$ of the influences associated with the permanent component to the generalized variance $|V(t)|=|V_\eta(t)+V_\varepsilon(t)|$ associated with both the permanent and transient influences of the multiple time series system (8) and (9). Thus when $\gamma(t)=0$ we have $V_\eta(t)=0$ and therefore $\eta(t)=0$ with probability 1 in (9). This means that the observed time series is influenced by transient components alone, i. e., the permanent component is a constant for all $t$. Alternatively, when $\gamma(t)=1$, $V_\varepsilon(t)=0$; then $Y(t)=P(t)$ with probability 1, and the permanent component $P(t)$ is a pure random walk without transient influences. It should be noted from (10) and (11) that because the variance-covariance matrices $V_\varepsilon(t)$ and $V_\eta(t)$ are proportional, the correlation matrix for $\eta(t)$, $R_\eta(t)$ in (7), is equal to the correlation matrix $R_\varepsilon(t)$.

We note that the model (8) and (9) differs from a standard vector ARIMA model because the permanent component $P(t)$ is unobserved. Although (8) and (9) could be rewritten so as to produce a vector ARIMA model in the observed $Y(t)$, a model which could be used for forecasting purposes, such a restatement would cause us to lose the opportunity of examining the behavior of the allocation parameter over time as well as the dynamic variance-covariance and correlation structure linking the components of the vectors $\varepsilon(t)$ and $\eta(t)$ in (8) and (9) — the desirability of which is a principal feature of our model and will be discussed in the next section.

The application of this model results in estimates of the allocation parameter $\gamma(t)$ and the variance-covariance matrices $V_\varepsilon(t)$ and $V_\eta(t)$ using maximum likelihood-based estimation methods developed by the authors in another context and presented in Enns, Machak, Spivey, and Wrobleski [1982]. Along with these estimates, the estimation procedure also produces a 1-month ahead forecast $\hat{P}(T+1)$ of the permanent component $P(T+1)$, jointly with the other model parameters. In other words, this estimate is a 1-step ahead forecast of both the permanent component $P(T+1)$ at $t=T+1$ and of the original observed series $Y(T+1)$. These estimation methods are described in the Appendix. A sequence of 1-month ahead forecasts is produced as follows. We begin by using four years of monthly data on the French monetary aggregates (February 1977 through January 1981) as the initial estimation period. The first of the 1-month ahead forecasts is made for February 1981; then the actual observation for February 1981 is appended to the estimation period and the observation for February 1977 is deleted from it. The model parameters are then reestimated using this next estimation period and a forecast for March 1981 is produced. Thus the estimation period (of length 48 months) "rolls through the data" in this manner and updated forecasts are generated for each month of the forecast period (which extends from February 1981 through October 1983). This model was previously applied to the analysis and forecasting of U.S. monetary aggregates and the results were published in Machak, Spivey, and Wrobleski [1983].

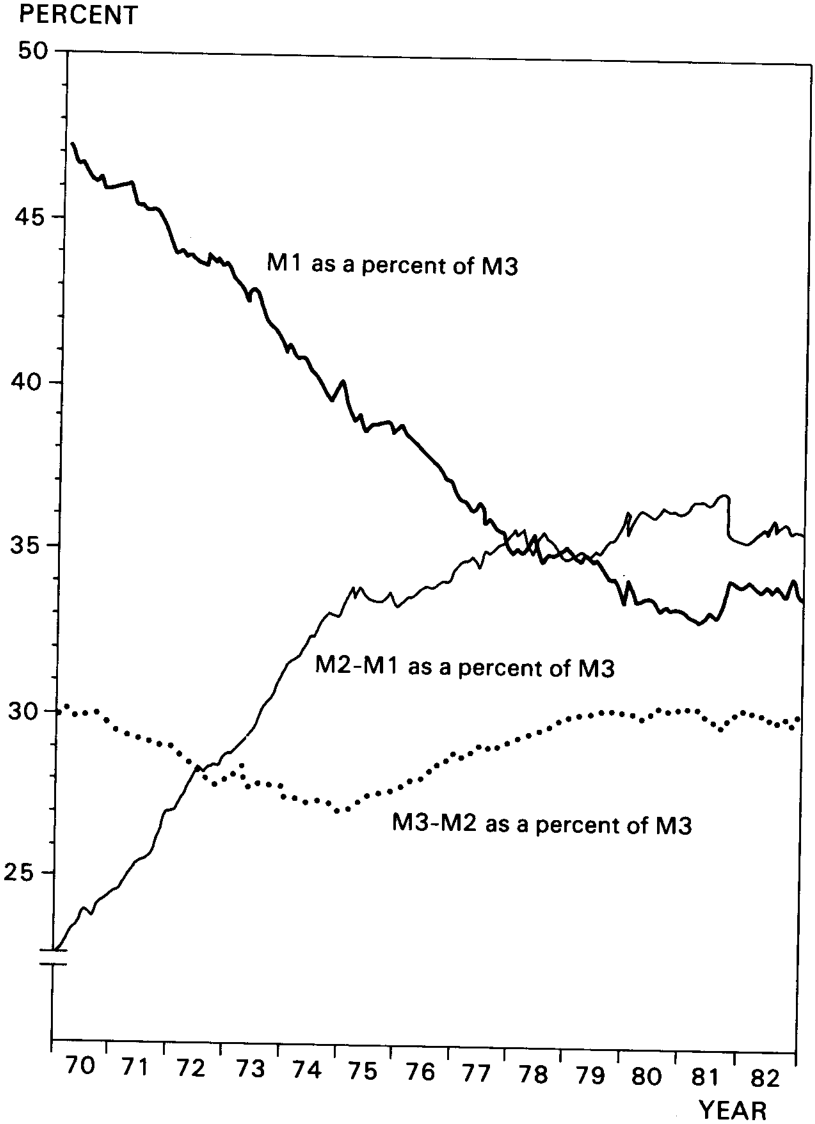
# 4 Analysis of French monetary aggregates

The flows of assets across the major French monetary aggregates were substantial over the 1970-1982 period. A convenient way to illustrate these flows is given in Figure 1, which displays M1, M2 − M1, and M3 − M2 as percents of M3 by month. M1 as a percent of M3 declined rapidly from early 1970 (when it was about 47 percent) until the end of 1981, when it began to level off at about 34 percent. M2 – M1 as a percent of M3 grew from about 23 percent in 1970 to about 36 percent by the end of 1982, after beginning to level off in early 1980, whereas M3 – M2 as a percent of M3 declined somewhat in the first half of the 1970s, then in mid-1975 began a slow rise so that by the end of the period shown it had reached its early 1970 figure of approximately 30 percent.

These shifts suggest that univariate forecasting approaches applied to the French monetary aggregates – which cannot directly assimilate the shifting mix between the aggregates – could be improved upon by a multiple time series approach of the kind used in this study. As indicated earlier, we have applied the multiple model (8) and (9) to the first differences of the logarithms of M1, M2 – M1, and M3 – M2 [see (1), (2), and (3)]. These

FIGURE 1

M1, M2 – M1, ***and*** M3 – M2 ***as a Percent of*** M3.

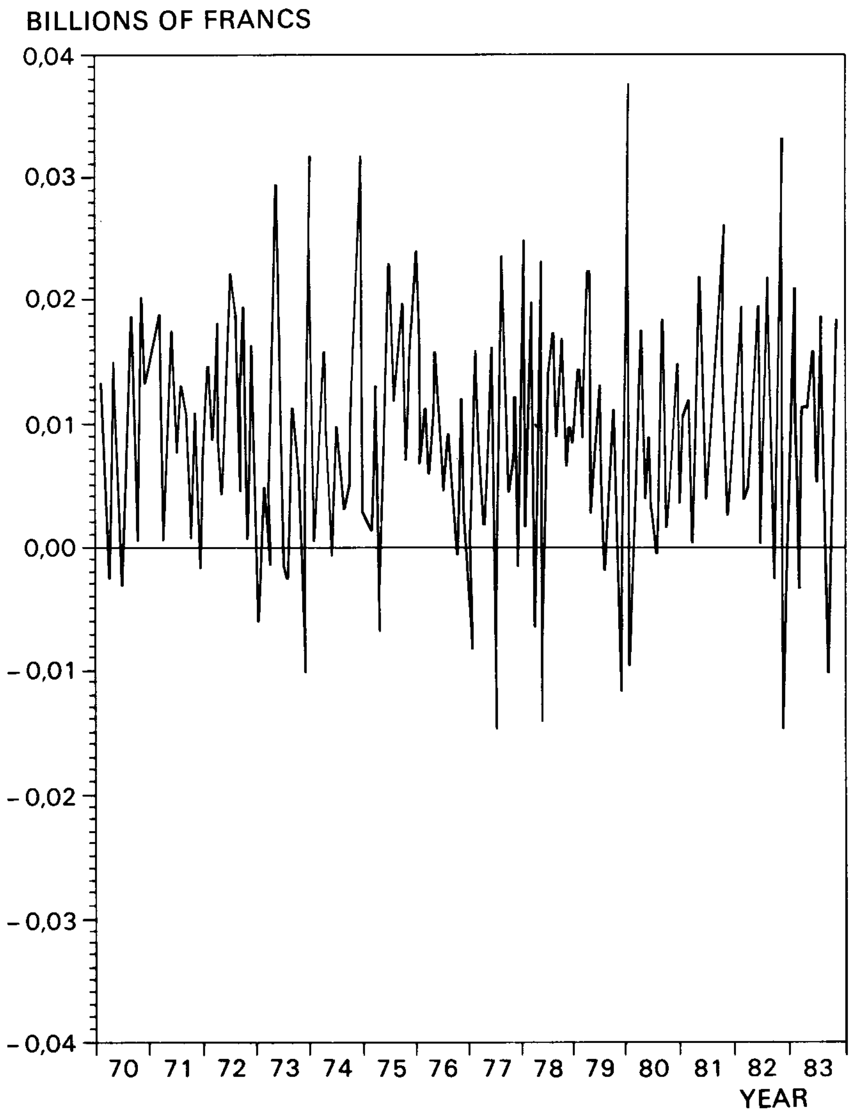


differences are plotted in Figures 2, 3, and 4, respectively. Figure 3 shows a remarkable "dip" in September 1981, which may be attributable to influences in existence in the early months following the change of governments which began in June of 1981. This dip occurs in the forecast period and adversely influences the forecasting performance of all the models considered in this paper.

The model (8) and (9) links the time series $Y_1(t)$, $Y_2(t)$, and $Y_3(t)$ in (1), (2), and (3) through its variance-covariance structure and by means of the allocation parameter $\gamma(t)$ given in (10), (11), and (12). The estimates $\hat{\gamma}(t)$ of this parameter are plotted in Figure 5; they decline from 0.85 in February 1981 to 0.46 for September 1983. If we consider (10), (11), and (12), this means that the estimated generalized variance of the disturbance

FIGURE 2

*First Differences of* log (M1).

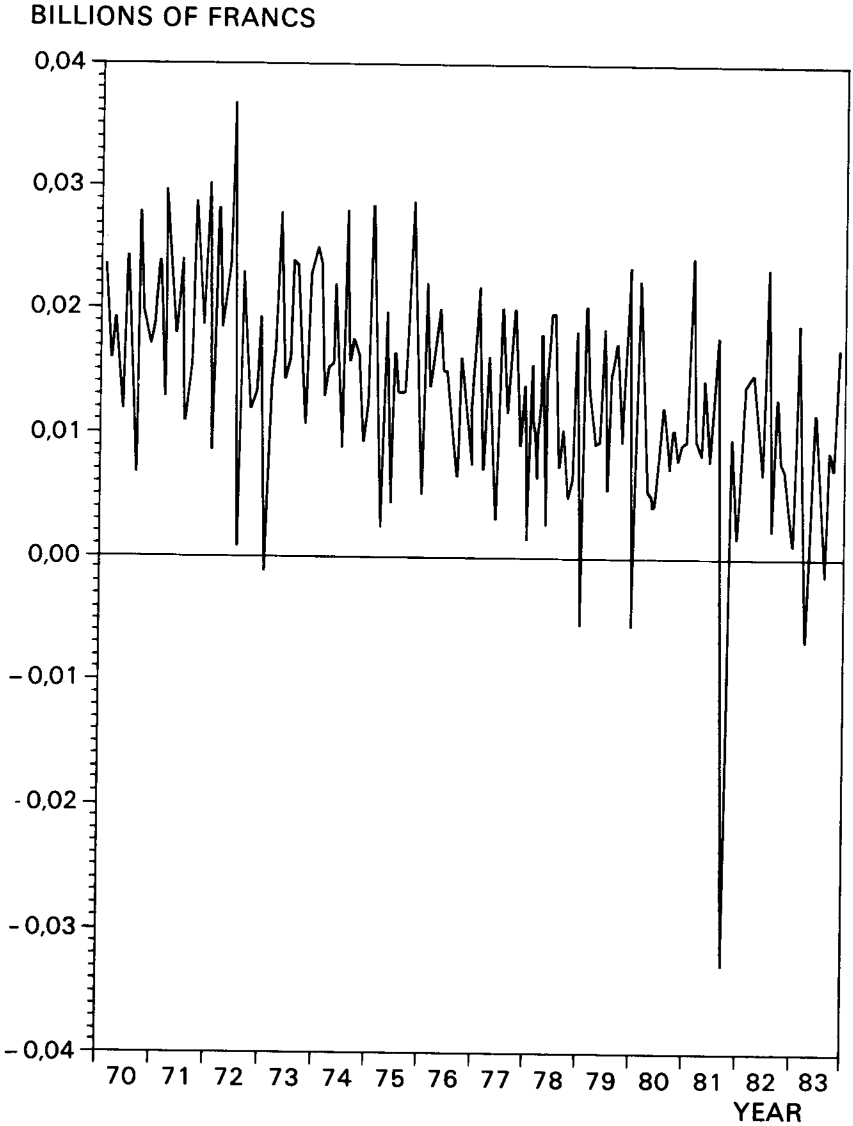


term associated with the permanent component P(t) declined as a percent of the total generalized variance over the period shown or that the generalized variance associated with the transient influences increased as a percent of the total estimated generalized variance. This suggests that the activities of market participants had an increasing influence on the rates of growth of the monetary aggregates whereas the monetary authorities had lessening influence on the observed growth rates over the period.

Figure 6 shows a plot of the sample estimates of the components of the correlation matrix $R_\eta(t)$ in (7) as estimated over each of the estimation periods as these periods roll forward in time. For example, the correlations for February 1981 were estimated from data from the first estimation period which extended from February 1977 through January 1981 and the

FIGURE 3

***First Differences of*** log (M2 – M1).



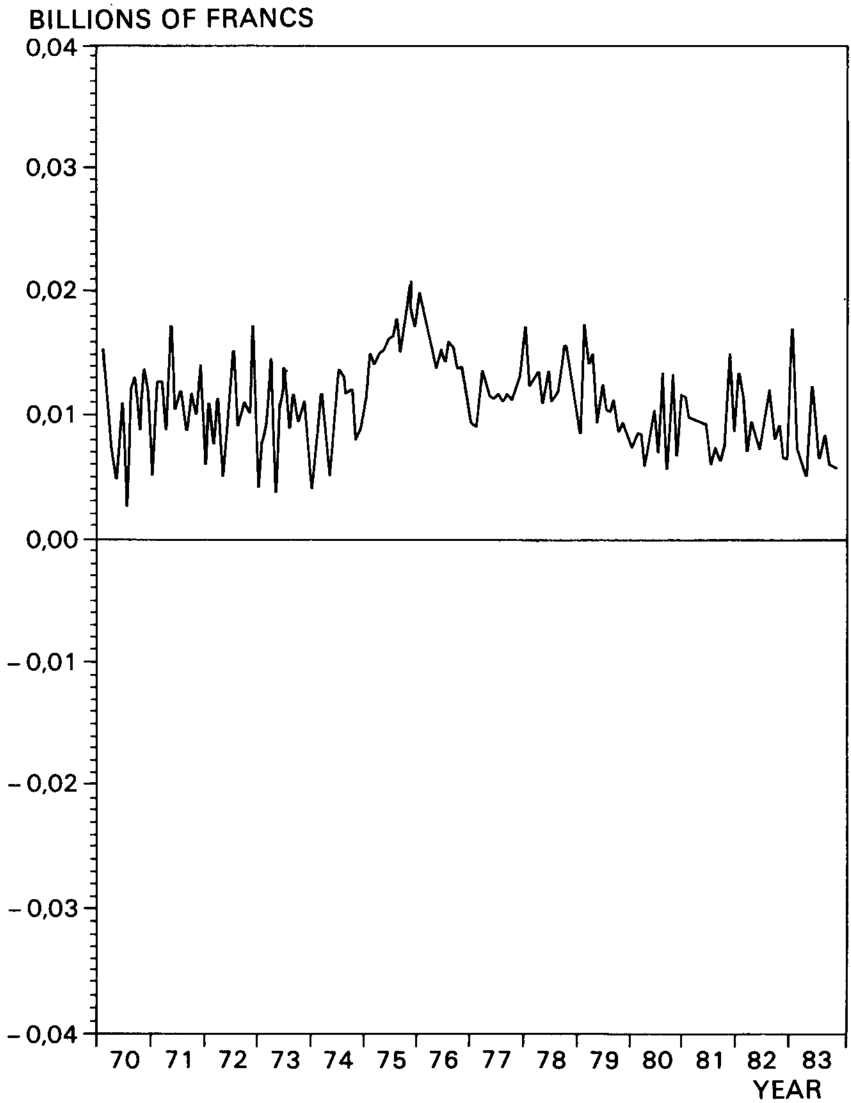
correlations for October 1983 were estimated from data from the last estimation period (October 1979 through September 1983).

The large negative correlations between the rates of change of M1 and M2 – M1 (the bottom time series in Figure 6) are consistent with the hypothesis that there is a net flow from M1 to M2 – M1 (the direction of this flow is suggested by Figure 1, in which the percent of the total money supply M3 allocated to M1 is falling at the same time that the percent allocated to M2 – M1 is rising). These correlations, moreover, are each significantly different from 0 at the .0,5 level. Unfortunately, a stronger statement than this concerning flows would require special studies of a sample of asset accounts across time.

The time series plotted in the center of Figure 6 shows the correlations

FIGURE 4

***First Differences of*** log (M3 – M2).

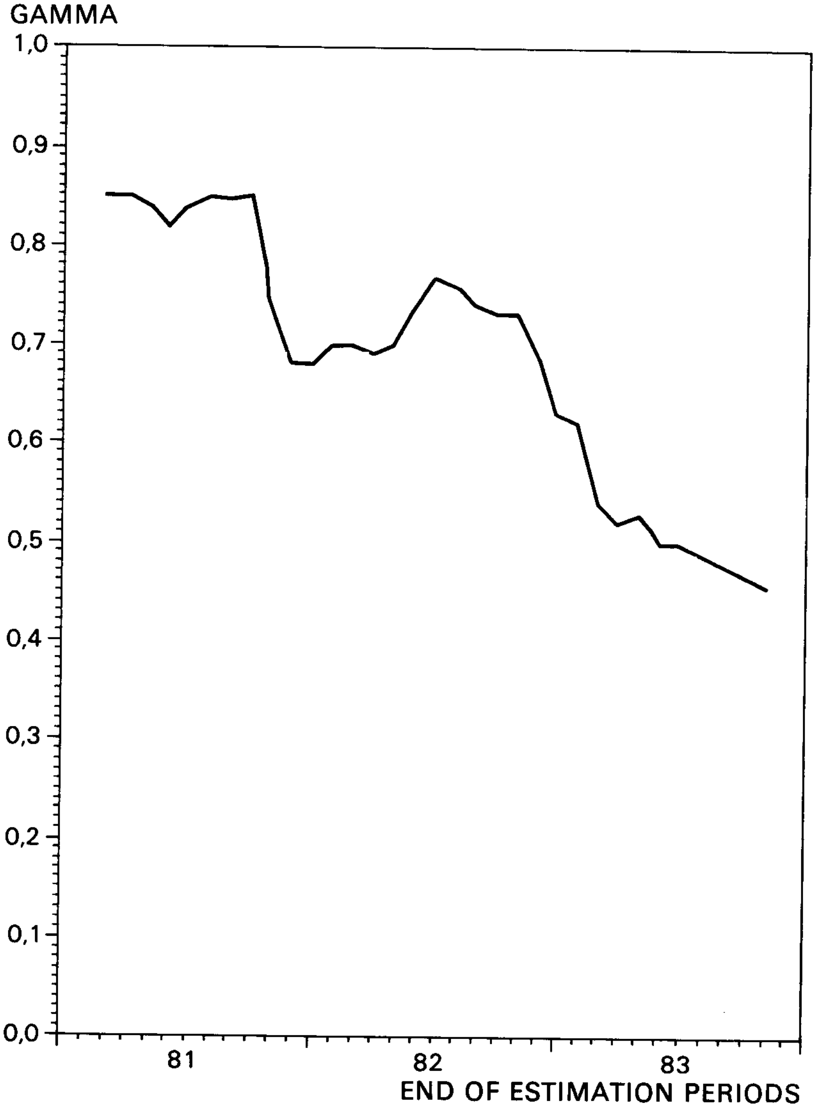


between the rates of change for M1 and M3 − M2. These fluctuate around −0.10 and none is significantly different from 0. Thus there appears to be no discernible net flow between these two aggregates, but this statement must be interpreted with care because there could have been considerable movement across these aggregates in either direction, resulting in no net flow because of cancellation.

The correlations between the rates of change of M2 − M1 and M3 − M2 are each positive (see top of Figure 6). Thus any flows across these aggregates are masked because the two appear to be moving in the same direction. Correlations between the rates of changes indicate that there was significant movement in the same direction for the estimation periods which ended in a given month of 1982.

FIGURE 5

***Estimated Values*** $\hat{\gamma}3\,(t)$ ***of the Allocation Parameter.***



# 5 Forecasting French monetary aggregates

Tables 1, 2, and 3 present descriptive statistics for the percent forecast errors for the levels of French M1, M2−M1, and M3−M2. The forecasts were one-month ahead forecasts. In terms of Mean Squared Error (MSE) the univariate random walk models turned in the poorest performance of

FIGURE 6

***Estimated Correlations*** $\hat{R}[\eta_i(t), \eta_j(t)]$.

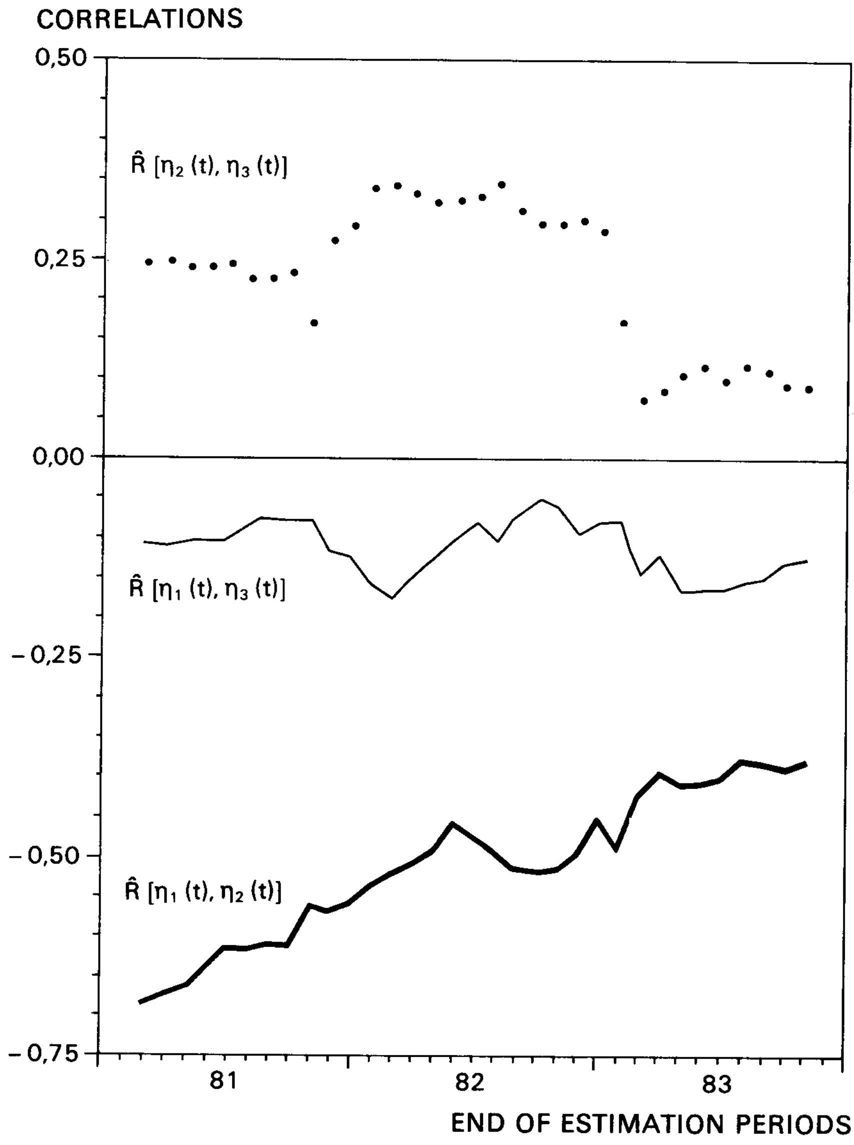


TABLE 1

***Descriptive Measures of Percent Forecast Errors for* M1**

| Measure | Multiple Model | Univariate ARIMA Model | Univariate Random Walk |
|---|---|---|---|
| Minimum | −4.0025 | −2.3060 | −4.8752 |
| Maximum | 3.2980 | 2.2014 | 3.5206 |
| Mean | −0.0138 | 0.1935 | 0.0098 |
| Lower Quartile | −1.3199 | −0.3534 | −1.2565 |
| Upper Quartile | 0.9488 | 1.0910 | 1.1005 |
| Median | 0.2126 | 0.1769 | 0.1375 |
| Midspread | 2.2687 | 1.4444 | 2.2615 |
| MSE | 0.0225 | 0.0118 | 0.0297 |

TABLE 2

***Descriptive Measures of Percent Forecast Errors for* M2 − M1**

| Measure | Multiple Model | Univariate ARIMA Model | Univariate Random Walk |
|---|---|---|---|
| Minimum | −4.5175 | −4.8977 | −4.4146 |
| Maximum | 3.1955 | 2.6345 | 4.1275 |
| Mean | −0.0288 | 0.0451 | 0.0333 |
| Lower Quartile | −0.5197 | −0.3847 | −0.7261 |
| Upper Quartile | 0.6374 | 0.7326 | 0.7703 |
| Median | 0.0173 | 0.2339 | −0.1133 |
| Midspread | 1.1571 | 1.1173 | 1.4964 |
| MSE | 0.0158 | 0.0169 | 0.0191 |

TABLE 3

***Descriptive Measures of Percent Forecast Errors for* M3 − M2**

| Measure | Multiple Model | Univariate ARIMA Model | Univariate Random Walk |
|---|---|---|---|
| Minimum | −0.6336 | −0.7840 | −0.9459 |
| Maximum | 1.0286 | 1.0824 | 1.0509 |
| Mean | −0.0172 | 0.0056 | −0.0122 |
| Lower Quartile | −0.2650 | −0.1642 | −0.2575 |
| Upper Quartile | 0.1288 | 0.0920 | 0.1987 |
| Median | −0.0746 | −0.0350 | −0.0096 |
| Midspread | 0.3938 | 0.2562 | 0.4562 |
| MSE | 0.0012 | 0.0013 | 0.0015 |

those considered for all three of the aggregates. The multiple model had the lowest MSE for M2 – M1 and M3 – M2, but the univariate ARIMA model had the lowest MSE for M1. The three models produced mixed results with respect to bias, which can be assessed in terms of the mean percent error. For example, for M1 the random walk model has the smallest bias, for M2 – M1 the multiple model bias is smallest, and for M3 – M2 the ARIMA model has the smallest bias.

In summary, the multiple model and the ARIMA models are to be preferred to the univariate random walk models, suggesting that it is useful to model the permanent component as a random walk rather than the observations themselves. With respect to the multiple and univariate ARIMA models, there is little reason to prefer one over the other based on forecast performance alone. In other words, the multiple model performs as well as ARIMA models, which, as shown by DEN BUTTER and FASE [1980], have performed remarkably well for France, and better than dynamic forecast simulations from money demand equations. Moreover, the multiple model is easier to construct and use than a collection of univariate ARIMA models, it enables one to incorporate a random walk into the model via the permanent component, and it can exploit the correlations between the monetary aggregates resulting from net flows across time.

## Estimation Methodology

Although the vectors $Y(t)$, $P(t)$, $\varepsilon(t)$, and $\eta(t)$ introduced in (8) and (9) all have dimension $3 \times 1$, this appendix will consider the slightly more general formulation of model (8) and (9) in which each of the above vectors will have dimension $k \times 1$. Thus, the model in (8) and (9) will represent a special case, corresponding to $k=3$, of the model considered here. In order to estimate the parameters in model (8) and (9) it will also be necessary to require $V_\varepsilon$, $V_\eta$, and $\gamma$ to be constant over the estimation period. Finally, because of the complicated notation introduced in this appendix, it will be convenient to denote the vectors $Y(t)$, $P(t)$, $\varepsilon(t)$, and $\eta(t)$ in subscripted fashion as $Y_t$, $P_t$, $\varepsilon_t$, and $\eta_t$. The same convention will apply to the time index $t$ of other vectors introduced in this appendix.

Using the above notation and conventions, model (8) and (9) can be restated as

(13) $$Y_t = P_t + \varepsilon_t$$

(14) $$P_t = P_{t-1} + \eta_t$$

where

$$\varepsilon_t \sim N(0, V_\varepsilon), \qquad \eta_t \sim N(0, V_\eta), \qquad E[\varepsilon_t \eta_t] = 0 \quad \text{for all } s, t.$$

Recall that in the Model Description section above, $\eta_t = P_t - P_{t-1}$ are assumed to be independent increments with conditioning being done with respect to time $t = T+1$ rather than time $t=0$ (as is often employed in the stochastic processes literature). In the conventional use of random walk models one is interested in computing the mean of $P_t$ with respect to $P_0$, when the realization of $P_t$ is subsequent to that of $P_0$. In our model, because we condition on $P_{T+1}$, we must compute the mean of $P_t$ with respect to $P_{T+1}$, where the realization of $P_t$ occurs prior to that of $P_{T+1}$. If the mean for $P_t$ conditioned on $P_{T+1}$ is computed for $t < T+1$ in a manner symmetric to the way it would be computed for $t > T+1$, then using (13) and (14), the mean of $Y_t$ conditional on $P_{T+1}$ can be written as

(15) $$E[Y_t \mid P_{T+1}] = P_{T+1}.$$

We stress that $E[Y_t \mid P_{T+1}]$ for $0 < t < T+1$ depends only on $P_{T+1}$ and not on both $P_{T+1}$ and $P_0$.

Next, let J be the $T \times 1$ vector, each of whose elements is the scalar 1, and define $\beta$ by

$$\beta = J \otimes P_{T+1},$$

Thus, $\beta$ is a $Tk \times 1$ vector consisting of T copies of the $k \times 1$ vector $P_{T+1}$ stacked on top of each other. If we write $Y = [Y_1', \ldots, Y_T']'$, it follows that Y, conditioned on β, has a multivariate normal density $N(\beta, \Phi)$, where the $k \times k$ blocks $\Phi_{st}$ of $\Phi$ are given by

$$\Phi_{st} = \mathrm{cov}[Y_s, Y_t \mid P_{T+1}] = \begin{cases} (T+1-t) V_\eta + V_\varepsilon, & s=t, \\ (T+1-t) V_\eta, & s<t, \\ (T+1-s) V_\eta, & s>t. \end{cases}$$

We note that the probability structure we have chosen for the $Y_t$ vectors results in conditional variances which grow larger as one moves backward in time from time $T+1$. This structure is similar to the traditional structure that one encounters when one moves forward in time with a random walk, except that the forward structure is now being reflected backwards in time. We make no assumptions about the value of $Y_t$ at time $t=0$. Because we are interested in using this model for forecasting, we believe that uncertainty should increase as one moves backwards from the end of the series at time $t=T+1$, rather than forwards from the beginning of the series.

Given $P_{T+1}$, the log likelihood function $L(V_\varepsilon, V_\eta \mid P_{T+1})$ can now be written as

$$(16) \quad \begin{cases} \log L(V_\varepsilon, V_\eta \mid P_{T+1}) = \log (2\pi)^{-Tk/2} |\Phi|^{-1/2} \exp\{-1/2 \| Y - \beta \|_{\Phi^{-1}}\} \\ \qquad \propto -1/2 \log |\Phi| - 1/2 \| Y - \beta \|_{\Phi^{-1}}. \end{cases}$$

The likelihood (16) can be rewritten if we make the following observations. First

$$(17) \qquad \Phi = P \otimes V_\eta + I \otimes V_\varepsilon,$$

where the elements of the $(T \times T)$ matrix P are given by

$$P_{st} = \min\{T+1-s, T+1-t\}.$$

Using the fact that

$$V_\varepsilon = (1-\gamma) V \qquad \text{and} \qquad V_\eta = \gamma V,$$

(17) can be rewritten as

$$(18) \qquad \begin{aligned} \Phi(\gamma, V) &= P \otimes \gamma V + I \otimes (1-\gamma) V \\ &= \gamma P \otimes V + (1-\gamma) I \otimes V \\ &= (\gamma P + (1-\gamma) I) \otimes V \end{aligned}$$

The log likelihood function can now be expressed in terms of $P_{T+1}$, γ, and V as

$$(19) \qquad \log L(\gamma, V \mid P_{T+1}) \propto -1/2 \log |\Phi(\gamma, V)| - 1/2 \| Y - \beta \|_{\Phi^{-1}(\gamma, V)}$$

In order to produce computationally tractable solution procedures for estimating the parameters $P_{T+1}$, γ, and V using (19), we employ a further reparameterization of (19) as follows. The matrix P, appearing in (18), satisfies an equation of the form $APA' = \Lambda$, where Λ is a diagonal matrix with diagonal entries $\lambda_j$ given by

$$\lambda_j = \left\{ 2 + \cos \left[ \frac{2\pi(T+1-j)(T+1-j)}{2T+1} \right] \right\}^{-1},$$

and the matrix A is orthogonal with entries given by

$$a_{ij} = \frac{2(-1)^{-j}}{\sqrt{2T+1}} \sin \left[ \frac{2\pi(T+1-i)(T+1-j)}{2T+1} \right].$$

If one now transforms Y by

$$\tilde{Y} = (A \otimes I) Y,$$

then $\tilde{Y}$ is multivariate normal with mean

$$E[\tilde{Y} \mid \beta] = (A \otimes I) E[Y \mid \beta] = (A \otimes I)(J \otimes P_{T+1}) = AJ \otimes P_{T+1},$$

and variance-covariance matrix

$$\begin{aligned} \mathrm{Var}[\tilde{Y} \mid \beta] &= (A \otimes I)[\gamma P + (1-\gamma) I \otimes V](A \otimes I)' \\ &= (\gamma \Lambda + (1-\gamma) I) \otimes V = D(\gamma) \otimes V \end{aligned}$$

where, of course, $D(\gamma) = \gamma \Lambda + (1-\gamma) I$. Thus the likelihood in (A7) can be expressed in terms of $\tilde{Y}$ as

$$(20) \qquad \log L(\gamma, V \mid P_{T+1}) \propto -\frac{1}{2} \log |D(\gamma) \otimes V| + \| \tilde{Y} - AJ \otimes P_{T+1} \|_{(D(\gamma) \otimes V)^{-1}}$$

$$= -\frac{k}{2} \log |D(\gamma)| - \frac{T}{2} \log |V| - \frac{T}{2} \sum_{t=1}^{T} \frac{1}{d_t(\gamma)} \| \tilde{Y}_t - a_t P_{T+1} \|_{V^{-1}}$$

where $d_t(\gamma)$ is the $t$-th diagonal element of $D(\gamma)$ and $a_t = \sum_{s=1}^{T} a_{ts}$, the sum of the elements of the $t$-th row of the matrix A.

We now introduce a nonlinear optimization procedure for obtaining estimates of the parameters $\gamma$, V, and $P_{T+1}$ in (20). Differentiating (20) with respect to $P_{T+1}$ and V gives the normal equations conditional upon $\gamma$, and solving these equations for the estimates $\hat{P}_{T+1}$ and $\hat{V}$ gives

$$(21) \qquad \hat{P}_{T+1}(\gamma) = \left( \sum_{t=1}^{T} \frac{a_s^2}{d_t(\gamma)} \right)^{-1} \sum_{t=1}^{T} \frac{a_t}{d_t(\gamma)} \tilde{Y}_t$$

and

$$(22) \qquad \hat{V}(\gamma) = \frac{1}{T} \sum_{t=1}^{T} \frac{1}{d_t(\gamma)} (\tilde{Y}_t - a_t \hat{P}_{T+1}(\gamma))(\tilde{Y}_t - a_t \hat{P}_{T+1}(\gamma))'.$$

If $\hat{P}_{T+1}(\gamma)$ and $\hat{V}(\gamma)$ are substituted into the log likelihood equation (19), the concentrated log likelihood function for $\gamma$ is obtained as

$$l(\gamma) = \log L(\hat{P}_{T+1}(\gamma), \gamma, \hat{V}(\gamma)) \tag{23}$$

and a simple grid search on $\gamma$ will produce the value $\hat{\gamma}$ of $\gamma$ which maximizes $l(\gamma)$ in (23). Using this value in (21) and (22) gives the estimates $\hat{P}_{T+1}(\hat{\gamma})$ and $\hat{V}(\hat{\gamma})$ which maximize (20).

# • References


BEGUIN, J.-M., GOURIEROUX, C. and MONTFORT, A. (1980). – "Identification of a Mixed Autoregressive-Moving Average Process: The Corner Method", in O. D. ANDERSON Ed., *Time Series*, Amsterdam: North-Holland Publishing Co., pp. 423-436.

BOUGHTON, J. M. (1979). – "Demand for Money in Major OECD Countries", in *OECD Economic Outlook*, Occasional Studies, January, pp. 35-57.

BOUGHTON, J. M. (1981). – "Recent Instability in the Demand for Money: An International Perspective", *Southern Economic Journal*, 47, pp. 579-597.

DEN BUTTER, F. A. G. and FASE, M. M. G. (1980). – "Forecasting the Money Stock in 6 EEC Countries", *mimeo*, Amsterdam: De Nederlandsche Bank NV.

CALLIARI, S., SPINELLI, F. and VERGA, G. (1984). – "Money Demand in Italy: A Few More Results", *The Manchester School*, 2, pp. 141-159.

DOOLEY, M. P. and SPINELLI, F. (1985). – "Financial Innovation, Deregulation, and Money Demand in France, Italy and Japan", to appear in *International Monetary Fund Staff Papers*.

ENNS, P. G., MACHAK, J. A., SPIVEY, W. A. and WROBLESKI, W. J. (1982). – "Forecasting Applications of an Adaptive Multiple Exponential Smoothing Model", *Management Science*, 9, pp. 1035-1044.

ENZLER, J., JOHNSON, L. and PAULUS, J. (1976). – "Some Problems of Money Demand", *Brookings Papers on Economic Activity*, 1, pp. 261-280.

FEROLDI, M. and MÉLITZ, J. (1982). – "The Franc and the French Financial Sector", *Annales de l'INSEE*, 47-48, pp. 9-42.

FROCHEN, P. (1983). – "Les innovations financières récentes en France et leur influence sur la demande de monnaie: un test économétrique", *mimeo*, Banque de France, octobre.

FROCHEN, P. (1984). – "La demande de liquidities M3, discussion des tendances récentes à l'aide de l'économétrie", *mimeo*, Banque de France, février.

GOLDFIELD, S. M. (1973). – "The Demand for Money Revisited", *Brookings Papers on Economic Activity*, 3, pp. 577-638.

GOLDFIELD, S. M. (1976). – "The Case of the Missing Money", *Brookings Papers on Economic Activity*, 3, pp. 683-730.

GOULD, J. P., MILLER, M., NELSON, C. R. and UPTON, C.W. (1978). – "The Stochastic Properties of Velocity and the Quantity Theory of Money", *Journal of Monetary Economics*, 4, pp. 229-248.

GOULD, J. P. and NELSON, C. R. (1974). – "The Stochastic Structure of the Velocity of Money", *American Economic Review*, 64, pp. 405-418.

GRANDMONT, J.-M. (1973). – "Sur la demande de monnaie de court terme et de long terme", *Annales de l'INSEE*, 9, pp. 65-85.

HAFER, R. W. and HEIN, S. E. (1980). – "The Dynamics and Estimation of Short-Run Money Demand", *Federal Reserve Bank of St. Louis Review*, March, pp. 26-35.

HAFER, R. W. and HEIN, S. E. (1982). – "The Shift in Money Demand: What Really Happened?" *Federal Reserve Bank of St. Louis Review*, February, pp. 11-16.

HENDRY, D. F. (1979). – "Predictive Failure and Econometric Modelling in Macroeconomics: The Transactions Demand for Money", in P. Ormerod Ed., *Economic Modelling: Current Issues and Problems in Macroeconomic Modelling in the U.K. and the U.S.*, London: Heinemann, pp. 17-242.

INSEE (1984). – *Tendances de la Conjoncture*, section on *politique monétaire*, février, pp. 20-22.

JUDD, J. P. and SCADDING, J. L. (1983). – "The Search for a Stable Money Demand Function: A Survey of the Post-1973 Literature", *Journal of Economic Literature*, XX, pp. 993-1023.

MACHAK, J. A., SPIVEY, W. A. and WROBLESKI, W. J. (1983). – "Analyzing Permanent and Transient Influences in Multiple Time Series Models", *Journal of Business and Economic Statistics*, 1, pp. 57-65.

MÉLITZ, J. (1976). – "Inflationary Expectations and the French Demand for Money, 1959-70", *The Manchester School*, XLIV, pp. 17-41.

MÉLITZ, J. (1982). – "The French Financial System: Mechanisms and Questions of Reform", *Annales de l'INSEE*, 47-48, pp. 361-387.

MELTZER, A. H. (1982). – "Money Growth in 1982", prepared for the Federal Reserve Academic Consultants, April 27, 1982, *mimeo.*

Ministère de l'Économie, des Finances et du Budget (1984). – "*La sur-estimation de la demande de monnaie par les modèles économétriques; doit-on incriminer uniquement le développement des FCP et SICAV de trésorerie?*" Paris, avril.

PIERCE, D. A. and PORTER, R. D. (1973). – "Linear Models and Linear Filters in the Analysis of Seasonal Time Series", *Proceedings of the Business and Economic Statistics Section, American Statistical Association*, pp. 537-542.

RAYMOND, R. (1983). – "The Formulation and Implementation of Monetary Policy in France", in P. Meek Ed., *Central Bank Views on Monetary Targeting*, New York: Federal Reserve Bank of New York, pp. 105-1145.

ROWLEY, V. V. (1985). – "Money Demand Predictability", *a paper* given at a Conference on Monetary Policy in a Changing Economic Environment, Washington, D.C.: American Enterprise Institute for Public Policy Research.

THORNTON, D. L. (1985). – "Money Demand Dynamics, Some New Evidence", *Federal Reserve Bank of St. Louis Review*, March, pp. 14-23.