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On the Efficacy of Fiscal Policy

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ABSTRACT. – This paper shows in a model of an economy with goods, labour, bond and money market where prices and wages are fixed but the interest rate is flexible that even in a Keynesian unemployment state an increase in government expenditure will not necessarily increase employment, if production takes time. The robustness of this proposition in regard to expectational assumptions is studied, too.

A propos de l'efficacité de la politique budgétaire

RÉSUMÉ. – Cet article montre, dans un modèle à quatre biens (produit, travail, titres et monnaie) où les prix et salaires sont fixes, mais le taux d'intérêt flexible, que, même en situation de sous-emploi keynésien, un accroisement des dépenses publiques ne résulte pas nécessairement en une hausse de l'emploi si la production exige un délai. On étudie également la robustesse de cette proposition vis-à-vis des hypothèses concernant les anticipations.

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1 Introduction

The question whether fiscal policy has a substantial impact on the performance of the economy is at the core of macroeconomic reasoning. The prevalent view of the sixties that fiscal policy, i. e. deficit spending, could enhance employment, if the economy were in a state of unemployment, came under increasing attack in the early seventies by the monetarists who claimed that government spending would "crowld out" private expenditure to a high degree and might possibly in the long-run worsen the situation. This problem was the main concern of an article by BLINDER and SOLOW [1973] [1] leading to the conclusion that the efficacy of fiscal policy is in principle an empirical matter (BLINDER, SOLOW [1973]), p. 336).

Starting with the work of BARRO and GROSSAMN [1976] and MALINVAUD [1977] these conclusions on the effectiveness of government expenditure programs had to be qualified from a theoretical point of view. In the context of fix-price equilibria in three goods models (one commodity, one type of labour, and money) it could be shown that fiscal policy will work only, if unemployment is of the Keynesian type, i. e. results from supply rationing in the goods market. More recent developments in this field which tended to include a fourth market for an interest bearing asset which is supposed to be cleared by the flexible adjustment of the interest rate to its equilibrium level confirmed these conclusions (compare HOOL [1979, p. 305], DANTHINE and PEYTRIGNET [1981, p. 9], GRANDMOND [1982, p. 24], BENASSY [1983, p. 10], BÖHM [1983, p. 38]. These results were derived without completely modelling the behaviour of the agents in the economy (the paper by Böhm is an exception), and in particular left expectation formation unspecified. Therefore, the influence of expectations on these results remains unclear.

A paper by NEARY and STIGLITZ [1983] addresses this issue particularly. The importance of expectations is stressed and it is shown that for the case of a rationally expected supply constraint equilibrium in the future the fiscal stimulus will even be enforced, if Keynesian unemployment prevails today. Thus, there scems to be no question that the government expenditure multiplier is positive in a Keynesian unemployment situation.

It is the purpose of this paper to show that the robustness of this result relies on the assumption that production of output is instantaneous. Thus, an increase in demand for output will translate immediately into an increased demand for the input labour. More precisely, if production takes time this immediate link between sales and production is broken. As a result government expenditure increases may well lead to a negative fiscal policy multiplier. The conditions for such a result are derived in this paper using a model with goods, labour, bond, and money market.

It is worth noting that the assumption that production takes time has been used extensively in temporary equilibrium models with and without rationing (e. g. SONDERMANN [1974], and GRANDMOND, LAROQUE

[1976]). But in spite of the fact that temporary equilibrium theory provides the microfoundations for fix-price macroeconomics, this structural assumption has never been incorporated into macroeconomic modelling. [2] Thus, the consequences of such an assumption were not explored so far. This paper wants to fill this gap.

To stress the generality of the result and to avoid that it may be ascribed to other features of a more specific model this paper is organised in two parts. In the first part (section 2 and 3), a very general outline of a four goods macroeconomic model is given and comparative static results are derived making the usual assumptions with respect to all behavioural functions except changing labour demand and output supply functions to reflect the intertemporal production structure. The second part, section 4 and 5, then provides a more specific microeconomic model of an economy which exhibits the crucial feature of intertemporal production. It is is in this more specific context that the influence of expectations on demand and supply functions is assessed. In particular, it can be shown that rational expectations will enhance the fiscal stimulus, a result similar to the NEARY and STIGLITZ [1983, p. 222] conclusion.

2. The Economy

The economy considered in this paper consists of four markets which open in a sequence of periods. There are three types of agents, consumers, firms, and government, trading in these markets. Prices on the goods and labour market are fixed at $p>0$ and $w>0$ respectively, and these markets are cleared by rationing demand or supply. Notice that all variables without a time index refer to period 1, the current period, on which the analysis of this paper focuses. Goods are assumed to be not storable and the only way to transfer wealth between periods consists in holding money which has a price of one in every period or trading a security at a price $q \in (0,1]$. The sale of such a security, called bond, obliges the seller to pay to the buyer one unit of money at the beginning of the following period. Thus, the rate of interest $\frac{1}{q} - 1$ is non-negative. The bond price is assumed to adjust in a way as to clear the bond market. There is neither rationing in the bond market nor in the money market.

In the following subsections the behaviour of a typical consumer and a typical firm will be sketched, but a more detailed analysis is postponed until section 4.

1. The article by BLINDER and SOLOW [1973] provides a good review of the controversies concerning the effectiveness of fiscal policies in the traditional IS-LM type analysis.
2. In the context of a traditional IS-LM type macroeconomic model SHALLER [1983] has addressed this question too. He shows as well that time consuming production may alter traditional conclusions on the efficacy of the fiscal stimulus.

2.1. Consumers

In every period a consumer has to choose the quantities of goods, $x \geqslant 0$, and bonds, $b \geqslant 0$, he wants to buy, the amount of labour, $l \geqslant 0$, he wants to sell, and the amount of money $m \geqslant 0$, he wants to hold over into the following period. Notice that a consumer will not issue securities, since b is assumed to be non-negative. Furthermore, it will be supposed that a consumer has no disutility of labour and will, therefore, supply his total endowment $L > 0$, if he is unconstrained in the labour market, or the maximum possible, if he faces a supply constraint λ in the labour market. In addition, his choice of (b, x, l, m) is constrained by his budget,

$$(1) \qquad m_0 + b_0 + \delta + (1-\tau)\, wl = px + qb + m,$$

where m_0 and b_0 denote the amount of money brought over from the previous period and the repayment from previously purchased bonds. δ stands for dividends paid to the consumer and τ is the proportional tax rate applied to the consumer's wage income. Both variables are not under the control of the consumer.

This choice problem gives rise to the following demand and supply functions of the consumer in the goods, money, and labour market:

$$(2) \qquad x^c = x^c(\underset{+}{q}, \underset{+}{\lambda}), \qquad m^c = m^c(\underset{+}{q}, \underset{+}{\lambda}), \qquad l^c = \min\{\lambda, L\}.$$

The signs of the partial derivatives of these functions which will be assumed throughout this and the following section are written below the respective variable. [3]. Since p and w are fixed, and m_0 and b_0 are predetermined they have been omitted from the list of arguments of these functions. These partial effects can be justified by noticing that an increase in q means a decrease in the interest rate which makes future consumption more expensive and bonds less attractive leading to a shift towards higher present consumption and higher money holdings. On the other hand, an increase in λ, i. e. reduced rationing in the labour market, increases wage income which stimulates consumption in the present and in the future, if goods are normal. Notice that expectations have been kept implicit so far. This problem will be dealt with in section 5.

2.2. Firms

While the described behaviour of a consumer conforms with the usual macroeconomic modelling, the model of the firm departs from this tradition assuming that production takes time. The idea that production does not take place instantaneously has been used quite extensively in the literature on temporary equilibria (e. g. GRANDMOND, LAROQUE [1976], SONDERMANN [1974]), but its implications for macroeconomic comparative static results have hardly been recognised.

The fact that production takes time implies immediately the following:

(i) output cannot be expanded instantaneously and is determined by previous decisions of the firm; [4]

(ii) sales revenues of the produced output cannot be balanced against the expenditures on inputs.

The latter of these properties makes the decision to produce an investment decision, since input expenditures must be prefinanced. Thus, production has its opportunity cost in terms of forgone interest earned on a financial investment.

According to the assumption that production takes time, a firm starts out with a certain amount of output $x_0 > 0$ produced in the previous period which can be sold in the present period. The firm will purchase inputs $l \geqslant 0$ and produce output $x \geqslant 0$ which can be sold only in the following period. The technology is assumed to satisfy the usual continuity and convexity conditions and will be represented by a production function $f(l)$.

Since a firm distributes an amount δ to the consumer, it can use its retained earnings or credit to finance its input expenditures. Credit can be obtained in this economy by selling bonds $b < 0$ at the going price q. If the firm is allowed to hold financial assets $b > 0$ and money $m > 0$ as well, its budget constraint takes the following form:

$$m_0 + b_0 - \delta + (1-\tau)ps = wl + qb + m, \tag{3}$$

where s denotes the sales of the firm which will equal its previously produced output x_0, if there is no sales constraint, or the level of such a constraint ξ.

If the firm maximizes its present value subject to its technological possibilities and its budget constraint, this gives rise to the following demand and supply functions:

$$s = \min\{\xi, x_0\}, \qquad m^f = \underset{+}{m^f}(\underset{+}{q}, \underset{+}{\xi}), \qquad l^f = \underset{+}{l^f}(\underset{+}{q}, \underset{+}{\xi}). \tag{4}$$

Again, as in the case of the consumer, all other arguments of these functions are suppressed and the influence of expectations is kept implicit. The signs of the partial derivatives given below the respective arguments follow from the fact that an increase in q (i. e. a decrease in the interest rate) reduces the financing costs of production, if the firm uses credit to finance its purchases of labour, or the opportunity cost of production, if it finances it out of retained earnings. Similarly, the opportunity costs of holding money will decline as q increases. On the other hand, an increase in a binding sales constraint ξ will yield more sales revenue to the firm inducing higher input purchases and higher money holdings, if money is held for transaction purposes.

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3. Notice that these assumptions on the partial derivatives correspond to those made by DANTHINE, PEYTRIGNET [1981], GRANDMONT [1982], BENASSY [1983], BÖHM [1984].

4. This aspect is neglected in the analysis of SHALLER [1983].

2.3. Government

As in macroeconomic models is common, "government" performs the role of a central bank, too, and is assumed to act exogenously but subject to a budget constraint. The government decides how much of the produced output to buy, $\gamma \geqslant 0$, and fixes a money supply $\mu \geqslant 0$. To finance these activities, it receives taxes from consumers and firms, T, and can buy or sell bonds β to balance its budget, i. e.

$$p\gamma + q\beta + \mu_0 = T + \beta_0 + \mu. \tag{5}$$

Notice that $\beta < 0$ holds, if a deficit $p\gamma - T$ is not financed by extra money supply $\mu - \mu_0$ and if there is no repayment from previous bond purchases, $\beta_0 \leqslant 0$. In this case the government has to raise the necessary funds by issuing bonds. Since government chooses (γ, μ), its bond transactions are determined by the budget constraint.

2.4 Equilibrium

An equilibrium in this economy is a bond price q and a set of constraints in the labour and goods market such that for a fixed price vector (p, w) and a government policy (γ, μ):

(i) all markets clear, and

(ii) at most one side of each market is constraint.

Since all agents choose their action subject to a budget constraint, Walras Law holds and the bond market is deleted from consideration following macroeconomic tradition.

According to the market side which is constraint in equilibrium, one can distinguish four types of constraint equilibria : [5]

(i) Keynesian Unemployment, if supply is rationed in both labour and goods market;

(ii) Classical Unemployment, if supply is rationed in the labour market and demand is constraint in the goods market;

(iii) Repressed Inflation, if demand is rationed in the labour and in the goods market;

(iv) Underconsumption, if demand is rationed in the labour market and supply in the goods market.

Notice that the last of these cases can arise in this model, for production is not instantaneous. Which one of these disequilibria will obtain depends on the level where prices and wages are fixed. In EICHBERGER [1983 *a*] the existence of such an equilibrium has been proved, and in EICHBERGER [1984 *a*] a detailed discussion of the price-wage constellations leading to these different disequilibrium states has been given.

Since it is the major thrust of this paper to show that government spending policies may have contractionary effects in Keynesian unemployment situations, this paper concentrates on this disequilibrium state and the reader is referred to EICHBERGER [1984 *a*] for an analysis of the other cases.

3. Keynesian Unemployment

To derive comparative static results the model of the previous section is simplified further by assuming that there is only one agent of each type. This allows to dispense with a detailed modelling of a rationing scheme. Given this assumption a Keynesian Unemployment equilibrium is a bond price and two supply constraints (q, λ, ξ) satisfying:

$$x^c(q, \lambda)+\gamma-\xi=0, \qquad \xi<x_0 \tag{6.1}$$

$$l^f(q, \xi)-\lambda=0, \qquad \lambda<L \tag{6.2}$$

$$m^c(q, \lambda)+m^f(q, \xi)-\mu=0. \tag{6.3}$$

The following theorem gives all the comparative static results where a variable as an index denotes the partial derivative with respect to this variable.

THEOREM: If the demand and supply functions are differentiable at the equilibrium (q, λ, ξ) and their partial derivatives have the signs given in (2) and (4), and if $x^c_\lambda\, l^f_\xi<1$ holds, then the functions $q(\gamma, \mu)$, $\lambda(\gamma, \mu)$, and $\xi(\gamma, \mu)$ which are implicitly defined by 6.1, 6.2, 6.3 have the following partial derivatives:

(i) $q_\gamma<0$, $q_\mu>0$, $\lambda_\mu>0$, $\xi_\gamma>0$, $\xi_\mu>0$, and

(ii) $\lambda_\gamma \gtreqless 0$ if and only if $l^f_q\, m^f_\xi - l^f_\xi (m^f_q+m^c_q) \gtreqless 0$.

Proof: By the implicit function theorem one has:

$$\begin{bmatrix} q_\gamma & q_\mu \\ \lambda_\gamma & \lambda_\mu \\ \xi_\gamma & \xi_\mu \end{bmatrix} = \begin{bmatrix} x^c_q & x^c_\lambda & -1 \\ l^f_q & -1 & l^f_\xi \\ m_q & m^c_\lambda & m^f_\xi \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

where $m_q \equiv m^c_q + m^f_q$,

$$\begin{bmatrix} x^c_q & x^c_\lambda & -1 \\ l^f_q & -1 & l^f_\xi \\ m_q & m^c_\lambda & m^f_\xi \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} -(m^f_\xi + m^c_\lambda l^f_\xi) & -(x^c_\lambda m^f_\xi + m^c_\lambda) & (x^c_\lambda l^f_\xi - 1) \\ -(l^f_q m^f_\xi - l^f_\xi m_q) & (x^c_q m^f_\xi + m_q) & -(x^c_q l^f_\xi + l^f_q) \\ (m^c_\lambda l^f_q + m_q) & -(x^c_q m^c_\lambda - x^c_\lambda m_q) & -(x^c_q + x^c_\lambda l^f_q) \end{bmatrix}$$

5. The first three of these disequilibrium states have received their name in MALINVAUD [1977, chapter 2] while the fourth one which arises only if output is storable or if production takes time was introduced by MUELLBAUER and PORTES [1978].

and

$$\Delta = -m_\xi^f (x_q^c + x_\lambda^c l_q^c) - m_\lambda^c (x_q^c l_\xi^f + l_q^f) + m_q (x_\lambda^c l_\xi^f - 1) < 0$$

holds. It is easy to check that by (2) and (4) and the premise of the theorem $x_\lambda^c l_\xi^f < 1$ the following sign pattern for the relevant first and third column of the inverse matrix holds:

$$\begin{bmatrix} - & \cdot & - \\ ? & \cdot & - \\ + & \cdot & - \end{bmatrix}$$

this completes the proof. □

Notice first that the premise $x_\lambda^c l_\xi^f < 1$ just states that the indirect effects of a change in the constraints (λ, ξ) do not outweigh their direct impact. Furthermore, it is worth noting that the effects of an increase in the supply of money on the endogenous variables (q, λ, ξ) are the same as in a model with atemporal production structure of the firm: interest rates decrease and supply constraints in both markets are relaxed.

The effect of an increase in government spending on the other hand leads to higher interest rates and a reduced sales constraint in the goods market as in traditional macroeconomic reasoning, but the effect of such a fiscal stimulus on employment is unclear. This is in stark contrast to conventional macroeconomic conclusions whether in the IS-LM tradition or in the more recent fix-price literature. The reason for this differing result lies in the intertemporal production structure. In models with atemporal production sales can be increased only, if production and input demand is increased. This, however, need not be the case in models with intertemporal production, since sales can be increased out of previously produced output. The decision to produce and to hire labour is independent from the present level of sales. Thus, the immediate link betwen sales and input demand is broken in this case.

As a consequence a change in government spending has a different impact on the economy. An increase in government expenditure raises the interest rate (decreases q) and thus increases the opportunity cost of production. Due to the fact that production is time-consuming, the decision to hire labour and to increase production which will yield sales revenues in the future only becomes an investment decision. The implicit return on investing in production has to be weighed against the return on a financial investment. The interest effect of a government expenditure increase, therefore, provides a disincentive for production and employment.

On the other hand, a firm might expect future sales constraints below the level of sales it would plan by return considerations only. In such a situation an increase in present government spending could make the firm believe that it will be less sales constraint in the future as well and induce it to raise production and employment.

These two effects run opposite to each other and create the ambiguity of the employment effect of government expenditure increases. Notice that it is the absence of such an interest effect on labour demand which renders the traditional results so unambiguous.

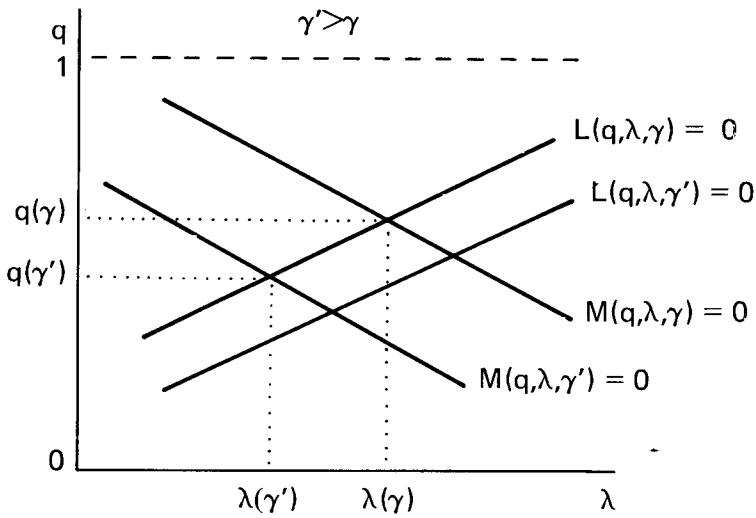
The following diagram may illustrate this result. Substituting the explicit solution of 6.1 for the sales constraints in equation 6.2 and 6.3 yields the following equilibrium loci for the labour and the money market in (q, λ) space:

$$(7.1) \qquad L(q, \lambda, \gamma) \equiv l^f(q, x^c(q, \lambda) + \gamma) - \lambda = 0$$

$$(7.2) \qquad M(q, \lambda, \gamma) \equiv m^c(q, \lambda) + m^f(q, x^c(q, \lambda) + \gamma) - \mu = 0.$$

It is strainght forward to check that given the signs of the partial derivatives in (2) and (4) $L(q, \lambda, \gamma)$ is increasing in q, decreasing in λ (for $1 > l^f_\xi x^c_\lambda$), and increasing in γ, while $M(q, \lambda, \gamma)$ is increasing in all its arguments. This leads to the Figure 1.

FIGURE 1



It is obvious from this diagram that an increase in government spending increases the interest rate, but may increase or decrease employment.

There is some other aspect worth mentioning. From the second statement of the theorem,

$$\lambda_\gamma \gtreqless 0 \quad \Leftrightarrow \quad l^f_q m^f_\xi - l^f_\xi (m^c_q + m^f_q) \gtreqless 0,$$

it is clear that $m^f_\xi > 0$ and $m_q = 0$ (given $l^f_q > 0$) is a sufficient condition for a negative effect of government spending increases on employment. But $m^f_\xi > 0$ and $m_q = 0$ just states that there is a transaction demand for money and no "speculative" demand. For such a case traditional macroeconomic theory predicts that fiscal policy becomes ineffective. With an intertemporal production structure, however, an increase in government expenditure will even worsen an existing unemployment situation.

4. A Microeconomically Explicit Model

So far the description of the private agents, consumers and firms, has remained rather heuristically. In this section a completely specified model of individual behaviour is presented which provides better insight into the assumptions made implicitly in the previous sections.

For this purpose one has to make precise what induces private agents to hold money – a no interest bearing asset – in an economy where there is an asset with a certain positive return rate $(1-q)\,q^{-1}$. Clearly, money will not be held as a store of value. This leaves the transaction motive for holding money. There are two principal ways to model this transaction demand of money:

(i) one may put money into the utility function directly, arguing that the agent has an immediate benefit from holding money balances because of "easier" trade, or

(ii) one may constrain the agent to use money for certain transactions by introducing an addtional constraint beside the budget, a so-called "Clower-" or "cash-in-advance" constraint. [6]

Both approaches sidestep a precise analysis of why agents may want to hold money. But it is beyond the scope of this paper to investigate this question further. In particular, the way chosen is of no consequence for the analysis of the efficacy of fiscal policy. Since the first approach, to put money into the utility function, just shifts the problem to an adequate description of preferences, the second approach has been preferred here (compare e. g. SVENSSON [1983]). This approach leads to a precise description of the payments necessary to perform certain transactions.

The cash-in-advance constraint requires an agent to pay for its expenditures on goods, labour, and bonds in cash which has to be available in advance at the beginning of the market period. The cash available at the beginning of each period consists of money held over from the previous period plus the repayments on previously purchased bonds which are due now. Since there is an asset with a strictly dominating return, the bond, the repayments of which are in money each agent will confine his money holdings to the minimum possible for him, [7] i. e. money holdings will equal the sales revenues from goods and labour modulo transfers like taxes and dividends.

Before consumers and firms are described a word on expectations seems to be in order. All agents in this economy will have to form expectations on future prices and eventually on market constraints. These expectations will be represented by a function Ψ indexed by the variable it is predicting and depending on all past and present information available. This information will include in particular past and present prices and market constraints,

initial asset holdings over the agents in the economy, and past, present, and eventually future exogeneous variables. Such a representation of expectations will be sufficiently general to deal with exogeneous expectation formation processes and with rational expectations. A more detailed description of the role of expectations will be postponed until the following section.

4.1. Consumers

In this subsection a consumer is considered who resembles the one discussed by MALINVAUD [1977, chapter II]. The major differences to his treatment of the consumer arise from the fact a consumer in the economy of this paper has an asset with a certain positive return rate besides money to store value.

The consumer plans for two periods only. His preferences on consumption in the two periods can be represented by a utility function $U(x, x_2) = xx_2$ and he has no disutility from working. [8] He starts the first period with some initial money holdings, $m_0 \geqslant 0$, and some repayments, $b_0 \geqslant 0$, on previously purchased bonds. His choices of consumption, labour, and bond and money holdings are subject to a budget constraint (1) and a cash-in-advance constraint

$$m_0 + b_0 \geqslant px + qb \tag{8}$$

in both periods. Since wage income in the second period cannot be consumed in this period because of the cash-in-advance constraint, $l_2 = 0$ will hold. Similarly, $m_2 = b_2 = 0$ will hold, which allows to collapse the budget and the cash-in-advance constraint of the second period to

$$m + b = \Psi_{p2}(\mathrm{I})\, x_2, \tag{9}$$

where Ψ_{p2} is the price expectation function of the consumer depending on all available information I.

The choice problem of the consumer can now be summarized as follows:

$$\text{Maximize } x \cdot x_2 \text{ by choosing } (b, x, l, x_2)$$

subject to

$$m_0 + b_0 + \delta + (1-\tau)\, wl = px + qb + m$$

$$m_0 + b_0 \geqslant px + qb$$

$$m + b = \psi_{p2}(\mathrm{I})\, x_2$$

$$b \geqslant 0, \qquad x \geqslant 0, \qquad \min\{\lambda, \mathrm{L}\} \geqslant l \geqslant 0, \qquad m \geqslant 0, \qquad x_2 \geqslant 0.$$

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6. "Cash-in-advance" constraints as means to introduce a demand for money in economies where there are other assets has become widespread over the last decade. Compare in particular the recent paper by SVENSSON [1983] which discusses this approach extensively and compares it to other approaches in the literature.

7. This implication can be avoided by making the preferences of the agent stochastic (compare LUCAS [1980]).

8. The assumption of "no disutility of labour" has been chosen here, since this paper focuses on supply constraint equilibria, i. e. the notional labour supply function does not influence the analysis in this paper.

It is immediately clear from this problem, that the consumer will work as much as possible, since this will increase his future consumption, and that he will hold the least possible amount of money, since bonds yield a higher return. Now it is easy to check that the following demand and supply system solves the consumer's choice problem (denote by $W_0 \equiv m_0 + b_0$ the cash available at the beginning of period 1):

$$l^c(q, \lambda) = \min\{\lambda, L\},$$

$$m^c(q, \lambda) = (1-\tau)wl + \delta,$$

$$b^c(q, \lambda) = \max\left\{0, \frac{1}{2}\left(\frac{W_0}{q} - m\right)\right\},$$

$$x^c(q, \lambda) = \min\left\{\frac{W_0}{p}, \frac{1}{2}\frac{W_0 + qm}{p}\right\},$$

$$x_2^c(q, \lambda) = \max\left\{\frac{m}{\Psi_{p2}(I)}, \frac{1}{2}\frac{W_0 + qm}{q\Psi_{p2}(I)}\right\}.$$

Obviously, the money demand and the consumption demand in both periods will be invreasing in λ, if there is a labour market constraint, and consomption demand in the first period is increasing in q, as long as the consumer has a positive demand for bonds. This confirms the signs of the partial derivatives in (2). Only the interest elasticity of the demand for money will be zero for this consumer, since money will be held as a mean of payment and, therefore, equals after tax wage income and dividend income. Notice that money demand would depend on the interest rate, if e. g. labour supply were not totally inelastic.

It should be noted that demand and supply functions in the current period do not depend on expectations due to the utility function used in this example. In a more general case expectations might influence these functions too.

4.2. Firms

To provide an example of a firm which has the properties outlined in the section 2.2, a producer is considered who has a technology which allows him to produce the output x with the only input labour according to the production function $\sqrt{l}$. This output is available for sale in the following period only, while he can sell out of his previously produced output x_0 today. Suppose that the producer plans for two periods and liquidates at the end of the second period. The objective of the firm, therefore, is to maximize its liquidation value which is equivalent to maximizing the sum of the discounted cash-flows given the budget constraints:

$$m_2 = (1-\tau)p_2 s_2 + b + m. \tag{10}$$

As in the present period, sales s_2 will be the minimum of the constraint ξ_2 and the product x. Notice, however, that p_2, ξ_2 are expected parameters and depend among other things on the expectations with respect to the rationing regime in the second period. If no rationing is expected, this

implies $\Psi_{p2} > x$. This aspect and its consequences will be taken up in the following section and discussed there in detail. Finally, the choices of the firm will be subject to a cash-in-advance constraint as in the case of the consumer:

$$m_0 + b_0 \geq wl + qb + \delta \tag{11}$$

This is necessary to model a consistent payment system, since the consumer receives his wage and dividend income in "cash".

These assumptions lead to the following optimization problem of the firm:

Maximize $(1-\tau)\,\Psi_{p2}(\mathrm{I})\,s_2 + b + m$ by choosing (b, s, l, m, x, s_2) subject to

$$x = \sqrt{l}$$

$$m_0 + b_0 + (1-\tau)ps = wl + qb + m + \delta$$

$$m_0 + b_0 \geq wl + qb + \delta$$

$$\min\{\xi, x_0\} \geq s \geq 0, \quad l \geq 0, \quad x \geq 0, \qquad \min\{\Psi_{\xi 2}(\mathrm{I}), x\} \geq s_2 \geq 0.$$

The following choices are immediately clear. The firm will sell in each period as much as possible and hold the minimum amount of money, namely exactly its after tax sales receipts. The only non-trivial choice is the production decision. If it expects no binding sales constraint, then the optimal choice requires the marginal product of labour $\frac{1}{2} l^{-1/2}$ to be equal to the relevant relative price, $\frac{w}{q(1-\tau)\Psi_{p2}(\mathrm{I})}$. Of course the next period's price net of taxes has to be discounted by q. On the other hand, if a constraint on sales is expected which does not allow to sell this amount, then the optimal production decision is to produce only as much as can be sold according to the expectations. The following demand and supply system reflects these considerations.

$$s^f(q, \xi) = \min\{\xi, x_0\},$$

$$m^f(q, \xi) = (1-\tau)ps$$

$$l^f(q, \xi) = \min\left\{\left(\frac{q(1-\tau)\Psi_{p2}(\mathrm{I})}{2w}\right)^2, (\Psi_{\xi 2}(\mathrm{I}))^2\right\},$$

$$b^f(q, \xi) = \frac{1}{q}(\mathrm{W}_0 - wl),$$

$$s_2^f(q, \xi) = \min\left\{\frac{q(1-\tau)\Psi_{p2}(\mathrm{I})}{2w}, \Psi_{\xi 2}(\mathrm{I})\right\},$$

where $\mathrm{W}_0 \equiv m_0 + b_0 - \delta$ holds.

The role of expectations becomes immediately clear by these functions. If a binding sales constraint is expected for the future, then labour demand responds to changes in exogenes variables and present price and constraint variables only to the extent that these variables affect constraint

expectations. In particular, a reduced sales constraint in the current period will stimulate production only if it leads to the expectation of an eventually reduced sales constraint tomorrow. On the other hand, if the firm expects not to be rationed in the future, then a reduced sales contraint will have no effect what so ever on production decisions, since these will be governed by return considerations only. Note that the fact that the firm may borrow and lend at the same rate (a common feature of macro models) rules out any effect through increased liquidity.

It is straightforward to check that the signs of the partial derivatives correspond to those assumed in (4) except that $m_q^f = 0$ follows here too as in the case of the consumer.

5. The role of expectations

The example of an economy discussed in the previous section made it perfectly clear that all demand and supply functions depend crucially on the way expectations are formed. This section tries to shed some light on this issue without attempting a complete analysis. In the first subsection, it will be shown that rational expectations – perfect foresight – will imply some restrictions on the expectation functions Ψ, but will leave the structure of the model unchanged otherwise. A second subsection, then, argues that the potential of a negative fiscal stimulus remains even under rational expectations.

5.1. The Expectation Function Ψ Under Rational Expectations

It is the basic idea of rational expectation models to use the structure of the model to put restrictions on expectations. To understand this procedure the following general model will be considered.

Denote by s_t the vector of market signals which are to be determined by the market equilibrium conditions. In the case of a fix-price model s_t would comprise the bond price q_t and demand and supply constraints for each market, in the case of a model with flexible prices this market signal would be the price vector. Let α_t be a vector of exogeneous variables in t, e. g. government policy instruments (γ_t, μ_t) and prices in the case of a fix-price model. Finally, let e_t be the vector of stock variables determined in period t and carried over to period $t+1$. In the case of the model of section 4, this would be the list of money, bond and commodity holdings for each agent. If only expectations with respect to the market signal of the following period matter [9], then the system of excess demand functions

9. Typically, the length of the individual planning horizon determines which future endogeneous variables matter.

for all markets can be written as $\Phi(s_t, s_{t+1}, \alpha_t, e_{t-1})$. Similarly, the demand functions for each agent with respect to the stocks e_t can be written compactly as $f(s_t, s_{t+1}, \alpha_t, e_{t-1})$. A rational expectation equilibrium can be defined now as a sequence of market signals $(s_t^*)_{t=1}^{\infty}$ such that for a given sequence $(\alpha_t)_{t=1}^{\infty}$ and a given e_0

$$\Phi(s_t^*, s_{t+1}^*, \alpha_t, e_{t-1}) = 0 \tag{12a}$$

$$e_t = f(s_t^*, s_{t+1}^*, \alpha_t, e_{t-1}) \tag{12b}$$

for all $t = 1, \ldots, \infty$ holds.

The question whether such an equilibrium exists and whether it is unique will not be treated here. For a rather general treatment compare e. g. BLANCHARD [1983]. Notice, however, that equation system (12 *a*) defines implicitly a difference equation of order one for each period which is linked to the one of the previous period by (12 *b*).

Clearly, the solution $(s_t^*)_{t=1}^{\infty}$ will be a function depending on the whole sequence of exogeneous variables $(\alpha_t)_{t=1}^{\infty}$ and on the starting stock position e_0. Therefore, the t-th component function $\Psi_{s_t}^r((\alpha_t^{\infty})_{t=1}, e_0)$ is the rational expectation of the market signal s_t in period t. Using such a "rational expectation" function ψ^r in the choice problems of all agents would allow to do the same kind of comparative static analysis as in section 2 of this paper.

5.2. The Impact of Rational Expectations

As shown in the previous subsection, rational expectations are just the special case of an expectation function determined by the structure of the model itself. To derive this function explicitly, however, is in general extremely difficult. Yet a method proposed and used by NEARY and STIGLITZ [1983] allows to provide some idea as to the consequences of rational expectations for the results derived in section 2. According to this approach a rational prediction of the endogenous variables is required for the second period, but exogeneous expectations are assumed for all periods following period 2. Therefore the system of equations (12) has to be solved for the first two periods only. This leads to a more tractable problem at least for structural models which are not too complicated.

From the analysis in section 3 it is clear that one has to assess the consequences of expectational assumptions on the demand and supply functions only. Yet, in the simplified model in section 4 only labour demand depends on the expected future price for the commodity and on the expectations in respect to future sales constraints. Therefore, it is necessary to analyse what "rational expectations" for these two variables imply.

For a model with fixed prices in the current period two options remain to be considered: either there are fixed prices in the future as well and agents have to predict future market constraints or prices will clear all markets in the future. These two cases will be considered consecutively for the simplified model in section 4.

If prices and wages are exogeneously determined in the second period as well, then the rationally expected market constraint ξ_2 is determined by

$$x_2^c(q, \lambda) + \gamma_2 = \xi_2. \tag{13}$$

Notice that the consumer does not work in the second period and the expected market constraint in the labour market is therefore immaterial for his decision. The expectation function $\Psi^r_{\xi 2}(q, \lambda, \gamma_2)$ under rational constraint expectations is implicitly defined by equation (13). It is easy to check that it is increasing in λ and decreasing in q given the partial derivatives of $x_2^c(.)$ (compare section 4.1), and from the analysis in section 4.2 it is clear that therefore labour demand becomes a decreasing function of q in this case where the firm expects to be sales constraint in the future. Under these circumstances an increase in government expenditure will unambiguously increase employment [theorem (ii)]. The channel of influence is the expectation of the firm that demand for its output in the second period will increase, since higher current interest rates will induce consumers to spend more in the future. Thus, rational expectations will contribute to the effectiveness of a government expenditure program.

If on the other hand prices are expected to clear the markets in the future, i.e. fix-prices are a purely temporary phenomenon, then the firm's rational expectations in respect to future prices, $\Psi^r_{p2}(q, \lambda, \gamma_2)$, are implicitly determined by

$$x_2^c(q, \lambda, p_2) + \gamma_2 - f(\lambda) = 0. \tag{14}$$

Given the partial derivatives of $x_2^c(.)$ from section 4.1 one gets immediately the result that Ψ^r_{p2} is decreasing in q and decreasing in λ, provided that $x^c_{2\lambda} - f' < 0$ holds. This condition just requires that the increase in consumption due to the increase in wage income will not exceed the marginal product of labour, an assumption which is usually taken for granted in fixed price models. As in the previous case a higher interest rate today will induce rational expectations of more demand and higher prices in the second period. But in this case this provides a counteracting effect to the immediate opportunity cost effect only, i.e. $l^f_q + l^f_{p2}(\partial \Psi^r_{p2}/\partial q)$ may be positive or negative depending on the relative strength of these effects.

The analysis in this section has shown that an increase in interest rates may stimulate future consumption through an intertemporal substitution effect. If this is "rationally" expected, then it will lead to the expectation of higher commodity prices or reduced sales constraints tomorrow. Both expectational effects tend to make labour demand an increasing function of the bond price, i.e. a decreasing function of the interest rate. If this expectational effect outweighs the direct opportunity cost effect, then the efficacy of the fiscal stimulus can be reestablished. It needs to be stressed, however, that a "true rational expectation analysis" would have to determine the future disequilibrium state endogenously. Clearly, the expectation of any disequilibrium state where sales of the firm are not rationed will render the effect of an increased demand in the second period immaterial. In such a case the negative impact of an increase in interest rates on labour demand would work with full force.

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