Business Formation and Cyclical Markups in the French Business Cycle

Franck PORTIER*

ABSTRACT. – Two main aspects of the business cycle have been recently considered as important lines of research: business formation and cyclical fluctuations of markups. In an explicitly imperfect competition framework, one needs to understand how entry and exit are related to markup fluctuations, and how this may account for the effect of aggregate demand on economic activity. The paper has two different goals. The first one is to document on French data the pro-cyclicality of business formation and the counter-cyclicality of markups. The second one is to build an intertemporal stochastic general equilibrium model which gives a common explanation to these two phenomena. A first (quite simple) model is proposed, with monopolistic competition between industries and Cournot competition with free entry inside each industry. A reduced-form model of more satisfactory intertemporal behavior of firms in their entry/exit and markups decisions is then proposed, which will motivate further work to give game-theoretic micro-foundations to these intertemporal behaviors.

Fluctuations des taux de marges et entrées-sorties de firmes; une approche théorique appliquée au cas français


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1 Introduction

Are markups variations and entry/exit decision key determinants of the propagation mechanism of exogenous supply and demand shock in the business cycle? We propose in this paper an empirical and theoretical answer to that question for the French business cycle.

As emphasized by Rotemberg and Woodford [1992b], markup fluctuations are needed to account for the observed positive response of hours, output and real wage to a demand shock. Counter-cyclical markups can explain such a stylized fact, as they lead to a labor demand shift which may offset the labor supply shift that generally occurs when the demand shock is a tax-financed government expenditures one.

Business formation is also often mentioned as a magnification and propagation mechanism of exogenous shocks. As documented by Davies and Haltiwanger [1992] and Chatterjee and Cooper [1993], net business formation is low during economic downturns, high during economic upturns. Chatterjee and Cooper [1993] and Devereux, Head and Lapham [1993] propose models where entry and exit magnify supply shocks effects, in a constant markup setting.

It is clear from Industrial Organization literature that the number of firms is correlated to the level of market power. The consequences of that link are explored in this paper. To that extent, one must first give a definition of what entry means. In a model of differentiated goods à la Dixit and Stiglitz [1977], entry can be understood as creation of a new differentiated good (extensive margin) or as creation of a new firm in the industry of a pre-existing good (intensive margin). In the first case (Chatterjee and Cooper [1993] and Devereux, Head and Lapham [1993]), the consequences of entry and exit behavior depend on the taste for diversity of consumers, as emphasized by Benassy [1993]. As we think that entry in the intensive margin is also an important phenomenon, which is directly linked to the level of markups, we will consider in this paper two-stage general equilibrium models, with competition, entry and exit inside industries and monopolistic competition between a fixed number of industries.

The paper is structured as follows. Section 2 presents some stylized facts for the French economy, and proposes a measure of markups fluctuations in the lines of Rotemberg and Woodford [1991]. Section 3 presents a model with Cournot-competition inside industries and free-entry led by an instantaneous zero-profit condition. It is shown that such a model can account for pro-cyclicality of business formation and counter-cyclicality of markups. Section 4 gives some insights of a more realistic forward behavior in the entry and exit decision. We first study an over-simplified two period-two firms partial equilibrium model. We then propose a reduced-form of markups and business formation decisions inside a general equilibrium model, and show how such a model can motivate further research on the micro-foundations of super-game entry and exit modelization.
2 Facts and Measure in the French Business Cycle

2.1. Business Formation

To measure the net business formation at date $t$, one needs to know the number of firms in the economy at date $t-1$, the number of business creation and the number of business destruction from $t-1$ to $t$. Unfortunately, there are no high-frequency and long-period data for these variables in the French statistics. The SIRENE $^1$ databank of Inséé gives in 1954, 1958, 1962, 1966, 1971 and annually since 1977 the number of firms in the French Economy. Since 1985, this figure is published monthly. Firm destructions are less easy to measure since there is no legal enforcement to inform SIRENE when a firm exits the market. One therefore needs data from bankruptcy to correct the SIRENE number for un-registered exits. In that case, the date of the judgment, and not the date of exit, is relevant for the SIRENE correction $^2$.

To (imperfectly) measure the number of firms, we then use the SIRENE file from 1977 to 1989, annual data. The number of firms ($n$) is the total number of firms less the number of self-employed, which represents more than 50% of the total. This series and the annual Gross Domestic Product series ($y$) are plotted on Figure 1, as log-deviations from a linear trend.

The procyclicality of the number of firms, which is clearly illustrated on Figure 1, is checked in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>GDP-Number of firms correlation.</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>cor($y$, $n_{t-1}$)</td>
</tr>
<tr>
<td></td>
<td>.42</td>
</tr>
</tbody>
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The contemporaneous correlation and the correlations at one lag and one lead are all positive $^3$. This fact can suggest the following (intuitive) mechanism: expansion creates business formation and business formation magnifies expansion. Such a mechanism will be more formally developed in sections 2 and 3.

1. Système Informatique pour le Répertoire des Entreprises et de leurs Etablissements.
3. One must consider these figures with caution, as the sample is small and the number of firms poorly measured.
2.2. Markups

2.2.1. Theoretical Framework

One needs a theoretical framework to measure markups of price over marginal cost, or markup of factors remuneration over their marginal productivity, since these variables are not observable. We follow HALL [1988] and ROTEMBERG and WOODFORD [1991] to give a measure of the average level of markup and of markup fluctuations for the French economy.

We assume that the macroeconomic value-added production function has increasing returns to scale, and is given by:

\[ Y_t = A_t F(K_t, x_t H_t) - N_t x_t \Phi \]

where \( K \) is the capital stock, \( H \) the level of worked hours, \( N \) the number of firms, \( \Phi \) a constant fixed-cost and where \( F \) is homogeneous of degree 1. \( x \) is a deterministic or stochastic non-stationary labor-augmenting technical progress and \( A \) is the deterministic or stochastic stationary level of total factor productivity. We assume that \( x, K \) and \( Y \) have a common (stochastic or deterministic) trend. With a fixed-cost per firm given by \( x_t \Phi \), there exists a stationary steady-state (with all variables deflated by \( x \) except \( H \) and \( N \)) where the number of firm is constant. Therefore, \( H, A \) and \( N \) are stationary variables.
Taking the log-linearization of (1) around the steady-state of the economy, one gets:

\[
\hat{y}_t = \frac{A}{Y}\hat{a}_t + \frac{AKF_1}{Y}\hat{k}_t + \frac{AxF_2 - N\Phi}{Y}\hat{x}_t + \frac{AxH^2}{Y}\hat{h}_t - \frac{xN\Phi}{Y}\hat{n}_t
\]

where the “hat” stands for relative deviation from steady-state and where \( F_i \) is the partial derivative of \( F \) with respect to its \( i \)th argument.

We assume that the factor markets are perfectly competitive, and that each firm has some market power. Each firm chooses a markup \((1 + \mu^*) \) and the aggregate factor demands at the steady-state are given by:

\[
AF_1 = (1 + \mu^*)z
\]

\[
AxF_2 = (1 + \mu^*)w
\]

where \( z \) is the real rental cost of capital and \( w \) the real wage rate (deflated by the general price level –i.e. the price of the composite good).

As \( F \) is homogeneous of degree 1, one has from equation (1)

\[
1 = \frac{AF_1}{Y}K + \frac{AxF_2}{Y}H - \frac{N\Phi}{Y}
\]

To avoid the difficulty of identifying separately the two components \( x \) and \( A \) of the technical progress, we assume in the following that all the technological progress is labor augmenting, which is equivalent to assuming \( \hat{a}_t = 0 \). With \( s_h = \frac{wK}{Y} \) and \( s_k = \frac{xL}{Y} \) being the share of labor and capital income in total value-added, and with \( s_\Phi = \frac{N\Phi}{Y} \), one gets from equations (2), (3), (4) and (5):

\[
\hat{x}_t = \frac{\hat{y}_t - (1 + \mu^*)s_k\hat{k}_t - (1 + \mu^*)s_h\hat{h}_t + s_\Phi\hat{n}_t}{1 - (1 + \mu^*)s_k}
\]

The true technological progress –i.e. not contaminated by demand shocks in that imperfectly competitive framework – can therefore be computed, knowing the average markup \( \mu^* \), the shares \( s_h \), \( s_k \) and \( s_\Phi \) and the deviations series \( \{\hat{n}\}, \{\hat{k}\}, \{\hat{h}\} \) and \( \{\hat{y}\} \).

From that \( \{\hat{x}\} \) series, one can recover a markup deviations series \( \{\hat{\mu}^*\} \).

One first needs to log-linearize (4):

\[
\frac{\mu^*}{1 + \mu^*}\hat{\mu}^*_t = \hat{x}_t - \hat{w}_t + \frac{(1 + \mu^*)s_k}{e}\left(\hat{k}_t - \hat{x}_t - \frac{(1 + \mu^*)s_h}{1 -(1 + \mu^*)s_h}\hat{h}_t\right)
\]

4. \( \mu^* \) is the markup of factor marginal productivity over factor marginal remuneration. It can be different from the markup of price over marginal cost \( \mu \) if materials enter the production process (see Rotemberg and Woodford [1992b]).
where \( e \) is the elasticity of substitution between factors in the production function. Using (6) for \( \hat{\gamma} \), one gets the following expression of markup fluctuations:

\[
(8) \quad \hat{\mu}_t = \frac{1 + \mu^*}{\mu^*} \left( \frac{e - (1 + \mu^*)s_k}{e(1 - (1 + \mu^*)s_k)} \hat{\gamma}_t + \frac{s_h(e - (1 + \mu^*)s_k)}{e(1 - (1 + \mu^*)s_k)} \hat{w}_t \right) \\
+ \frac{1 + \mu^*}{\mu^*} \left( \frac{(1 - e)(1 + \mu^*)s_k}{e(1 - (1 + \mu^*)s_k)} \hat{\gamma}_t - \frac{(1 + \mu^*)s_h}{1 - (1 + \mu^*)s_k} \hat{w}_t - \hat{w}_t \right)
\]

This expression is the one of Rotemberg and Woodford [1991] corrected for the variations of the number of firm. Because of the lack of data for the rental rate of capital and the number of firms at a quarterly frequency and over a large sample, we will proxy the variations of markups by

\[
\hat{\mu}_t = \frac{1 + \mu^*}{\mu^*} \left( \frac{e - (1 + \mu^*)s_k}{e(1 - (1 + \mu^*)s_k)} \hat{\gamma}_t - \frac{(1 + \mu^*)s_h}{1 - (1 + \mu^*)s_k} \hat{w}_t - \hat{w}_t \right)
\]

To measure an average markup rate \( \mu^* \) of labor productivity over the wage rate, we use Hall’s method. \( \mu^* \) must be such that the measure of the technical progress \( \hat{\gamma} \) given by equation (6) is orthogonal to a pure demand shock, e.g. the innovation \( \varepsilon_{\hat{g}} \) of government expenditures process \( \hat{g} \). One must therefore choose \( \mu^* \) such that

\[
\text{cov}(\hat{\gamma}(\mu^*), \varepsilon_{\hat{\mu}t}) = 0
\]

### 2.2.2. Results on French Data

We assume that the labor-augmenting technical progress follows a stationary process around a deterministic trend, and we compute \( \{\hat{h}\}, \{\hat{y}\}, \{\hat{w}\}, \{\hat{g}\} \) as log-deviation from that common deterministic trend. We use French National Account quarterly data on the sample 1970:1-1991:4. \( y \) is the Gross Domestic Product, \( h \) is the total hours worked per quarter, \( w \) is computed as the labor share in GDP series times GDP and divided by the number of worked hours. We assume zero-profit at the steady-state, and \( s_k \) is therefore given by \( 1 - s_h \). \( g \) is the consumption of public and private administrations. We estimate the government expenditures innovation \( \varepsilon_g \) as the innovation of an AR(1) process on \( \hat{g}_t \):

\[
\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_{g,t}
\]

With these series, one gets the following average markup:

**Table 2**

**Average markup** (\( \text{cov}(\hat{\gamma}(\mu^*), \varepsilon_{\hat{\mu}t}) = 0 \)).

<table>
<thead>
<tr>
<th>( \rho_g )</th>
<th>( \mu^* )</th>
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<tr>
<td>.94</td>
<td>.373</td>
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</table>

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This average markup seems relatively weak (with respect to measures on the U.S. economy), and will need to be confirmed by studies at the industry level. As shown in Rotemberg and Woodford [1992a] and [1995], this measure underestimates the market power if materials are needed for production. We will therefore study the robustness of the markup fluctuations measure to this parameter \( \mu^* \). One also needs an estimation of the capital-labor elasticity \( e \). As we will choose \( F \) to be Cobb-Douglas in the theoretical model, we set \( e = 1 \). We also check the results for \( e \) in the interval \([1/2, 2]\).

With the benchmark calibration \( (\mu^* = .373 \text{ and } e = 1) \), the series \( \hat{\gamma} \) and \( \hat{\mu}^* \) are plotted on Figure 2.

One must notice that this measure of average markup is not robust to a change in the measure of the demand shock \( \varepsilon_y \). For that reason, the sensibility of markups-output correlation to the average markup level is studied in Figure 3.

The market power is defined as a decreasing function of the price-elasticity of demand.

The plotted series of markups has been rescaled, and is equal to the actual series multiplied by \( \frac{\mu^*}{1 + \mu^*} \).

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**Figure 2**


One can clearly see the counter-cyclicality of markups in the French business cycle, and Table 3 gives the lagged, contemporaneous and lead correlations:

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5. One must notice that this measure of average markup is not robust to a change in the measure of the demand shock \( \varepsilon_y \). For that reason, the sensibility of markups-output correlation to the average markup level is studied in Figure 3.

6. The market power is defined as a decreasing function of the price-elasticity of demand.

7. The plotted series of markups has been rescaled, and is equal to the actual series multiplied by \( \frac{\mu^*}{1 + \mu^*} \).
Expansions are correlated with a low level of markups (–.67), and lead to a lower level of markups the next period (–.73). When markups are low, output of the next period is high (–.60).

The counter-cyclicality of markups is robust to the choice of $e$ and $\mu^*$, as shown on Figure 3, which plots the correlation between GDP and markups for $e \in [1/2, 2]$ and $\mu^* \in [1, 2]$.

For a given level of $e$, the higher is $\mu^*$ the stronger is the counter-cyclicality of markups. For a given level of $\mu^*$, the higher is $e$ the weaker is the counter-cyclicality of markups. Nevertheless, even for very conservative values (no average markups and $e = 2$), the correlation is negative.

Is that negative correlation robust to the approximation we made by assuming $\hat{n} = 0$, $\hat{K} = 0$? As the sign of the number of firms is the same as the one of output in equation (8) and as it is pro-cyclical, the
correlation will be more negative if one adds a series of cyclical fluctuations of \( n \). As investment is pro-cyclical, including \( \hat{k} \) will also increase the counter-cyclicality of markups 8.

2.3. Markups and Business Formation

From the last two subsections, one can draw the following conclusions:

- there is a positive correlation between the number of firms and the level of activity,
- there is a negative correlation between markups and the level of activity.

One can therefore draw from these results a negative correlation between markups and the number of firms, which may be an important propagation mechanism in the business cycle. This will be true if correlations are respectively +1 and −1. In our case, the results are less affirmative, as it can be seen on Figure 4 and Table 4:

**FIGURE 4**
Number of Firms and Markups Cyclical Fluctuations (France, 1977-1989, annually).

8. See Rotemberg and Woodford [1991] for a detailed discussion of the consequences of taking into account the discrepancy between average and marginal wage (Bils [1987]) or adjustment costs for hours.
There is no clear contemporaneous relation between the number of firms and markups (.015), and between markups and lagged number of firms (.13). Nevertheless, when markups are high, the number of firms is high one year after (.28), which can be explained by a free-entry mechanism. Once again, these results, on a small sample with a proxy of a number of firms, must be considered with caution. At this stage of our research, we can find five different explanations for this absence of correlation, under the null hypothesis that there must be a negative correlation between these two variables. The three first rely on the data treatment: (i) the sample is too small for the result to be significative, (ii) the number of firms is too badly measured, (iii) the series are too badly detrended i.e. the trend-stationary assumption is incorrect. For this first panel of explanation, one can only answer with new data. Two more economic explanations can be found: (iv) they are two sources of impulse in the economy, one that gives a positive correlation between markups and the number of firms, one that gives a negative one, with these two effects cancelling in the business cycle; (v) there is a relation between “potential” entry and exit and markups, but not between “effective” entry and exits and markups, since some entry-deterrence mechanisms can occur. Such mechanisms will be developed more explicitly in section 4. For the moment, we restrict our attention on models where the correlation is negative.

3 A Model of the Interaction between Business Formation and Markups

We propose in this section a simple theoretical link between business formation and markups, in a model where industries are monopolistic competitors, where firms, producing an identical good, play Cournot inside each industry and where entry and exit occur to fulfill a zero-profit condition.

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9. This absence of relation between markups and the number of firms is also true in the contestable market literature.
3.1. The Model

Technology and Preferences: The economy is composed of industries indexed by $i$ and $N_{i,t}$ firms indexed by $j$ in each industry. The value-added production function of firm $j$ in industry $i$ is given by

$$Y_{i,j,t} = A_tF\left(K_{i,j,t}, \kappa^t H_{i,j,t}\right) - \kappa^t \Phi$$

where $\kappa$ is the rate of labor-augmenting technical progress. $A_t$ is given by the following stochastic process:

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log A + \varepsilon_{A,t}$$

where the $\varepsilon_{A,t}$ are i.i.d. stochastic variables following a $N(0, \sigma^2_A)$, and where $\rho_A < 1$. $A$ is the steady state level of $A_t$. The total factor productivity $A_t$ is a technological shock common to all firms of the economy, and we assume that $A_t$ is observed at the beginning of period $t$. $\Phi$ is a fixed cost of production, expressed in terms of each firm output.

We will assume in the following that $F$ is Cobb-Douglas, with respective weights $\alpha$ (on capital) and $(1 - \alpha)$ (on hours).

There is a representative household whose preferences are given by the utility function $U_0$ which represents the expectation of the discounted sum of instantaneous utility flows $u$, conditional on the information available at date $t = 0$:

$$U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - H_t) \right]$$

$$= E_0 \left[ \sum_{t=0}^{\infty} \beta^t \log C_t + v(1 - H_t) \right]$$

where $\beta \in [0, 1[$. $C_{i,t}$ is a CES basket of the different goods produced in the economy (one in each industry):

$$C_t = \left( \sum_{i=1}^{n} C_{i,t}^{1+\gamma} \right)^{1+\gamma}$$

where $\frac{1+\gamma}{\gamma}$ is the elasticity of substitution between the different goods. We can define a price index $P_t$:

$$P_t = \left( \frac{1}{n} \sum_{j=1}^{n} P_{j,t} \right)^{-\gamma}$$

10. Except for the competition inside industries, this economy is close to HAURAUT and PORTIER [1993].
which verifies by construction the equation:

\[ P_t C_{ik,t} = \sum_{j=1}^{n} P_{j,t} C_{ij,j,t} \]

\( H_t \) is the level of worked hours of the household, and we assume that \( v(1 - H_t) \) is concave and that the elasticity of its derivative with respect to \( H_t \) is constant:

\[ \frac{(1 - H_t) v''(1 - H_t)}{v'(1 - H_t)} = \zeta \quad \forall H_t \]

This elasticity \( \zeta \) is inversely related to the intertemporal substitution elasticity of leisure.

The household accumulates capital and rents it to firms. Capital is a composite good, given as a CES index of the different produced goods. To simplify the computation of the demand function addressed to industry \( i \), we assume that the investment index has the same structure as the consumption one \(^{11}\). The accumulation technology is then given by the following equation:

\[ K_{t+1} = (1 - \delta)K_t + \left( \sum_{i=1}^{n} I_{i,t}^{1+\gamma} \right)^{1+\gamma} = (1 - \delta)K_t + I_t \]

where \( I_{i,t} \) is the investment index. One unit of each consumption good at period \( t \) is necessary to get one unit of capital at period \( t + 1 \); the existing capital depreciates at constant rate \( \delta \).

**Government expenditures:** The government levies lump sum taxes \( T_t \) to finance its expenditures \( G_t \). For simplicity, it is assumed that these expenditures do not enter the production or the utility function. The period \( t \) budgetary constraint of the government is then given by:

\[ G_t = T_t \]

\( G_t \) follows the stochastic process:

\[ \log G_t = \rho_G \log G_{t-1} + (1 - \rho_G) \log G + \varepsilon_{G,t} \]

where the \( \varepsilon_{G,t} \) are i.i.d. stochastic variables following a \( \mathcal{N}(0, \sigma_G^2) \), and where \( \rho_G < 1 \). \( G \) is the steady state level of \( G_t \). We assume that \( G_t \) is observed at the beginning of period \( t \).

\(^{11}\) See Gali [1991] for the consequences of relaxing that assumption.
Optimal Behavior of the Household: The optimal choice of the consumption and investment index composition can be viewed as a static choice. For given levels of consumption and investment $C_t$ and $I_t$, household $i$ maximizes respectively these index by choosing $C_{i,t}$ et $I_{i,t}$ for all $i$, taking as given prices $P_{i,t}$ in each industry. Solving these two programs gives the following consumption and investment demands of the household to industry $i$:

\begin{align}
C_{i,t} &= P_{i,t}^{1 + \gamma} P_t^{1 + \gamma} C_t \\
I_{i,t} &= P_{i,t}^{1 + \gamma} P_t^{1 + \gamma} I_t
\end{align}

The household enters in period $t$ with a predetermined level of capital $K_t$. In period $t$, the household is taxed in a lump sum way at a level $T_t$, receives its wage income, the remuneration of its capital and the profits of all the firms. It chooses its level of consumption, its supply of labor and the quantity of capital $K_{t+1}$ it will transfer next period. With $w_t$ the real wage and $z_t$ the capital gross rental rate, the budget constraint of the household is given by:

\begin{equation}
P_t K_{t+1} + P_t C_t \leq P_t (1 - \delta + z_t) K_t + P_t w_t H_t + P_t \Pi_t - P_t T_t \tag{21}
\end{equation}

The optimal solution of the household problem verifies the Bellman equation:

\begin{equation}
V_t(K_t, I_t) = \max_{C_t, H_t, K_{t+1}} \{\log C_t + v(1 - H_t) + \beta E_t[V(K_{t+1}, I_{t+1})]\}
\end{equation}

with respect to the budget constraint (21). Let $I_t$ be the informational set of the household at period $t$: $I_{t} = \{G_t, A_t, \{P_{i,t}\}_{i=1}^{N}, w_t, z_t\}$. We solve this problem to get the first order conditions of the household program. We also impose the transversality conditions:

\begin{equation}
\lim_{\tau \to +\infty} E_t \left[ \beta^{t+\tau} K_{t+\tau+1} \frac{\partial V_{t+\tau+1}}{\partial K_{t+\tau+1}} \right] = 0 \tag{22}
\end{equation}

With

\begin{equation}
\lambda_t = \beta E_t \left[ \frac{\partial V(K_{t+1})}{\partial K_{t+1}} \right]
\end{equation}

12. The interest rate is then given by $\eta_t = z_t - \delta$. 

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one gets the following Euler equations:

\begin{align*}
(23) & \quad \frac{1}{C_l} = \lambda_l \\
(24) & \quad \nu'(1 - H_l) = \lambda_l w_l \\
(25) & \quad 1 = \beta E_t \left[ (1 + z_{t+1} - \delta) \frac{\lambda_{t+1}}{\lambda_t} \right]
\end{align*}

**Demand Addressed to Industry \( i \):** From (19) and (20) one gets the inverse demand function addressed to industry \( i \):

\[ P_{i,t} = P_t B_{1,t} y_{i,t}^{-\theta} \]

with \( B_{1,t} = \frac{Y_t^0}{n} \) and where \( \theta \) is the inverse of the price-elasticity of demand \( (\theta = \frac{1}{1+\gamma}) \)

**Optimal factor demands of firm \( j \):** The optimal factor demands of firm \( j \) are given by the resolution of the following problem:

\[
\begin{align*}
\min_{K_{i_{t+1}},J_{i_{t+1}}} & \{ z_t K_{i_{t+1}} + w_l H_{i_{t+1}} \} \\
\text{s.t.} & \quad A_t K_{i_{t+1}}^{\alpha} (H_{i_{t+1}})^{1-\alpha} - \kappa^t \Phi \geq Y_{i_{t+1}} 
\end{align*}
\]

One gets from this program:

\begin{align*}
(26) & \quad \kappa^t H_{i_{t+1}} = \left( \frac{\alpha}{1 - \alpha} \right)^{-\alpha} A_t^{-1} \left( \frac{w_l}{z_t} \right)^{-\alpha} (Y_{i_{t+1}} + \kappa^t \Phi) \\
(27) & \quad K_{i_{t+1}} = \left( \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} A_t^{-1} \left( \frac{w_l}{z_t} \right)^{1-\alpha} (Y_{i_{t+1}} + \kappa^t \Phi)
\end{align*}

One can therefore give the expression of the cost function of firm \( j \):

\[
C(Y_{i_{t+1}}) = B_{2,t} (Y_{i_{t+1}} + \kappa^t \Phi)
\]

with \( B_{2,t} = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha} A_t^{-1} w_l^{1-\alpha} z_t^{\alpha} \). \( B_{2,t} \) is the constant (at the firm level) marginal cost of production.

**Competition in Industry \( i \):** We assume that each firm \( j \) of industry \( i \) plays Cournot with the other firms of the industry (denoted as \(-j\)), and thinks that each industry has a negligible effect on the aggregate variables. Plugging the inverse demand function in the definition of firm \( j \) profit, one gets the following program:

\[
\max_{Y_{i_{t+1}}} \{ P_t B_{1,t} (Y_{i_{t+1}} + Y_{i_{t+1}} - j) - P_t B_{2,t} (Y_{i_{t+1}} + \kappa^t \Phi) \}
\]
At the symmetrical equilibrium in industry \( i \), one has \( N_{it} Y_{it} = Y_{it} \), and the optimality conditions of the firms’ program lead to:

\[
Y_{it} = \left( \frac{B_{2t}}{B_{1t}} \right)^{-1/\theta} \left( \frac{N_{it}}{N_{it} + \theta} \right)^{-1/\theta}
\]

\[
\Pi_{it} = (1 - B_{2t}) Y_{it} - B_{2t} N_{it} k^d \Phi
\]

**Free-Entry and Zero-Profit Condition:** We assume that entry and exit are free in each industry, and that a zero-profit condition is verified in each period:

\[
\Pi_{it} = 0 \quad \forall \ i, j, t
\]

This extreme assumption of instantaneous entry or exit could be relaxed and substituted with an error-correcting mechanism on the number firm, as in ROTEMBERG and WOODFORD [1992a], without affecting the qualitative results of the model.

**Symmetrical Equilibrium:** With the normalization \( n = 1 \) and \( P_i = 1 \), \( \forall t \), the symmetrical equilibrium of the economy is given by equations (9), (16), (17), (21), (23), (24), (25), (28), (29) and (30).

From equation (28), one can recover the level of markup over marginal cost \( B_{2t} \), which is given by:

\[
\mu_t = \frac{1}{B_{2t}} - 1 = \frac{\theta}{N_t - \theta}
\]

The markup is a decreasing function of the number of firms in the economy, and for \( N_t = 1 \), \( \mu_t = \gamma \), which is the constant level of markup of the standard Dixit-Stiglitz model of monopolistic competition. As profits are pro-cyclical in this economy (without free-entry), the number of firms will be pro-cyclical and markups will be counter-cyclical.

**Linearization and Resolution:** Deflating all variables (except hours and the number of firms) by the labor-augmenting technical progress \( x \), one gets a stationary system that can be log-linearized around its steady-state. The steady-state cannot be computed analytically since the zero-profit condition induces a non-linearity in the number of firms. Nevertheless, we can solve numerically for the steady-state, and therefore we can log-linearize the system defining the equilibrium to obtain the following linear system (where \( \tilde{v}_t \) is the percentage of deviation of the variable \( V_t \) from its stationary value \( \bar{V} \)):

\[
M_1 \tilde{q}_t = M_2 \tilde{\theta}_t + M_3 \tilde{e}_t
\]

\[
M_4(L) E_t \begin{bmatrix} \tilde{\theta}_{t+1} \\ \tilde{e}_{t+1} \end{bmatrix} = M_5(L) E_t \begin{bmatrix} \tilde{\theta}_{t+1} \\ \tilde{e}_{t+1} \end{bmatrix} + M_6(L) E_t \begin{bmatrix} \tilde{e}_{t+1} \end{bmatrix}
\]
where the $M_j$ are real matrix and the $M_j(L)$ are polynomial matrix of order less or equal to one. $L$ is the lag operator and

\[
\hat{\Theta}_t = \begin{pmatrix} \hat{k}_t \\ \lambda_t \end{pmatrix}, \quad \hat{\Omega}_t = \begin{pmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{\omega}_t \\ \hat{\varphi}_t \\ \hat{\eta}_t \\ \hat{\mu}_t \end{pmatrix}, \quad \hat{e}_t = \begin{pmatrix} \hat{u}_t \\ \hat{g}_t \end{pmatrix}
\]

This system is solved following BLANCHARD and KAHN [1980] and KING, PLOSSER and REBELO [1987] and the solution is given by the set of equations:

\[
\begin{align*}
\hat{s}_{t+1} &= M\hat{s}_t + \varepsilon_{t+1} \\
\hat{d}_t &= \Pi\hat{s}_t
\end{align*}
\]

(33) \hspace{1cm} (34)

where $\hat{s}_t$ is the vector of state variables (pre-determinate or exogenous) and $\hat{d}_t$ is the vector of controls:

\[
\hat{s}_t = \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix}, \quad \hat{d}_t = \begin{pmatrix} \hat{c}_t \\ \hat{h}_t \\ \hat{\omega}_t \\ \hat{\varphi}_t \\ \hat{\eta}_t \\ \hat{\mu}_t \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} 0 \\ 0 \\ \varepsilon_{A,t} \\ \varepsilon_{f,t} \end{pmatrix}
\]

The $\Pi$ matrix is the optimal policy rule matrix. Its $\pi_{jl}$ coefficients are instantaneous elasticities whose values will be crucial in the dynamics. Equations (33) and (34) allow us to simulate the model and to compute impulse responses functions.

### 3.2. Responses to Technological and Government Expenditures Shocks

The model is calibrated according to HAIRAULT and PORTIER [1993] and LAFFARGUE, MALGRANGE and PUJOL [1990] (Table 5).

<table>
<thead>
<tr>
<th>Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>.46</td>
</tr>
<tr>
<td>$\rho_A$</td>
</tr>
<tr>
<td>.9</td>
</tr>
</tbody>
</table>
The markup in a model with one firm per industry is equal to $\gamma = .4$, and one must notice that the effective equilibrium markup is only $\mu = .29$, because of the strictly greater than one number of firms $N$. The elasticity of labor supply $1/\zeta$ is chosen relatively low, as we think that the “good results” of a model cannot rely on a unrealistic high level of labor supply elasticity.

Following a technological shock, potential profits increase, which leads to entry and low markups: business formation is pro-cyclical and markups are counter-cyclical (Figure 5).

![Figure 5](image)

**Figure 5**

*Response of Markups and Number of Firms to a Technological Shock.*

In such a case, as stressed in Rotemberg and Woodford [1992]a, counter-cyclical markups magnify output and hours response (Figure 6) relatively to a constant markup model. Therefore, if imperfect competition with markup fluctuations reduces the measured variability of the technological shock, it implies a propagation mechanism which leads, for a given variability of exogenous technological shock, to a higher variability than in the perfect competition case.

The response of the economy to a government expenditures shock (Figures 7 and 8) clearly illustrates the difficulty of imperfect competition models in accounting for a *keynesian* effect of demand shock. The response of the economy for the first four quarters is coherent with stylized facts for output, hours and the number of firms (pro-cyclical) and for the markups (counter-cyclical). Nevertheless, real wage instantaneously

---

13. See Portier [1993] (chapter 3) for a discussion on that point.
Figure 6
Response of Output, Hours and Real Wage to a Technological Shock.

Figure 7
Response of Markups and Number of Firms to a Government Expenditures Shock.
decreases, which means that the wealth effect of tax increases induces a shift of the labor supply curve which dominates the shift of the labor demand curve induced by the reduction of markups.

At the same time, consumption and investment are strictly crowded-out by government expenditures. Therefore, capital stock decreases and output and hours become negative —i.e., are below their steady-state level — after five periods for output and seventeen for hours. As markup counter-cyclicality shifts the labor demand curve, the response of hours is less negative than in a perfect competition model.

The key determinant of that instantaneous elasticity of wage to a government expenditures shock are the elasticity of labor supply (which is related to the inverse of $\zeta$) and the elasticity of substitution between goods $\gamma$, which is a measure of the level of markups.

Figure 9 shows that wage elasticity (in absolute terms) is a decreasing function of the degree of imperfect competition and the elasticity of labor supply. For high levels of markup and low elasticity of labor supply, this elasticity can be found positive. Nevertheless, in such a case, the crowding-out effect of private expenditures is maximum, and output and hours quickly go below their steady-state values.

This Cournot model succeeds in explaining markups and business formation cyclical behavior, but does not allow for a large enough shift in labor demand following a demand shock, and therefore cannot account for pro-cyclicality of real wage.
4 Towards an Intertemporal Relation Between Business Formation and Markups

The essential shortcomings of the free-entry Cournot model of section 3 are first the very mechanic link between the number of firms and the level of markup, and second the absence of any strategic behavior in the entry/exit decisions of firms. The importance of the former point is stressed by ROTemberg and Woodford [1992a], where several theories of markup fluctuations are developed and tested. Much of the Industrial Organization literature is devoted to the latter—i.e. the analysis of entry deterrence strategies of incumbent in product markets. Since Bain [1956], barriers to entry, defined as anything that allows incumbent firms to earn supranormal profits without threat of entry, have been a key phenomenon in the understanding of market structure 14.

Since we are at the moment unable to model a super-game with incumbents, entry-deterrence and forward-looking behavior in general

equilibrium, we will adopt in this section a reduced-form of the markup-entry decision block of a model which is close to the one of section 3. Before, we give some insights of the link between current profits, expected profits and entry/exit decisions in a very simple two period model.

4.1. Some Insights in a Stylized Model

Let us consider an industry with the inverse demand function:

\[ p_t = 1 - \frac{1}{a_t} q_t \]

where \( q \) is the aggregate output in the industry and \( a_t \) a scale parameter of the demand function. Booms are periods with high values of \( a \), recessions are periods with low values 15. The industry is composed of an incumbent (firm 1) and a potential entrant (firm 2) in period 1, and the world ends at the end of period 2. Marginal cost is null for both firms, but firm 2 has to pay an installation cost \( \phi \) to enter in period one. If this cost is paid, firms will be similar in period 2. For simplicity, we assume that firm 2 cannot produce in period 2 if it was not producing in period one 16. Firms discount future at rate \( \frac{1}{1+r} \). The presence of an installation cost is crucial since it leads to an explicit forward behavior of firms: the entry decision is made with respect to the comparison between the expected discounted sum of profits and the cost \( \phi \).

The game is played as follows: firm 1 decides how much to produce in period one \( (q_{1,1}) \), and it is assumed that this decision cannot be changed 17. After having observed \( q_{1,1} \), firm 2 decides whether or not it pays the fixed cost \( \phi \) and enters. If it enters, firm 2 will be a Stackelberg follower in the quantity game of period 1, and Cournot competitor in period 2. If not, firm 1 will be monopolistic in period 1 and 2.

Profits of period 2 are given by the resolution of the Cournot duopoly game if firm 2 enters in period 1 (profits \( \pi_{2,1}^d \) for firm 1 and \( \pi_{2,2}^d \) for firm 2), or by the resolution of the monopolistic problem of firm 1 in the other case (profit \( \pi_{2,1}^m \)):

\[
\begin{align*}
\pi_{2,1}^d &= \frac{a_2}{9} \\
\pi_{2,2}^d &= \frac{a_2}{9} \\
\pi_{2,1}^m &= \frac{a_2}{4}
\end{align*}
\]

15. One can define booms and recessions in terms of technological shocks without affecting the behavior of the model.

16. Relaxing this hypothesis would not alter the qualitative results of the model, and would simply reduce the range of parameters in which firm 1 is alone in period 2.

17. The decision of firm 1 is then credible. Relaxing this hypothesis will need an explicit treatment of credible strategies, e.g. sunk capacities.
If it enters in period 1, firm 2 takes \( q_{1,1} \) as given, and its best response function is given by:

\[
q_{1,2}(q_{1,1}) = \frac{a_1}{2} - \frac{q_{1,1}}{2}
\]

This best response function is used by firm 1 to decide its production level and to compute the profits of first period \( \pi_{1,1}^d, \pi_{1,2}^d \) and \( \pi_{1,1}^m \) in the monopoly case. The relevant profits for the entry decision and the entry deterrence decision are the discounted sum of profits of each period:

\[
\pi_1 = \begin{cases} 
\pi_1^d = \pi_{1,1}^d + \frac{1}{1 + r} \pi_{2,1}^d & \text{if firm 2 enters} \\
\pi_1^m = \pi_{1,1}^m + \frac{1}{1 + r} \pi_{2,1}^m & \text{if firm 2 does not enter}
\end{cases}
\]

\[
\pi_2 = \begin{cases} 
\pi_{1,2} - \phi + \frac{1}{1 + r} \pi_{2,2} & \text{if firm 2 enters} \\
0 & \text{if firm 2 does not enter}
\end{cases}
\]

There is a simple graphical resolution of the model. If firm 1 chooses a large quantity in period 1, price will be low and firm 2 intertemporal profit may be negative. In such a case, entry is deterred. Therefore, to decide whether or not it deters entry, firm 1 compares its monopoly profit \( \pi_{1,1}^m \) at the quantity \( q_{1,1} \) where \( \pi_2 \) is null with its profit in the duopoly case \( \pi_{1,1}^d \). In the case depicted on Figure 10, entry is deterred.

**Figure 10**

*Entry Deterrence.*
One can do some comparative static with that model. Let us assume that \( a_1 \) is increased \(^{18}\), which means that the economy will experience a boom in period 1. The effect of the shock on the model is depicted on Figure 11. Profits in period 1 will be higher, and in the case of Figure 11, they are now high enough to finance the payment of the installation cost in period 1 for the potential entrant. Firm 1 cannot deter entry. In that model, a boom causes output growth and a price fall large than the one implied by the increase in \( a_1 \) because the market structure changes from monopoly to duopoly. In terms of a more general model, output increases, the number of firms is pro-cyclical and the markup (here the price as marginal cost is null) is counter-cyclical.

**Figure 11**

*Entry Accommodation with a Boom in Period 1.*

Is the result of Figure 11 robust to a general equilibrium modelization? One can show in the simple model that entry may not occur, but that the pro-cyclicality of markups still holds. One can reasonably assess that, in a general equilibrium model, the occurrence of a boom today will increase the interest rate, and therefore reduce the discounted sum of future profits. If the increase in the interest rate is high enough, it may reverse the positive effect of the boom in period 1 on the profit of firm 2. This is the case on Figure 12, where the boom increases output and decreases markup (prices), but where entry does not occur.

---

\(^{18}\) The same analysis can be done for a boom in period 2.
This simple model has shown that markups may fall even with a constant number of firms, as long as the threat of entry is taken into account, and that forward-looking behavior is important in the understanding of business formation and markups cyclical movements. These insights are developed in a general equilibrium model with a reduced-form treatment of markups and entry behavior.

### 4.2. An Intertemporal Reduced-Form Model

The model is close to the one of section 3. The economy is composed of one representative household and \( n \) industries in monopolistic competition. Technology and preferences are the same, and we limit our analysis to symmetrical equilibria. The only difference concerns the markup and entry/exit behavior. We first assume that the number of firms is determined by the following equation:

\[
N_t = NY_t^{\nu_1}X_t^{\nu_2}
\]

where \( N \) is a scale parameter and \( X_t \) is the expected discounted sum of future profits if no firms ever enter the market. \( X \) is therefore the value of incumbents if no entry occurs in the future. As it has been stressed in the simple two period model, this variable is a key determinant of the behavior of incumbents. The model of section 3 corresponds to the case \( \nu_1 > 0 \) and \( \nu_2 = 0 \).
As markups are function of the number of firms, but can also vary without any entry or exit, because markups undercutting can be considered as barriers to entry (see the example of Figure 12, the markup behavior is given by the following equation:

\[ \mu_t = \mu N^{\mu_1} Y_t^{\mu_2} X_t^{\mu_3} \]

where \( \mu \) is a scale parameter. The model of section 3 corresponds to the case \( \mu_1 < 0 \) and \( \mu_2 = \mu_3 = 0 \).

To compute the path of \( X \), one must solve the model in two stage. First, we solve all the model \( \text{i.e.} \) find the saddle-path \( \text{for a constant number of firms} \) \( \text{i.e.} \) without equation (35), where \( X \) is given by the following recursive equation:

\[ X_t = \Pi_t + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} X_{t+1} \right] \]

It is assumed that firms and the representative household have access to a complete set of frictionless securities markets, and \( \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\} \) is a pricing kernel for contingent claims. In this first stage, \( X \) and \( \lambda \) are forward variables and \( k \) is backward. Once solved, this model with constant number of firms gives \( X_t \) as an function of the states \( (k_t, A_t, G_t) \).

We then plug that expression of \( X \) in a model with equation (35) to compute the effective dynamical behavior of the economy. This allows us for impulse response function analysis.

### 4.3. Responses to Technological and Government Expenditures Shocks

We calibrate the model such that steady-state level of markup and number of firms are the same as in the pure Cournot competition model. Nevertheless, to magnify the effect of future profits, we allow in this economy for a 2% share of profits in value-added.

We study four different models \( \text{19, models I, II, III and IV, which correspond} \) to the values of elasticities \( \mu_i \) and \( \nu_i \) (equations (35) and (37)) given in Table 6.

<table>
<thead>
<tr>
<th>Model</th>
<th>( \nu_1 )</th>
<th>( \nu_2 )</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>III</td>
<td>.9</td>
<td>.1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IV</td>
<td>.5</td>
<td>-.5</td>
<td>-1</td>
<td>-.4</td>
<td>.4</td>
</tr>
</tbody>
</table>

19. A first best strategy would have been to estimate these elasticities. This has not be done for data availability reasons.
Model I is a model with imperfect competition, constant number of firms and constant markups. Model II is close to the model of section 3, where markups depend only and negatively on the number of firms, and where the number of firms depends only and positively on contemporaneous output, which is positively correlated with contemporaneous profits. Model III allows for a positive impact of future profits on entry decisions. Model IV corresponds to the Rotemberg and Woodford [1992b] model of implicit collusion plus entry/exit. For a given number of firms, markups depend positively on $\frac{X}{Y}$, with elasticity .4: “an increase in $\frac{X}{Y}$ raises the size of the punishment (the foregone profits represented by $X$) relative to the size of the current sales (as represented by $Y$). It thus allows the firms in each oligopolistic industry to charge higher markups without fearing deviations” Rotemberg and Woodford [1992b], p. 40)

We also allow in this model markups to depend negatively on the number of firms. The entry/exit decision is assumed to depend negatively on the ratio $\frac{X}{Y}$, with elasticity -.5. When future profits of incumbents if they stay alone ($X$) increases, the incitation to deter entry and to push out of the market marginal firms is higher, and the number of firms decreases. With that calibration (other calibration may be considered), this effect dominates the negative effect of $\frac{X}{Y}$ on markups.

Models II, and III lead to counter-cyclical markups when a technological shock occurs (Figure 13). Because of the crowding-out effect of government expenditures, these three models lead to pro-cyclical markups after 5 periods when such a demand shock occurs. The case of model IV is more interesting. When a positive technological shock occurs, markups are pro-cyclical: the perspective of high future profits leads to an aggressive behavior of incumbents and some firms are thrown out of the market. On the contrary, a government expenditures shock, because of the potential crowding-out effect, reduces future profits, leads to entry and reduces markups.

Figure 14 shows that imperfect competition and counter-cyclical markups (models II and III) magnifies the effect of a technological shock relative to a constant markups model (model I). In model IV, as future profits $X$ increases relatively to $Y$, markups are pro-cyclical and the response of output is lower.

In models I, II and III, private expenditures crowding-out leads to a negative deviation of output from its steady-state after ten quarters. In model IV, entries counter-balance this crowding out effect and lead to a more realistic and always positive deviation of output from its steady state level (Figure 15). Business formation is pro-cyclical following a government shock in model IV, but is counter-cyclical when a positive technological shock occurs (which seems counter-factual). Such a model can therefore display a null correlation between markups and the number of firms, for suitable variances and persistence of the two shocks.

Consequently, as the labor demand shift is higher in model IV, real wage is less counter-cyclical (Figure 16). Nevertheless, one cannot get a positive response of wage without assuming a completely inelastic labor supply.
**Figure 13**

Response of Markups to Technological and Government Expenditures Shocks.

**Figure 14**

Response of Output to a Technological Shock.
Figure 15
Response of Output to a Government Expenditures Shock.

Figure 16
Response of Real Wage to a Government Expenditures Shock.
5 Conclusion

We show in this paper that correlations between activity, markups and business formation can be observed in the French business cycle, and that models with entry/exit in the intensive margin —i.e. inside industries — can account for such stylized facts. Our first model is a simple Cournot competition plus instantaneous free-entry condition, while a second reduced-form one deals with forward looking behavior in entry/exit and markups decisions. These developments suggest two necessary extensions. The first one is, keeping the reduced-form model, to estimate the full model using Generalized Method of Moments, in order to shed a light on the signs of the $\mu_t$ and $\nu_t$ coefficients. This has not yet been done because we lack of large sample series of firms number for the French economy. A direct extension will be there to estimate the model on US data. The second extension we plan is to give some micro-foundations to entry/exit decisions in an explicit game-theoretic setting. These two extensions are work in progress.

References


