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The effects of the dispersion of shareholdings on performance of owner controlled firms

Paul A. GROUT, François LAISNEY *

ABSTRACT. – The paper considers a model of the firm in which management acts to maximize share-holder welfare (i. e. owner controlled) and bargains over wages and employment with existing workers. The main result of the paper is that the performance of owner controlled firms will be independent of the dispersion of shareholdings for all values of shareholders non-firm wealth if and only if shareholders are risk neutral.

Effets de la dispersion des actions sur les performances d'une entreprise contrôlée par ses actionnaires

RÉSUMÉ. – L'article considère un modèle de l'entreprise où la direction maximise l'utilité des actionnaires et négocie le salaire et l'emploi avec un syndicat. Le résultat central est que dans cette situation les performances de l'entreprise seront indépendantes de la dispersion des actions pour toute valeur de la fortune des actionnaires extérieure à la firme si et seulement si les actionnaires sont neutres face au risque.

* P. A. GROUT: University of Bristol, Department of Economics, Alfred Marshall Building, 40 Berkeley Square, Bristol BS8 1HY, Royaume Uni; F. LAISNEY: Universität Heidelberg, Institut fur Statistik, Grabengasse 14, D6900 Heidelberg 1, République fédérale d'Allemagne. The authors wish to thank Parkash CHANDER and the participants to the 1985 European Winter Meeting of the Econometric Society, especially Birgit GRODAL, Martin HELLWIG, Mervyn KING and Jean-Jacques LAFFONT for remarks and comments, as well as an anonymous referee of the Annales. Paul GROUT wishes to thank the E.S.R.C. for financial support (Grant B00242003).

1 Introduction

The relationship between the dispersion of shareholdings and the behaviour and performance of firms has received considerable attention over the years. Stemming initially from the analysis of BERLE and MEANS [1932], the emphasis has been placed on the type of control of the firm – managerial control, owner control, etc. – and its effect on the objectives that the firm pursues (e. g. FLORENCE [1961], HINDLEY [1970], KAMERSCHEN [1968], MONSEN, CHIU and COOLEY [1968] and PALMER [1973]). The fact that managerial control may take different forms and have differing effect on objectives has been considered in detail (e. g. WILLIAMSON [1964]). However, the implicit assumption concerning owner controlled firms is that, at least in a world of certainty, the performance and behaviour is independent of the dispersion of shareholdings.

In this paper we focus attention on owner controlled firms and analyze why and how the performance of such firms may be influenced by the dispersion of shareholdings. A critical feature of owner controlled firms (indeed this is usually taken as the definition) is the fact that the majority of shares are in the hands of a small number of individuals. For most of these individuals their shareholdings in the company will form a significant part of their total wealth. Consequently their total wealth will be significantly affected if the company does badly and this weakens the position of management (who by definition act to maximize the welfare of shareholders) when negotiating with the workforce.

We show that if the labour market is not perfectly competitive, there is no reason to expect that the performance and behaviour is independent of the dispersion of shareholdings. The basis of our argument is the following simple idea. Suppose the firm bargains with workers over the wage and employment level. An increase in the dispersion of shareholdings will limit the extent to which a shareholder can lose if management adopt a strong bargaining position with the workfore. Of course, the maximum potential loss is the aggregate effect on all shareholders and, since dispersion increases the number of shareholders, we are aggregating a larger number of smaller potential losses. The aggregate effect is unclear but intuitively the spreading of such losses may improve the position of management when bargaining. We formalize this notion using conventional management-union efficient bargaining and show when this intuition holds and when it does not. The main feature of management-union efficient bargaining is that bargaining takes place over wages and employment. We show later that this is essential for the results to hold. The particular solution concept we employ is the symmetric Nash solution. This is by far the most commonly used concept in management-union bargaining (see, e. g. MACDONALD and SOLOW [1981]). It can be justified both on axiomatic grounds and from sequential offer bargaining (see RUBINSTEIN [1982]). A comparison with an inefficient bargaining solution is given by ARTUS and NOROTTE [1985]; we also comment on that solution at the end of this paper and justify our choice of the Nash bargain. Theoretically, it is well known that a bargainer's attitudes to risk will influence the outcome in bargaining situations (e. g. CRAWFORD and VARIAN [1979], ROTH and ROTHBLUM [1982] and SOBEL [1981]). To the extent that shareholders aggregate attitude to risk is influenced by the dispersion of shareholdings we can expect that the behaviour and performance of the firm may depend on the dispersion of shareholdings even if management always acts to maximise shareholders' welfare. It is this formal effect that we utilise in the paper. The following section presents the model, Section 3 states and proves the theorems and Section 4 discusses the results and the assumptions of the model.

2 The Model

The firm uses capital, k, and labour, l, both of which are essential for production and not firm specific. [1] The firm has a strictly convex profit function $\Pi(c, w)$ (i.e. $\Pi(c, w)$ is the maximum profit attainable if the firm can purchase capital at c per unit and labour at w per man) with cross partial derivatives of constant sign. Workers are identical and each has a concave utility function, $u(w)$. Shareholders are identical and each has a concave utility function v. Any worker not employed by the firm has a utility level of $u(\bar{w})>0$ earned from working elsewhere in the economy. Thus in bargaining terms $u(\bar{w})$ is the *status quo* utility level of a worker. Workers act to maximise their aggregate utility $l\,u(w)$.

The firm can remove the existing workers and replace them with workers from the competitive labour market but we assume this is costly, giving the shareholders an aggregate return of $\bar{\Pi}=\Pi(c, \bar{w})-z>0$ where $z>0$ is the cost of removing the existing workers. It is the fact that z is positive that gives the workers some monopoly power, obviously the closer z is to zero the less the final wage will deviate from the competitive wage. There are n identical shareholders each of which owns $1/n$ of the firm. This simple form of ownership allows us to talk unambiguously of a fall or increase in dispersion of shareholdings. We assume throughout that, irrespective of the dispersion of shareholdings, management acts to maximise the aggregate welfare of shareholders $nv\,\Pi(c, w)/n)$. Each shareholder has a level of wealth θ which comes from sources other than the firm in question. We assume that the firm's behaviour has no significant effect on the rest of the economy, thus θ is independent of the behaviour of the firm.

We represent the outcome of the bargain between management and workers by the Nash bargain. The solution to the bargain determines a wage w paid to each worker and a number of workers employed. If $z>0$ this employment level will not be on the firm's demand curve for labour. Using the envelope theorem, if the level of employment is fixed at l we can associate to the level of employment a virtual wage λ such that $l=-\Pi_2(c, \lambda)$. Thus knowledge of the true wage paid to each worker, w, and the virtual wage that determines employment, λ, fully characterises the solution. It is the effect of changes in n and θ on w and λ that we wish to investigate. The level of profit in the firm can be viewed in two parts.

1. This is necessary to ensure that the virtual price of capital is equal to c (see GROUT [1982, 1984, 1985]).

First, we have the level of profit the firm would receive if it actually paid workers λ, i. e. $\Pi(c, \lambda)$. Second, we must reduce this quantity because the firm pays each worker a wage of w and not λ. The total profit of the firm, therefore, is:

$$\Pi(c, \lambda)-(w-\lambda) l=\Pi(c, \lambda)+(w-\lambda) \Pi_2(c, \lambda)$$

In this one period model this is an exact measure of the performance of the firm. The behaviour of the firm is described by its choice of labour and investment, i. e.

$$l=-\Pi_2(c, \lambda)$$

and

$$k=-\Pi_1(c, \lambda).$$

We now describe the bargaining solution. The *status quo* level of welfare for the shareholders is $nv(\bar{\Pi}/n)$ and the *status quo* level of welfare for the workers is $\bar{l}u(\bar{w})$ where $\bar{l}$ is the number of workers who are taken into account. [2] If a bargain is struck with wage w and virtual wage λ, then the level of shareholder welfare is:

$$(1) \qquad nv\left(\frac{\Pi(c, \lambda)+(w-\lambda) \Pi_2(c, \lambda)}{n}\right)$$

and the level of worker welfare is

$$(2) \qquad (\bar{l}-l) u(\bar{w})+lu(w)=(\bar{l}+\Pi_2(c, \lambda)) u(\bar{w})-u(w) \Pi_2(c, \lambda)$$

The Nash bargaining solution is the (w, λ) pair that maximises the product of (2) minus $\bar{l}u(\bar{w})$ and (1) minus $nv(\bar{\Pi}/n)$ [see Grout (1984*a*)]. The solution has first order conditions:

$$(3) \qquad \frac{-(u(w)-u(\bar{w})) \Pi_2(c, \lambda)}{u'(w)}=\frac{n[v((\Pi(c, \lambda)+(w-\lambda) \Pi_2(c, \lambda))/n)-v(\bar{\Pi}/n)]}{v'((\Pi(c, \lambda)+(w-\lambda) \Pi_2(c, \lambda))/n)}$$

and

$$(4) \qquad w-\lambda=\frac{u(w)-u(\bar{w})}{u'(w)}.$$

The second equation is the standard result in McDonald and Solow [1981]. Notice that if workers are risk neutral then $\lambda=\bar{w}$ but if workers are risk averse then $\lambda<\bar{w}$.

3 The Performance and Behaviour of the Firm

We begin by considering the case when all the shareholders' wealth is contained in the firm (i. e. $\theta=0$).

THEOREM 1 : If $\theta=0$ then the performance of the firm is independent of the dispersion of shareholdings if and only if shareholders utility exhibits constant relative risk aversion.

Proof: The proof is in two parts. In part *a* we show that the performance of the firm will be independent of n if and only if the right hand side of (3) is independent of n. We then show, in part *b*, that a necessary and sufficient condition for this to hold is that shareholders utility exhibits constant relative risk aversion.

a From (4) we obtain:

$$\frac{\partial \lambda}{\partial w}=1-\frac{(u'(w))^2-u''(w)\,(u(w)-u(\bar{w}))}{(u'(w))^2}$$

which is negative if workers are risk averse ($u''(w)<0$) and zero if workers are risk neutral, so any change in both w and λ induced by a change in n will have opposite effects on w and λ. Furthermore, from (1) shareholders welfare is decreasing in w and increasing in λ:

$$v'(-)\Pi_2(c,\lambda)<0$$

and

$$v'(-)\,(w-\lambda)\Pi_{22}(c,\lambda)>0$$

respectively; $\Pi_{22}(c, \lambda)$ is positive because the profit function is strictly convex. If the right hand side of (3) is independent of n, clearly so will be the performance of the firm. Conversely, if the right hand side varies with n then λ must either rise or fall, with w moving in the opposite direction. Therefore, if the right hand side of (3) varies with n the performance of the firm will change. Consequently, the performance of the firm will be independent of the dispersion of shareholdings if and only if the right hand side of (3) is independent of n.

b We now prove that constant relative risk aversion is necessary for the right hand side of (3) to be independent of n. Denoting

$$\Pi^*=\Pi(c,\lambda)+(w-\lambda)\Pi_2(c,\lambda)$$

and supposing n is continuous, we differentiate the right hand side of (3) with respect to n to obtain:

2. Following McDONALD and SOLOW [1981] we assume $l \leqslant T$ is not a binding constraint.

$$(5)\quad \frac{\delta}{\delta n}\mathrm{RHS}=\frac{1}{v'(\Pi^*/n)^2}((v(\Pi^*/n)-v(\bar{\Pi}/n))[v'(\Pi^*/n)+(\Pi^*/n)v''(\Pi^*/n)] -\frac{1}{n}v'(\Pi^*/n)[\Pi^* v'(\Pi^*/n)-\bar{\Pi}v'(\bar{\Pi}/n)]).$$

The right hand side of (3) will be independent of n if (5) is equal to zero, i. e. if

$$(6)\qquad \frac{1}{n}\left(\frac{\Pi^* v'(\Pi^*/n)-\bar{\Pi}v'(\bar{\Pi}/n)}{v(\Pi^*/n)-v(\bar{\Pi}/n)}\right)=1+\frac{(\Pi^*/n)v''(\Pi^*/n)}{v'(\Pi^*/n)}.$$

The right hand side of (6) does not depend on $\bar{\Pi}$ and the same must then hold for the left hand side. A symmetry argument shows that this does not depend on Π^* either. Thus $xv''(x)/v'(x)$ is constant, which requires v to exhibit constant relative risk aversion.

Constant relative risk aversion is also sufficient. To see this assume $v(-)$ does exibit constant relative risk aversion and consider (3). Clearly n will cancel out of (3) altogether which shows the performance of the firm cannot depend on n in this case. □

We have the following corollary to Theorem 1.

COROLLARY 2 : If $\theta=0$ then:

i. if workers are risk averse the level of employment is independent of the dispersion of shareholdings if and only if shareholders utility exhibits constant relative risk aversion;

ii. if workers are risk averse the level of investment is independent of the dispersion of shareholders if and only if one of the following holds:

(*a*) shareholders utility exhibits constant relative risk aversion; or

(*b*) $\Pi_{12}(c, w)=0$.

Proof: Obviously if (5) is zero then λ is constant for all n hence investment and employment are independent of n. If (5) is not zero and $u''(w)\neq 0$ then λ will depend on n and, because $\Pi(c, w)$ is strictly convex, $-\Pi_2(c, \lambda)$ depends on λ. If $\Pi_{12}(c, w)=0$ then investment will be independent of n irrespective of λ. □

We now consider the effect of $\theta>0$ and again we can be specific if the shareholders exhibit constant relative risk aversion.

THEOREM 3 : If $\theta>0$, shareholders are risk averse and exhibit constant relative risk aversion then the performance of the firm improves as shareholdings become more disperse.

Proof: If shareholders exhibit constant relative risk aversion but $v(x)\neq \ln x$ then without loss of generality we can denote the right hand side of (3) as

$$Q=\frac{n}{1-\rho}((\Pi^*/n)+\theta)\left(1-\left(\frac{((\bar{\Pi}/n)+\theta)}{((\Pi^*/n)+\theta)}\right)^{1-\rho}\right)$$

using $v(x)=x^{1-\rho}/(1-\rho)$, $\rho \geq 0$, $\rho \neq 1$. Rewriting Q as

$$\frac{1}{1-\rho}((\Pi^*+n\theta)-(\bar{\Pi}+n\theta)^{1-\rho}(\Pi^*+n\theta)^{\rho})$$

we have

$$\frac{\partial Q}{\partial n}=\frac{\theta}{1-\rho}(1-(1-\rho)A^{\rho}-\rho A^{\rho-1}) \tag{7}$$

where

$$A=\frac{(\Pi^*+n\theta)}{(\bar{\Pi}+n\theta)}.$$

Denoting the right hand side of (7) by F(A) we have

$$F'(A)=\frac{\theta}{1-\rho}(-\rho(1-\rho)A^{\rho-1}-\rho(\rho-1)A^{\rho-2})=\rho\theta A^{\rho-2}(1-A)$$

which is strictly negative if ρ is positive, since A is strictly greater than one. As $F(1)=0$, it follows that $F(A)$ is negative for A greater than one, and thus the right hand side of (3) is decreasing in n. Using the argument given in the proof of Theorem 1, this implies that the profit of the firm increases as shareholdings become more dispersed.

The above argument does not hold if shareholders exhibit constant relative risk aversion and $v(x)=\ln x$. In this case the right hand side of (3) is

$$Q=(\Pi^*+n\theta)(\ln((\Pi^*/n)+\theta)-\ln((\bar{\Pi}/n)+\theta)).$$

$$\partial Q/\partial n=\theta[\ln A+1-A]$$

which is negative since $\ln A < A-1$. Thus the effect of an increase in the dispersion of shareholdings is the same for $v(x)=\ln x$ as for $v(x)=(1/1-\rho)x^{1-\rho}$. □

COROLLARY 4 : If $\theta > 0$, shareholders are risk averse and exhibit constant relative risk aversion then:

i. if workers are risk averse employment declines as the dispersion of shareholdings increases;

ii. if workers are risk averse investment increases (declines, stays constant) as the dispersion of shareholdings increases if $\Pi_{12}(c, w)<0(>0, =0)$.

Proof: The result in (i) follows from the proof of Theorem 1. Using the fact that $\Pi_1(c, \lambda)=-k$ and that the proof of Theorem 1 shows that λ is increasing in n if shareholder profit is increasing in n, we see that an increase in dispersion of shareholdings (if shareholders exhibit constant relative risk aversion and are not risk neutral) will increase k if $\Pi_{12}(c, \lambda)<0$ and reduce k if $\Pi_{12}(c, \lambda)>0$. □

Theorems 1 and 3 together give the main result of the paper:

THEOREM 5 : The performance of the firm is independent of the dispersion of shareholdings for all values of θ if and only if shareholders are risk neutral.

Finally it is of interest to ask what is the effect of different values of θ. In order to derive specific results, we introduce $D = 1 - 1/n$ as a measure of dispersion. The degree of dispersion is zero if the firm has a single owner and takes on the value one in the limit as the number of shareholders goes to infinity.

COROLLARY 6 : If shareholders exhibit constant relative risk aversion then the effect of a marginal increase in the dispersion of shareholdings is equivalent to the effect of a marginal increase in θ times the aggregate non-firm wealth of all shareholders, i. e. $n\theta$.

Proof: Using the inverse function theorem we have $dn/dD = n^2$ where D is the measure of dispersion of shareholdings. From the definition of Q in the proof of Theorem 2 we know that if shareholders exhibit constant relative risk aversion then

$$\frac{\partial Q}{\partial n}\frac{1}{\theta} = \frac{\partial Q}{\partial \theta}\frac{1}{n}$$

i. e.

$$\frac{\partial Q}{\partial n} = \frac{\partial Q}{\partial \theta}\frac{\theta}{n}$$

$$\frac{\partial Q}{\partial D} = \frac{\partial Q}{\partial n}\frac{dn}{dD}$$

$$= \frac{\partial Q}{\partial \theta} n\theta,$$

as required. □

4 Discussion

In this paper we have shown that if a firm faces an imperfectly competitive labour market there is no reason to suppose that even if the firm is owner controlled, the performance, investment and employment of the firm will be independent of the dispersion of shareholdings. The basic idea that an increase in the dispersion of shareholdings is desirable because it limits the extent to which a shareholder can lose if management adopt a strong bargaining position with the workforce is simple and appealing. We have shown that this idea can be formalized and have analyzed the circumstances when the simple intuition is correct. The model is complementary to the existing literature since our results on the effects of a change in the dispersion of shareholdings assume that the firm is owner controlled for all dispersion levels. Once the dispersion of shareholdings reaches some level the management will no longer act to maximise the welfare of shareholders. Therefore we can combine our results here with the idea that once owners lose control the performance of the firm will fall. Taken together this would suggest that dispersion initially improves the performance of the firm but eventually the performance may deteriorate once dispersion reaches some critical level. The argument rests on the assumption of an imperfectly competitive (probably unionised) workforce, therefore the arguments would seem to be more appropriate for European countries than for the USA.

Having concentrated on identical shareholders we have been able to avoid difficulties of defining an increase or decrease in dispersion. Obviously once we move away from this special case the measure of dispersion becomes important. However, the main result of the paper, that there is no reason to suppose performance is independent of dispersion of shareholdings in owner controlled firms, will still hold.

The results present a reason for diversification of shareholdings which is in addition to the conventional risk spreading one. However, it is not completely independent since it is exactly the effect of risk spreading, by changing the aggregate shareholders attitude to risk, that benefits shareholders in the bargain with existing workers. We have concentrated on the effect of changes in shareholding dispersion for all levels of n. The main reason is that if a firm is owner controlled we may well expect n to be small and, as a result, the attitude to risk of these shareholders to be quite important. However, for almost all utility functions the profit of the firm will be maximised as n approaches infinity, whether or not it increases or decreases monotonically with n for smaller values. The reason is that the second derivative of the aggregate shareholders welfare function, $(1/n)\,v''((\Pi^*/n)+\theta)$ will converge to zero in the limit as n approaches infinity if $\theta>\bar{\theta}$ and $v''(\theta)$ is bounded away from infinity for all $\theta>\bar{\theta}$. As we have already mentioned as n approaches infinity the firm is unlikely to remain owner controlled so this particular general result is of little practical interest and has not been pursued.

Finally the role of the imperfect labour market should be discussed. We have assumed that workers and management bargain over wage and employment. An alternative model which is frequently employed is that where the union sets the wage and the firm chooses employment, i. e. the union chooses w and the firm sets union employment equal to

$$l=-\Pi_2(c,w)$$

if

$$\Pi_2(c,w)\geqslant\Pi_2(c,\bar{w})-z$$

and union employment equal to zero if

$$\Pi(c,w)<\Pi_2(c,\bar{w})-z.$$

The union's objective is to maximise

$$(\bar{l}+\Pi_2(c,w))\,u(\bar{w})-\Pi_2(c,w)\,u(w)$$

subject to

$$\Pi(c,w)\geqslant\Pi(c,\bar{w})-z.$$

If the constraint is not binding then the firm's profit is $\Pi(c, w^*)$ where w^* is given by [3]

$$u(w^*)=u(\bar{w})-u'(w^*)\frac{\Pi_2(c,w^*)}{\Pi_{22}(c,w^*)}.$$

If the constraint is binding then w^* is given by

$$\Pi(c,w^*)=\Pi(c,\bar{w})-z.$$

In either case w^* is greater than $\bar{w}$ but is independent of the number of shareholders. Of course, the model where the union fixes the wage is the correct model if a single union faces an almost infinite number of small firms so that it is impossible for the union to bargain over employment and wages with each firm individually. Whenever it is feasible for the union to bargain with individual firms the outcome of bargaining can be better for both parties than the outcome if the union fixes the wage and the firm fixes employment. This result is well known and was initially given in LEONTIEF [1946]. In the analysis of the effects of the dispersion of shareholdings it seems inappropriate to concentrate attention on perfectly competitive firms of almost zero size and therefore we believe the bargaining model of the labour market is the appropriate one for the issue we are analysing.

Bargaining with employee shareholding is another problem that deserves attention and has been analyzed by GROUT [1986], and ESTRIN, GROUT, and WADHWANI [1986]. However, the impact of the dispersion of shareholdings in that context has not been studied yet.

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3. Assuming the second order conditions are satisfied.